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ABSTRACT

This teacher's guide and student guide are designed to accompany a mathematics textbook that contains supplemental readings, activities, and methods adapted for secondary students who have disabilities and other students with diverse learning needs. The materials are designed to help these students succeed in regular education content courses and include simplified text and smaller units of study. The curriculum correlates to Florida's Sunshine State Standards and is divided into the following 5 units of study: (1) number sense, concepts, and operations; (2) measurement and scale drawings; (3) geometry; (4) creating and interpreting patterns and relationships, including creating and interpreting tables, identifying and plotting pairs, using concepts about numbers, and using and writing algebraic expressions; and (5) probability and statistics, including experimental and theoretical probability, mean, mode, median, and range, graphing data, collecting, organizing and displaying data, circle graphs, stem-and-leaf plotting, and solving problems involving statistics and probability. For each unit, the teachers guide includes a general description of the unit's content and the unit's focus, provides suggestions for enrichment, and contains an assessment to measure student performance. Appendices in the teacher's guide describe instructional strategies, list enrichment suggestions, contain suggestions for specific strategies to facilitate inclusion, and contain a

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chart describing standards and benchmarks. The student guide contains vocabulary lists, explanation of content, and practice exercises designed to evaluate comprehension. (Contains 11 references.) (CR)

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Mathematics 2.
Teacher's Guide [and Student Guide].
Parallel Alternative Strategies for Students (PASS).

Linda Walker and Sue Fresen

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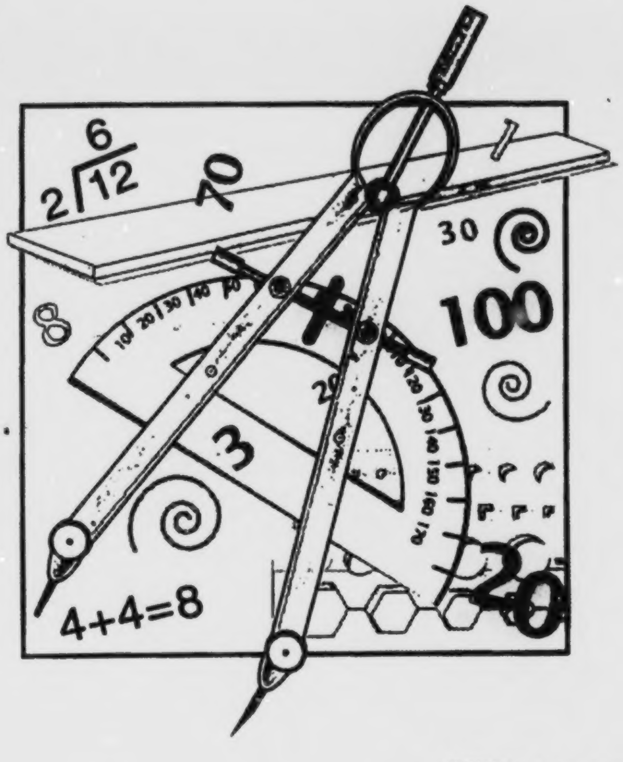
Mathematics 2

Course No. 1205040



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Bureau of Instructional Support and Community Services
Division of Public Schools and Community Education
Florida Department of Education
2001

2A

Parallel
Alternative
Strategies for
Students

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Parallel Alternative Strategies for Students

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PASS Volume Title: ______________________________ Date: ______________

Your Name: ______________________________ Your Position: ______________

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____

Directions: *We are asking for your assistance in clarifying the benefits of using the PASS book as a supplementary text. After using the PASS book with your students, please respond to all the statements in the space provided; use additional sheets if needed. Check the appropriate response using the scale below. Then, remove this page, fold so the address is facing out, attach postage, and mail. Thank you for your assistance in this evaluation.*

## Content	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1. The content provides appropriate modifications, accommodations, and/or alternate learning strategies for students with special needs.	☐	☐	☐	☐	☐
2. The content is at an appropriate readability level.	☐	☐	☐	☐	☐
3. The content is up-to-date.	☐	☐	☐	☐	☐
4. The content is accurate.	☐	☐	☐	☐	☐
5. The content avoids ethnic and gender bias.	☐	☐	☐	☐	☐
Presentation					
6. The writing style enhances learning.	☐	☐	☐	☐	☐
7. The text format and graphic design enhance learning.	☐	☐	☐	☐	☐
8. The practice/application activities are worded to encourage expected response.	☐	☐	☐	☐	☐
9. Key words are defined.	☐	☐	☐	☐	☐
10. Information is clearly displayed on charts/graphs.	☐	☐	☐	☐	☐
Student Benefits					
11. The content increases comprehension of course content.	☐	☐	☐	☐	☐
12. The content improves daily grades and/or tests scores.	☐	☐	☐	☐	☐
13. The content increases mastery of the standards in the course.	☐	☐	☐	☐	☐

Usage

The simplified texts of *PASS* are designed to be used as an additional resource to the state-adopted text(s). Please check the ways you have used the *PASS* books. Feel free to add to the list:

☐ additional resource for the basic text
☐ pre-teaching tool (advance organizer)
☐ post-teaching tool (review)
☐ alternative homework assignment
☐ alternative to a book report
☐ extra credit
☐ make-up work
☐ outside assignment
☐ individual contract
☐ self-help modules
☐ independent activity for drill and practice
☐ general resource material for small or large groups
☐ assessment of student learning
☐ other uses: ______________________

Overall

Strengths:

Limitations:

Other comments:

***Directions:** Check each box that is applicable.*

I have daily access at school to:	☐ A computer	☐ A printer	☐ The Internet	☐ A CD-ROM drive
All of my students have daily access at school to:	☐ A computer	☐ A printer	☐ The Internet	☐ A CD-ROM drive
I would find it useful to have PASS on:	☐ The Internet	☐ CD-ROM	☐ Mac	☐ PC/IBM

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Mathematics 2

Teacher's Guide

Course No. 1205040

Bureau of Instructional Support and Community Services
Division of Public Schools and Community Education
Florida Department of Education

2001

This product was developed by Leon County Schools, Exceptional Student Education Department, through the Curriculum Improvement Project, a special project, funded by the State of Florida, Department of Education, Division of Public Schools and Community Education, Bureau of Instructional Support and Community Services, through federal assistance under the Individuals with Disabilities Education Act (IDEA), Part B.

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Mathematics 2

Teacher's Guide

Course No. 1205040

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Foreword

Parallel Alternative Strategies for Students (PASS) books are content-centered packages of supplemental readings, activities, and methods that have been adapted for students who have disabilities and other students with diverse learning needs. *PASS* materials are used by regular education teachers and exceptional education teachers to help these students succeed in regular education content courses. They have also been used effectively in alternative settings such as juvenile justice educational programs and second chance schools, and in dropout prevention and other special programs that include students with diverse learning needs.

The content in *PASS* differs from standard textbooks and workbooks in several ways: simplified text; smaller units of study; reduced vocabulary level; increased frequency of drill and practice; concise directions; less cluttered format; and presentation of skills in small, sequential steps.

PASS materials are not intended to provide a comprehensive presentation of any course. They are designed to *supplement* state-adopted textbooks and other instructional materials. *PASS* may be used in a variety of ways to augment the curriculum for students with disabilities and other students with diverse learning needs who require additional support or accommodations in textbooks and curriculum. Some ways to incorporate this text into the existing program are as

- a resource to supplement the basic text
- a pre-teaching tool (advance organizer)
- a post-teaching tool (review)
- an alternative homework assignment
- an alternative to a book report
- extra credit work
- make-up work
- an outside assignment
- part of an individual contract
- self-help modules
- an independent activity for drill and practice
- general resource material for small or large groups
- an assessment of student learning

The initial work on *PASS* materials was done in Florida through Project IMPRESS, an Education of the Handicapped Act (EHA), Part B, project funded to Leon County Schools from 1981–1984. Four sets of modified content materials called *Parallel Alternate Curriculum* (*PAC*) were disseminated as parts two through five of *A Resource Manual for the Development and Evaluation of Special Programs for Exceptional Students, Volume V-F: An Interactive Model Program for Exceptional Secondary Students.* Project IMPRESS patterned the *PACs* after curriculum materials developed at the Child Service Demonstration Center at Arizona State University in cooperation with Mesa, Arizona, Public Schools.

A series of 19 *PASS* volumes was developed by teams of regular and special educators from Florida school districts who volunteered to participate in the EHA, Part B, Special Project, Improvement of Secondary Curriculum for Exceptional Students (later called the Curriculum Improvement Project). This project was funded by the Florida Department of Education, Bureau of Education for Exceptional Students, to Leon County Schools during the 1984 through 1988 school years. Regular education subject area teachers and exceptional education teachers worked cooperatively to write, pilot, review, and validate the curriculum packages developed for the selected courses.

Beginning in 1989 the Curriculum Improvement Project contracted with Evaluation Systems Design, Inc., to design a revision process for the 19 *PASS* volumes. First, a statewide survey was disseminated to teachers and administrators in the 67 school districts to assess the use of and satisfaction with the *PASS* volumes. Teams of experts in instructional design and teachers in the content area and in exceptional education then carefully reviewed and revised each *PASS* volume according to the instructional design principles recommended in the recent research literature. Subsequent revisions have been made to bring the *PASS* materials into alignment with the Sunshine State Standards.

The *PASS* volumes provide some of the text accommodations necessary for students with diverse learning needs to have successful classroom experiences and to achieve mastery of the Sunshine State Standards. To increase student learning, these materials may be used in conjunction with additional resources that offer visual and auditory stimuli, including computer software, videotapes, audiotapes, and laser videodiscs.

User's Guide

The *Mathematics 2 PASS* and accompanying *Teacher's Guide* are supplementary resources for teachers who are teaching mathematics to secondary students with disabilities and other students with diverse learning needs. The content of the *Mathematics 2 PASS* book is based on the *Florida Curriculum Frameworks* and correlates to the Sunshine State Standards.

The Sunshine State Standards are made up of *strands, standards,* and *benchmarks.* A *strand* is the most general type of information and represents a category of knowledge. A *standard* is a description of general expectations regarding knowledge and skill development. A *benchmark* is the most specific level of information and is a statement of expectations about student knowledge and skills. Sunshine State Standards correlation information for *Mathematics 2,* course number 1205040, is given in a matrix in Appendix D.

The *Mathematics 2 PASS* is divided into five units of study that correspond to the mathematics strands. The student book focuses on readings and activities that help students meet benchmark requirements as identified in the course description. It is suggested that expectations for student performance be shared with the students before instruction begins.

Each unit in the *Teacher's Guide* includes the following components:

- **Unit Focus:** Each unit begins with this general description of the unit's content and describes the unit's focus. This general description also appears in the student book. The Unit Focus may be used with various advance organizers (e.g, surveying routines, previewing routines, paraphrasing objectives, posing questions to answer, developing graphic organizers such as in Appendix A, sequencing reviews) to encourage and support learner commitment.

- **Suggestions for Enrichment:** Each unit contains activities that may be used to encourage, to interest, and to motivate students by relating concepts to real-world experiences and prior knowledge.

- **Unit Assessments:** Each unit contains an assessment with which to measure student performance.

- **Keys:** Each unit contains an answer key for each practice in the student book and for the unit assessments in the *Teacher's Guide*.

The appendices contain the following components:

- **Appendix A** describes instructional strategies adapted from the Florida Curriculum Frameworks for meeting the needs of students with disabilities and other students with diverse learning needs.
- **Appendix B** lists teaching suggestions for helping students achieve mastery of the Sunshine State Standards and Benchmarks.
- **Appendix C** contains suggestions for specific strategies to facilitate inclusion of students with disabilities and other students with diverse learning needs. These strategies may be tailored to meet the individual needs of students.
- **Appendix D** contains a chart that correlates relevant benchmarks from the Sunshine State Standards with the course requirements for *Mathematics 2*. These course requirements describe the knowledge and skills the students will have once the course has been successfully completed. The chart may be used in a plan book to record dates as the benchmarks are addressed.
- **Appendix E** contains the glossary from the *Florida Curriculum Framework: Mathematics*
- **Appendix F** contains two sheets of graph paper that can be duplicated as needed.
- **Appendix E** lists reference materials and software used to produce *Mathematics 2*.

Mathematics 2 is designed to correlate classroom practices with the Florida Curriculum Frameworks. No one text can adequately meet all the needs of all students—this *PASS* is no exception. *PASS* is designed for use with other instructional materials and strategies to aid comprehension, provide reinforcement, and assist students in attaining the subject area benchmarks and standards.

Unit 1: Number Sense, Concepts, and Operations

This unit emphasizes how numbers and number operations are used in various ways to solve problems.

Unit Focus

Number Sense, Concepts, and Operations

- Associate verbal names, written words, and standard numerals with whole numbers, integers, and decimals; numbers with exponents; and numbers in scientific notation. (A.1.3.1)
- Understand relative size of integers, fractions, and decimals. (A.1.3.2)
- Understand concrete and symbolic representations of rational numbers in real-world situations. (A.1.3.3)
- Understand that numbers can be represented in a variety of equivalent forms, including fractions, decimals, and percents. (A.1.3.4)
- Understand and use exponential and scientific notation. (A.2.3.1)
- Understand the structure of number systems other than the decimal system. (A.2.3.2)
- Understand and explain the effects of addition, subtraction, multiplication, and division on whole numbers, fractions, mixed numbers, and decimals, including the inverse relationship of positive and negative numbers. (A.3.3.1)
- Select the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of rational numbers and percent, including the appropriate application of the algebraic order of operations. (A.3.3.2)

- Add, subtract, multiply, and divide whole numbers, decimals, and fractions to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to predict results and to check the reasonableness of results. (A.4.3.1)
- Use concepts about numbers, including primes, factors, and multiples. (A.5.3.1)

Algebraic Thinking

- Describe a wide variety of patterns and relationships. (D.1.3.1)

Data Analysis and Probability

- Understand and apply the concepts of range and central tendency (mean, median, and mode). (E.1.3.2)

Lesson Purpose

Lesson One Purpose

- Understand the relative size of fractions and decimals. (A.1.3.2)
- Understand concrete and symbolic representations of rational numbers in real-world situations. (A.1.3.3)
- Understand that numbers can be represented in a variety of equivalent forms, including fractions and decimals. (A.1.3.4)
- Understand and explain the effects of addition, subtraction, multiplication, and division on whole numbers, fractions, mixed numbers, and decimals, including the inverse relationship of positive and negative numbers. (A.3.3.1)

- Add, subtract, multiply, and divide decimals and fractions to solve problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)

Lesson Two Purpose

- Understand and use exponential and scientific notation. (A.2.3.1)
- Use concepts about numbers, including primes, factors and multiples. (A.5.3.1)
- Understand and explain the effects of multiplication on whole numbers and decimals, including inverse relationships. (A.3.3.1)
- Associate verbal names, written word names, and standard numerals with integers, fractions, and decimals; numbers expressed as percents; numbers with exponents; numbers in scientific notation; and absolute value. (A.1.3.1)
- Understand that numbers can be represented in a variety of equivalent forms, including integers, fractions, decimals, percents, scientific notation, exponents, and absolute value. (A.1.3.4)

Lesson Three Purpose

- Understand the structure of number systems other than the decimal system. (A.2.3.2)
- Use concepts about prime numbers. (A.5.3.1)
- Select the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of rational numbers and percents. (A.3.3.2)
- Add, subtract, multiply, and divide whole numbers and decimals to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)

- Use estimation strategies to predict results and to check the reasonableness of results. (A.4.3.1)
- Associate verbal names, written word names, and standard numerals with whole numbers and decimals. (A.1.3.1)
- Understand the relative size of integers and decimals. (A.1.3.2)
- Describe a wide variety of patterns and relationships. (D.1.3.1)
- Understand that numbers can be represented in a variety of equivalent forms, including fractions, decimals and percents. (A.1.3.4)
- Understand and apply the concepts of range and central tendency (mean, median, and mode). (E.1.3.2)

Lesson Four Purpose

- Understand and use exponential and scientific notation. (A.2.3.1)
- Understand the structure of number systems other than the decimal system. (A.2.3.2)
- Use concepts about numbers, including primes, factors, and multiples. (A.5.3.1)
- Associate verbal names, written word names and standard numerals with integers; numbers with exponents; and numbers in scientific notation. (A.1.3.1)
- Understand that numbers can be represented in a variety of equivalent forms. (A.1.3.4)
- Select the appropriate operation to solve problems, including the appropriate application of the algebraic order of operations. (A.3.3.2)

Lesson Five Purpose

- Select the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of numbers, ratios, proportions and percent. (A.3.3.2)
- Add, subtract, multiply, and divide numbers to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to check the reasonableness of results. (A.4.3.1)

Suggestions for Enrichment

1. Have students cut strips of paper $8\frac{1}{2}$" long and 1" wide. Then have students make a set of fraction strips for each of the following, using number sense and folding.

    a. $\frac{1}{2}$
    b. $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$
    c. $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}$
    d. $\frac{1}{3}, \frac{2}{3}$
    e. $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}$
    f. $\frac{1}{9}, \frac{2}{9}, \frac{3}{9}, \frac{4}{9}, \frac{5}{9}, \frac{6}{9}, \frac{7}{9}, \frac{8}{9}$

2. Make a deck of 42 cards for playing an equivalent fraction game. Each card must contain one fraction. Construct two cards each of the following: $\frac{1}{2}, \frac{1}{4}, \frac{2}{3}, \frac{3}{2}, \frac{3}{5}, \frac{5}{4}$ and $\frac{1}{5}$. Then construct one card each of the following:

    $\frac{6}{12}, \frac{3}{6}, \frac{4}{8}, \frac{8}{16}, \frac{4}{16}, \frac{3}{12}, \frac{2}{8}, \frac{6}{24}, \frac{8}{12}, \frac{10}{15}, \frac{6}{9}, \frac{4}{6}, \frac{18}{12}, \frac{9}{6}, \frac{12}{8}, \frac{24}{16}, \frac{6}{10}, \frac{12}{20}, \frac{9}{15}, \frac{15}{25}, \frac{10}{8}, \frac{15}{12}, \frac{20}{16},$

    $\frac{30}{24}, \frac{3}{15}, \frac{4}{20}, \frac{2}{10},$ and $\frac{5}{25}$.

Have groups of four students play to get two "books," with each book containing three cards: one with the fraction reduced to lowest terms and two cards with fractions equivalent to the first card. Each student is dealt six cards. Students take turns drawing and discarding cards until one student gets two books. When a student gets a book, those cards are placed face up on the table. The winner is the student who first collects two books. (Optional: Make other sets of cards to play using equivalent decimals and percentages.)

3. Have students find examples in the real world of fractions, decimals, percents, numbers expressed in words, numbers expressed in scientific notation, and numbers expressed as comparisons of relative size of numbers.

   Ask students to make a display of their examples to share with others.

4. Ask students to gather data on the number of students in their school in categories such as grade, race, and gender. Have students determine the percentages and display the data to share with others.

5. Have students pick five whole numbers. Multiply each by 0.07, and 0.47, and 0.87. Ask students to write a sentence each time describing the relationship between the chosen number and the product.
   Example: When my whole number was multiplied by 0.07 the product was ____________ .
   When my whole number was multiplied by 0.47 the product was ____________ .
   When my whole number was multiplied by 0.87 the product was ____________ .

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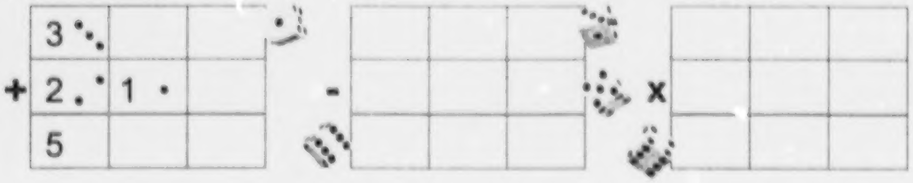
6. Decide which computation skill to practice: addition, subtraction, or multiplication. Ask students to draw 3 x 3 grids on their papers as you draw one on the board.

   Example:



   Roll a die and ask students to write the number rolled into any one of the top two rows. (The bottom row is for the answer.) Once a number is written in one of the top two rows of squares, it cannot be moved. Continue to roll the die until each of the top two rows are filled. Have students work the problem created on their grid. The object of the game is to get the highest number if adding or multiplying or the lowest number if subtracting.

   You are also playing on the board, and the students are trying to beat your answers. Survey the class and write best answer on the board, awarding a point to anyone with that answer. (Grids can be adapted to students' levels.)

7. Roll four dice and post the numbers on the board. Ask students to find as many ways as possible to obtain 24 using the given numbers using any or all of the four operations.

8. Have students list in 10 minutes the different ways to produce a result of 100 using addition, subtraction, multiplication and/or division.

9. Write on the board, "The answer is 16. [Or any number you choose.] What is the question?" Set a time limit and ask students to write as many questions as possible to fit that answer.

10. Have groups of students play *Monopoly* with no cash, making journal entries instead to keep track of cash flow.

11. Have groups number off as person one, two, three, and four. Roll two dice and have all the number one people do the same computation as a journal entry; continue with the other numbered players. After playing a few rounds, have all the same-numbered people get together to post and compare their calculations.

12. Have students estimate the fraction of the day they spend eating, sleeping, talking on the phone, reading, watching television, showering or bathing, using the computer, at work, or at school. Would these fractions add up to one whole? Why or why not? About how many hours does each fraction represent? How many hours a year do you spend sleeping or on the phone? If you needed more time for studying, what activities could most easily be adjusted? (Optional: Have students construct a circle graph of their day's activities.)

13. Construct Bingo grids that include five rows and five columns with one middle free space. Design each Bingo card with a random selection of fractions, decimals, and/or percents to reinforce equivalents (or any concept of choice). Create teacher-held flash cards that are designed to match equivalent answers on the students' Bingo cards. Keep track of cards revealed to aid in checking answers. Determine difficulty by regulating amount of student computation required to generate equivalents. Have students cover the quantity announced or its equivalent with a marker. First student with five in a row calls "Bingo" and upon substantiation, wins the round.

14. Have groups of students plan and build the highest freestanding structure at the lowest cost using teacher-predetermined materials and costs (e.g., 20 plastic drinking straws @ $1.00 each; 20 small paper cups @ $1.00; 10 small paper clips @ $.20 each; 1 roll of masking tape @ $.20 an inch; 1 yard/meter stick).

    After the structure is built, ask students to discuss and answer the following: What was the most difficult part of building the structure? What problems did you have to solve? What changes did you have to make after beginning construction? If you could do it again, what changes would you make?

15. Have students access online recipes and convert all ingredient amounts into tablespoons or into teaspoons.

16. Have students increase or decrease ingredients in recipes by a specific amount. (Optional: Have students convert measurements—such as cups to pints, pounds to ounces.) Recipes can be found on the Internet.

17. Have students convert recipes to feed the class. For example, if there are 20 students in a class and if the recipe feeds eight, the conversion ratio is a ratio of the new yield (how many people you want to feed) divided by the old yield (how many people the recipe was written for). In this case, the ratio would be $\frac{20}{8} = 2\frac{1}{2}$, and everything in the recipe would need to be increased $2\frac{1}{2}$ times. A conversion may require the rounding off of some amounts (e.g., it would not be possible to have $2\frac{1}{2}$ eggs).

18. Have students use the newspaper food sections and convert recipes in standard measurements to their metric equivalents.

19. Select a recipe and list prices and sizes of all the ingredients. Have students determine preparation cost of the recipes based only on the amounts used in the recipe.

20. Discuss the history of Roman numerals using charts, photos, or pictures.

21. Have students make individual charts showing each Roman numeral symbol and its equivalent in Arabic numbers.

22. Discuss addition and subtraction in the Roman numeral system. Have students practice using Roman numerals to add and subtract.

23. Give students 20-30 flat toothpicks to form Roman numerals for given Arabic numerals.

24. Have students write Roman numerals for family members' birth years, number of students in the school, or other large numbers of interest to them.

25. Have students bring in examples or pictures of Roman numerals (e.g., book preface, book chapter, watch, a clock, building erection date, statue or monument, outline topic numbers.)

26. Tell students to suppose they have $1,000 to spend only on things advertised in today's newspaper. Ask students to list the items they would purchase and add their total cost (including any sales tax), coming as close to $1,000 as possible.

27. Have students look at entertainment listings from announcements, advertisements, and the newspaper and plan an entire day of entertainment. Have students calculate the amount of time and money the day would cost.

28. Have students choose 10 interesting newspaper advertisements, decide if they advertise goods or services, and then classify the products advertised as needs or wants in today's society. Then ask students to choose 10 items from their advertisements that they would like to own and calculate their total cost, including any sales tax.

29. Have students find a newspaper story containing numerical data. Ask students to consider what the data helps to describe or support and why it is important. Then have students write a word problem with this data.

30. Have students analyze advertisements for the same product and decide which stores offer the best buys. (Optional: Tell each student to use the classified advertisements to find the best used car for $1,000. Have students compare choices and present their reasons as to why the car they chose was the best.)

31. Have the students imagine that they work 40 hours a week and earn \$2.00 per hour above minimum wage. Have students calculate their monthly income. Tell students they may spend between $\frac{1}{4}$ to $\frac{1}{3}$ of their monthly income for food. In groups have students brainstorm one week (7 days) of menus for three meals a day, then break down what food items they need to buy. Have students use the Internet to find the cost of each item, add the cost up for the week, and then multiply the total by four for the month.

32. Have students imagine they are high school graduates who have chosen to live together for awhile before going on to postsecondary school. Have pairs use the classified ads to complete the activity.

    - Look in the "help wanted" section for a job for a high school graduate and find a job and a monthly salary. What is the total monthly income for two roommates minus 25 percent for taxes?

    - Use the total final income as a guide to find an affordable apartment in the "apartments for rent" section. List the number of bedrooms and rent for one month.

    - Estimate the cost of water, electricity, and gas as 10 percent of the rent money. Plan to spend about \$25 a month for a telephone without long distance calls.

    - List the costs for one month.

    - If you've run out of money here, go back and start over.

    - How much furniture can you buy with the money left over after rent, utilities, and telephone costs are subtracted from the income figure? List the furniture, add their prices, and total the cost.

    - How much money do you have left over for food and entertainment?

33. Have groups use the Internet or a map to track a trip from Tallahassee, Florida, northwest across the United States to Olympia, Washington, stopping at each state capital along the way, and record miles traveled from capital to capital on a chart or table. The group to reach Olympia, Washington in the fewest miles traveled wins.

    Then have groups calculate the following: At 23 miles per gallon of gas at $1.37, how much would gas for their trip cost? If rate of speed averaged 60 miles per hour (mph) and nine hours a day were spent driving, how many days would it take to make their trip? How long would the trip take traveling seven hours a day at 60 mph? Traveling nine hours a day, how much would food cost per person if breakfast cost $4.50, lunch cost $5.25, and supper cost $7.35? How much would hotel expenses be for the whole trip at $39.00 per person a night? What was the total cost of gas, meals, and hotel expenses one-way? Round trip? (Optional: Have students use the Internet to compare prices for flying coach from Tallahassee, Florida to Olympia, Washington. Considering all costs, what would have been the least expensive method of travel?)

34. Have students use the Internet to find the 100 top national advertisers to calculate percent increases (or percent decreases) rounded to the nearest tenth of a percent for the top 25 companies in terms of advertising dollars spent during the year.

35. Have students use baseball statistics and compute the percentage of each teams' won-lost records; place teams in order based on won-lost records; arrange teams based on over .500 and under .500; compute total runs scored by teams; and determine pitchers' records and winning percentages.

36. Have students use the Internet to research population statistics pertaining to a country of their choice. Have students use this data to set up and solve ratios. (Optional: Have students calculate area comparisons between countries or make a travel brochure based on data collection for their country.)

37. Have groups create word problems using the classified section and commercial advertisements, news articles, and graphics from the newspaper.

38. Have students calculate payments on a $1,000 credit card bill with a minimum repayment term of 2.5 percent per month, 5 percent per month, and 10 percent per month. Ask students to obtain information about annual percentage rates (APR) and fees for various credit cards or provide them with the information.

Have students make three tables with the following information. The tables will include an opening balance, the interest charge for the month (APR divided by 12), payment for the month (first table with 2.5 percent, second table with 5 percent, third table with 10 percent), and an ending balance. Have students do this for 12 months and then determine the following for each table.

- How much debt was paid?
- How much was paid in total?
- How much was interest and principal?
- What was the proportion of interest and principal to total payments?
- Construct a pie chart for each table.
- Find the total time it would take to pay off each credit card at the given payment rates.
- Find the total amount of interest you paid to borrow $1,000 in each case.
- Does it take four times as long to pay off a credit card at 2.5 percent each month as it does at 10 percent each month?
- Discuss the pros and cons of credit cards.

39. Have students use the Internet to investigate certificate of deposit (CD) rates for $1,000 invested in a seven-year CD. Ask students to show their data in a table for the seven years and show total earnings including the $1,000. Then have students show earnings on a bar graph. (Optional: Calculate the difference if compound interest is paid monthly as opposed to annually.)

40. Have students discuss companies that manufacture popular products, such as running shoes or soft drinks. Ask students to choose a stock and follow its progress each day for a week (or longer). Have students calculate whether they would have earned or lost money if they sold their stock at the end of the week.

41. Have students use the Internet to research five foreign countries and their exchange rate in United States dollars. Ask students to create a graph that compares the value of the dollar in the five chosen foreign countries' currencies. Ask students to determine how much $1000 in United States currency would be worth today in each of their chosen countries.

42. Review concepts of the unit through a silent *Jeopardy* activity. Select 10 categories of topics, five for the first round and five for the second round. Have each student divide a piece of paper into the first and second round of *Jeopardy*. Assign point values of 1, 2, 3, 4, 5 for the first round and 2, 4, 6, 8, 10 for the second round. Randomly read questions from any topic and ask students to silently write the answers on the divided paper. After a set time, do a final *Jeopardy* question and allow students to wager for 0-10 points. Check papers and tally scores.

43. Have students design flowcharts of correct sequencing for working problems (e.g., fractions, measurement).

44. Have groups select a mathematical term (e.g., decimals, fractions, percents) and outline the basic categories involved with the term (e.g., basic categories for decimals: meaning and purpose of decimals, changing decimals to common fractions, changing common fractions to decimals, adding and subtracting decimals, division with decimal fractions, multiplying with decimal fractions and/or percentages). Ask each group to determine the basic operating principle, concepts, and content for each category listed. Have groups design a lesson in the form of a filmstrip on the chosen math concepts.

45. Have students interview a person who uses math in his or her occupation and find out specifically how he or she uses math in this occupation. Ask students to obtain an example of a typical math problem (worked out with a solution).

46. Have students research famous female mathematicians on the Internet (http://www.agnesscott.edu/lriddle/women/women.htm). Ask students to present information (e.g., a poem, an interview, a puppet show) on a selected female mathematician's life, struggles, and desire to study math.

47. Have students use the Internet to research a famous mathematician (*female mathematicians*: Maria Gaetana Agnesi, Mary Everest Boole, Ada Byron, Emille du Chatelet, Winifred Edgerton, Sophie Germain, Caroline Herschel, Grace Murray Hopper, Hypatia, Sophia Kovalevskaya, Florence Nightingale, Elena Lucrezia Cornaro Piscopia, Mary Fairfax Somerville, Theano; *male mathematicians*: Archimedes, Charles Babbage, Rene Descartes, Euclid, Euler Leonhard, Leonardo Fibonacci, Johan Carl Friedrich Gauss, Hippocrates, William Jones, Ernest Eduard Kummer, John Napier, Blaise Pascal, Plato, Pythagoras, Robert Recorde, James Joseph Sylvester, John Wallis, Johannes Widman).

Ask students to fill out a chart recording the following information.

- year of mathematician's birth and death
- country of birth or primary residence throughout life
- notable contributions to the fields of mathematics
- obstacles in early life that could have kept them from success
- obstacles that occurred in later private life
- prejudice faced (in regard to sex, religion, politics, or other matters)
- whether they worked on mathematics (or taught mathematics) for an income or for a hobby

Optional: Have students enter the data into a class data bank (i.e., Filemaker Pro) with the above categories. Give each group a copy of the completed database to analyze the results and come up with answers to the following.

- Find mathematicians who faced similar obstacles in early life. Did they use similar approaches to solve those problems? Why or why not?
- Do you think there were mathematicians on the list who would have liked to earn an income teaching or researching mathematics, but who could not do so? Why couldn't they?
- Find a mathematician whose early life is in some way similar to yours (or who faced prejudices similar to those you have faced) and describe the comparison.
- Look at the dates of the lives of the mathematicians and their accomplishments and explain whose work depended upon the work of someone who came before him or her or whose work was parallel to someone else's work.

Have each group choose three mathematicians who shared at least one common trait and write a report on the mathematicians' similarities, differences, and contributions, and an oral report which includes visual aids, worksheets or other handouts, and a visual display of at least one mathematician's life and area of study.)

48. Read *Math Curse* by John Sciesczka to the class and discuss the real and unrealistic math problems used to create a humorous story. Show how the illustrations enrich and expand the story. Allow groups to generate ideas about math problems and then review math concepts recently covered in class by creating their own illustrated books.

49. Have students select content-related activities and write the processes used to complete each activity. Have students scan the Sunshine State Standards and identify all standards that apply to the student behavior demonstrated in completing the selected activities. Ask students to then revise their written explanations to describe how each activity developed or reinforced each identified standard. Collect the students' work samples and the written reflections to form a student portfolio.

50. When giving a quiz, consider announcing a special number, which is the sum of the answers to all the problems.

51. See Appendices A, B, and C for other instructional strategies, teaching suggestions, and accommodations/modifications.

Unit Assessment

Match each **number** *with the correct* **name or equivalent form**. *Write the letter on the line provided.*

______ 1. 4.9 million

______ 2. four billion two hundred ninety-eight thousand

______ 3. 2×3^2

______ 4. $|-16|$

______ 5. $\frac{7}{100}$

______ 6. 4.298×10^6

______ 7. 0.24

______ 8. $2^3 \times 3$

______ 9. $4 \times 10^3 + 2 \times 10^2 + 9 \times 10 + 8 \times 1$

______ 10. 2^5

A. 4,298,000

B. XXIV

C. 4,900,000

D. 32

E. 18

F. 4,000,298,000

G. 16

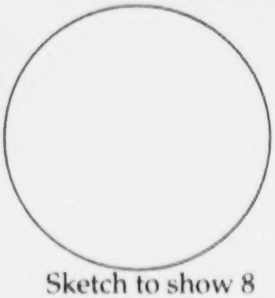
H. 4,298

I. 24 percent

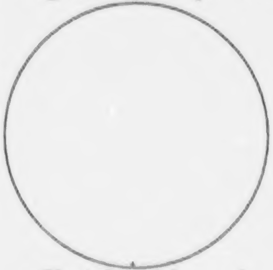
J. 0.07

Answer the following.

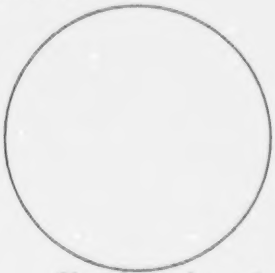
11. The three circles represent three pizzas. The first is to be cut into 8 equal parts considered small slices, the second into 4 equal parts considered large slices, and the third into 6 equal parts considered medium slices. Show your work and explain how 3 small slices, 2 medium slices, and 1 large slice compare to each other.



Sketch to show 8
equal slices
(small)



Sketch to show 4
equal slices
(large)



Sketch to show 6
equal slices
(medium)

12. One large slice would be ____________ (fractional part) of a pizza.

13. One medium slice would be ____________ (fractional part) of a pizza.

14. One small slice would be ____________ (fractional part) of a pizza.

15. Three small slices would be ____________ (<, >, =) 2 medium slices.

16. Three small slices would be ____________ (<, >, =) 1 large slice.

17. Two medium slices would be ____________ (<, >, =) 1 large slice.

18. Two small slices would be ____________ (<, >, =) 1 large slice.

19. One medium slice would be ____________ (<, >, =) 2 small slices.

20. 5 small slices would be ____________ (<, >, =) 2 large slices.

21. Choose any three of the six problems to solve.

- For each problem chosen, show all work.
- For each problem chosen, either provide an estimated answer before you solve or explain why your answer is reasonable upon completion.

Problem One

In 1996 the United States used 93.36 quadrillion (93,360,000,000,000,000) British thermal units (BTUs) of energy and produced 72.58 quadrillion BTUs. Find the difference in usage and production and express your answer in scientific notation.

Problem Two

A resort hotel has 2,500 rooms, and 25 percent of the rooms have one bed while the others have two beds. If the hotel uses two sheets per bed, find the total number of sheets needed to make up all the beds.

Problem Three

One dollar in American money was equivalent to one and one-half dollars in Australian money when Monique visited there. If she purchased an item valued at $30 in Australian dollars, what was the value in American dollars?

Problem Four

Norman Rockwell, a well-known American artist whose sketches were used on covers of the *Saturday Evening Post* magazine for years, sketched with Ticonderogas No. 2 pencils. George Lucas used the same kind of pencils more recently when he drew Anakin Skywalker, the Jedi Knights, and Jar Jar Binks for a *Star Wars* movie. The pencil manufacturer reports it produced more than 90 million No. 2 pencils in 1998 having a retail price from $1.50 to $4.00 per dozen. If the mean (average) of this reported minimum and maximum is used to determine the total retail value of pencils manufactured, what would that amount be?

Problem Five

In 1999 there were approximately 150 IMAX theaters in 22 countries, and 28 of them had 3-D capacity. Find the percent having 3-D capacity rounded to the nearest whole number.

Problem Six

(You will not provide an estimate for this problem. You will use Clue #2 to check the number you believe to be the secret number.) Use these clues to determine the mystery number.

Clue 1: This two digit number is an even number.
Clue 2: It has twelve factors including 1 and itself.
Clue 3: The tens digit is a prime number, and the units digit is a prime number.
Clue 4: The tens digit is 1 more than 3 times the units digit.

Write **True** *if the statement is correct. Write* **False** *if the statement is* not *correct.*

________ 22. When adding $\frac{3}{4}$ and $\frac{1}{2}$, the sum is the same as when adding $\frac{1}{2}$ and $\frac{3}{4}$.

________ 23. When multiplying $\frac{3}{4}$ and $\frac{1}{2}$, the product is the same as when multiplying $\frac{1}{2}$ and $\frac{3}{4}$.

________ 24. When dividing $\frac{1}{2}$ by $\frac{3}{4}$ the quotient is the same as when dividing $\frac{3}{4}$ by $\frac{1}{2}$.

________ 25. The greatest possible sum when adding any two of the following fractions ($\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{3}{4}$,) is $\frac{8}{12}$.

________ 26. The product of 0.25 and 0.85 will be less than 1.

You may *not* use a calculator for this final part of this assessment.

Answer the following.

27. $2 + 4 \times 8 - 6 =$ ________________

28. $(13 + 22) + 1 - 6^2 =$ ________________

29. $5 \times 8 + 2 - 6 \times 4 + 3^3 =$ ________________

30. $6 - 4 + 8 - 2^3 =$ ________________

31. $(4 \times 9 + 6) - 2 - 4 =$ ________________

Keys

Lesson One

Practice (p. 11)

1. $\frac{1}{5}$
2. $\frac{1}{4}$
3. $\frac{1}{3}$
4. $\frac{1}{2}$
5. $\frac{2}{3}$
6. $\frac{3}{4}$

Practice (p. 12)

1. $\frac{30}{60}$
2. $\frac{20}{60}$
3. $\frac{15}{60}$
4. $\frac{45}{60}$
5. $\frac{12}{60}$
6. $\frac{40}{60}$
7. a. $\frac{1}{5} = \frac{12}{60}$
    b. $\frac{1}{4} = \frac{15}{60}$
    c. $\frac{1}{3} = \frac{20}{60}$
    d. $\frac{1}{2} = \frac{30}{60}$
    e. $\frac{2}{3} = \frac{40}{60}$
    f. $\frac{3}{4} = \frac{45}{60}$
8. Work should support answers in practice page 11.

Practice (pp. 13-14)

1. 0.50
2. 0.33
3. 0.25
4. 0.75
5. 0.20
6. 0.67
7. a. $\frac{1}{5} = 0.20$
    b. $\frac{1}{4} = 0.25$
    c. $\frac{1}{3} = 0.33$
    d. $\frac{1}{2} = 0.50$
    e. $\frac{2}{3} = 0.67$
    f. $\frac{3}{4} = 0.75$
8. Work should support answers in practice page 11.
9. Answers will vary.

Practice (p. 15)

1. decimal number
2. fraction
3. least common multiple (LCM)
4. denominator
5. numerator

Practice (pp. 16-17)

1. $\frac{17}{12}$ or $1\frac{5}{12}$; Answers will vary but should include adding largest two.
2. $\frac{9}{20}$; Answers will vary but should include adding smallest two.
3. does not exceed
4. $\frac{11}{20}$; Answers will vary but should include subtracting smallest from largest.
5. b

Practice (pp. 18-19)

1. $\frac{5}{6}$
2. $\frac{6}{8}$ or $\frac{3}{4}$
3. $\frac{7}{10}$
4. $\frac{7}{6}$ or $1\frac{1}{6}$
5. $\frac{5}{4}$ or $1\frac{1}{4}$
6. $\frac{7}{12}$
7. $\frac{8}{15}$
8. $\frac{3}{3}$ or 1

Keys

9. $\frac{13}{12}$ or $1\frac{1}{12}$
10. $\frac{9}{20}$
11. $\frac{11}{12}$
12. $\frac{4}{4}$ or 1
13. $\frac{13}{15}$
14. $\frac{19}{20}$
15. $1\frac{5}{12}$
16. greater than either addend
17. Answers will vary.

Practice (pp. 20-21)

1. $\frac{1}{6}$
2. $\frac{1}{8}$
3. $\frac{1}{10}$
4. $\frac{2}{6}$ or $\frac{1}{3}$
5. $\frac{3}{8}$
6. $\frac{1}{12}$
7. $\frac{1}{15}$
8. $\frac{2}{9}$
9. $\frac{3}{12}$ or $\frac{1}{4}$
10. $\frac{1}{20}$
11. $\frac{2}{12}$ or $\frac{1}{6}$
12. $\frac{3}{16}$
13. $\frac{2}{15}$
14. $\frac{3}{20}$
15. $\frac{1}{2}$
16. less than either of the factors; no
17. Answers will vary.

Practice (p. 22)

1. E
2. A
3. D
4. C
5. G
6. B
7. F
8. H

Practice (p. 23-24)

1. $\frac{1}{6}$
2. $\frac{1}{4}$
3. $\frac{3}{10}$
4. $\frac{1}{12}$
5. $\frac{2}{15}$
6. $\frac{1}{20}$
7. $\frac{1}{6}$
8. $\frac{1}{3}$
9. $\frac{5}{12}$
10. $\frac{7}{15}$
11. $\frac{1}{4}$
12. $\frac{5}{12}$
13. $\frac{2}{4}$ or $\frac{1}{2}$
14. $\frac{11}{20}$
15. $\frac{1}{12}$
16. Answers will vary.

41

Keys

Practice (pp. 25-28)

1. $\frac{3}{2}$ or $1\frac{1}{2}$
2. $\frac{4}{2}$ or 2
3. $\frac{5}{2}$ or $2\frac{1}{2}$
4. $\frac{3}{4}$
5. $\frac{4}{6}$ or $\frac{2}{3}$
6. $\frac{4}{3}$ or $1\frac{1}{3}$
7. $\frac{5}{3}$ or $1\frac{2}{3}$
8. $\frac{3}{6}$ or $\frac{1}{2}$
9. $\frac{4}{9}$
10. $\frac{5}{4}$ or $1\frac{1}{4}$
11. $\frac{3}{8}$
12. $\frac{4}{12}$ or $\frac{1}{3}$
13. $\frac{3}{10}$
14. $\frac{4}{15}$
15. $\frac{8}{9}$
16. $\frac{2}{3}$
17. $\frac{2}{4}$ or $\frac{1}{2}$
18. $\frac{2}{5}$
19. $\frac{4}{3}$ or $1\frac{1}{3}$
20. $\frac{6}{4}$ or $1\frac{2}{4}$ or $1\frac{1}{2}$
21. $\frac{3}{4}$
22. $\frac{3}{5}$
23. $\frac{6}{3}$ or 2
24. $\frac{9}{4}$ or $2\frac{1}{4}$
25. $\frac{4}{5}$
26. $\frac{8}{3}$ or $2\frac{2}{3}$
27. $\frac{12}{4}$ or 3
28. $\frac{15}{4}$ or $3\frac{3}{4}$
29. $\frac{10}{3}$ or $3\frac{1}{3}$
30. $1\frac{1}{8}$
31. b
32. The sums are the same. The order of the addends does not affect the sum.
33. The products are the same. The order of the factors does not affect the product.

Practice (p. 29)

1. 1.42
2. 0.45
3. smaller
4. 0.55
5. b

Practice (p. 30)

Answers will vary but should meet the following criteria: Sketches of three circles, three squares, and three octagons each divided into four equal parts.

Practice (p. 31)

1. addends
2. fraction
3. sum
4. decimal number
5. denominator
6. least common multiple (LCM)
7. numerator
8. positive numbers
9. proper fraction
10. improper fraction
11. mixed number
12. multiples

Practice (p. 32)

1. E
2. B
3. F
4. H
5. G
6. A
7. C
8. D

Keys

Lesson 2

Practice (p. 36)

1. 2^3 x 3
2. 3^4
3. 5^3
4. 2^3 x 5
5. 2 x 3 x 7
6. 8
7. 6
8. 120
9. 840

Practice (p. 37)

1. 81
2. 1,000
3. 512
4. 64
5. 64
6. Answers will vary.

Practice (p. 39)

1. $(4 \times 10^6) + (5 \times 10^5) + (9 \times 10^4) + (8 \times 10^3) + (6 \times 10^2) + (4 \times 10^1) + (4 \times 10^0)$
2. 2×10^3
3. $(5 \times 10^6) + (8 \times 10^5) + (6 \times 10^2) + (4 \times 10^1)$
4. $2 \times 10^{10} + (4 \times 10^9)$
5. 832,686
6. 4,010,205

Practice (p. 41)

1. $16,200,000; sixteen million, two-hundred thousand dollars
2. $4,500,000,000; four billion, five-hundred million dollars
3. $5.9 million
4. $8.3 billion

Practice (p. 43)

1. True
2. True
3. True
4. True

Practice (p. 45)

1. True
2. True
3. True
4. True
5. True
6. False; it would be 10 billion.

Practice (p. 49)

1. 860,000,000
2. 4,300,000
3. 3,700,000,000
4. 487,500
5. 61,544,000
6. .000041
7. .00000036
8. .000005567

Practice (p. 50)

1. 10^6
2. 10^{10}
3. 10^5
4. 4.348×10^9
5. 6.0×10^6
6. 8.75×10^{-1}
7. 9.5×10^{-3}
8. 8.63×10^{-9}

Practice (p. 52)

Percent	Decimal	Fraction
25%	0.25	$\frac{25}{100}$ or $\frac{1}{4}$
40%	0.40	$\frac{40}{100}$ or $\frac{2}{5}$
10%	0.10	$\frac{10}{100}$ or $\frac{1}{10}$
50%	0.50	$\frac{50}{100}$ or $\frac{1}{2}$
75%	0.75	$\frac{75}{100}$ or $\frac{3}{4}$
3%	0.03	$\frac{3}{100}$
20%	0.20	$\frac{20}{100}$ or $\frac{1}{5}$
33% or $33\frac{1}{3}$%	0.33	$\frac{33}{100}$ or $\frac{1}{3}$
90%	0.90	$\frac{90}{100}$ or $\frac{9}{10}$
15%	0.15	$\frac{15}{100}$ or $\frac{3}{20}$

Keys

Practice (p. 54)

1. 18
2. 2
3. 400
4. 900
5. 621

Practice (pp. 55-56)

1. percent
2. standard form
3. absolute value
4. prime factorization
5. scientific notation
6. expanded form
7. exponent (exponential form)
8. prime number
9. composite number
10. greatest common factor (GCF)
11. pattern
12. whole number
13. table

Lesson Three

Practice (pp. 59-61)

1. a. 400; $400
   b. 370; more; less
2. a. 480
   b. men
3. 470.16; Answers will vary.
4. men; $26.33; women

Practice (pp. 62-63)

1. 9,600,000
2. 25
3. 0.25
4. 10,000,000
5. 2,500,000
6. 2,400,000
7. 100,000; more

Practice (pp. 64-65)

row 4 = 20
row 5 = 22
row 6 = 24
row 7 = 26
row 8 = 28
row 9 = 30
row 10 = 32

1. 22; 24; 26; 28; 30; 32; 230

Practice (p. 67)

1. 46
2. 46
3. 46
4. 46
5. 46
6. 46; 230
7. 230

Practice (pp. 68-69)

1. 2,000; 16; 10,200; 28
2. 10,000; 30
3. 333, or accept numbers between 300 and 350
4. 364

Practice (pp. 70-71)

1. 2,004
2. Answers will vary.
3. Answers will vary.

Practice (p. 72)

1. the strength of an earthquake
2. 10
3. 10
4. 100
5. 1; 1; 2
6. 100

Keys

Practice (p. 73)

1. four billion, ninety-three million, seven hundred thirty-nine thousand, six hundred five
2. billion; 80 billion
3. 80 billion
4. 100 billion

Practice (pp. 74-75)

1. 2,000
2. 40,000
3. 20
4. 2,500; 40,000
5. 16
6. yes
7. Answers will vary.

Practice (p. 76)

1. 2, 3, 5, 7, 11, 13, 17, 19, 23
2. 3, 3, 19; 3, 5, 17; 5, 7, 13; 7, 7, 11

Practice (p. 77)

1. 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 = 1023
2. 100; $1000
3. aunt; uncle; $23

Lesson Four

Practice (p. 79)

1. 78; LXXVIII
2. 210

Practice (p. 80)

1. 1,895
2. 571

Practice (p. 81)

1. 6,471,000,000; 6.471 x 10^9; six billion, four hundred seventy-one million; 6.5 billion

Practice (p. 82)

Answers will vary.

Practice (pp. 83-84)

1. multiplication; addition
2. multiplication; subtraction
3. division; multiplication
4. subtraction; addition

Practice (p. 88)

1. 10
2. 9
3. 4
4. 1
5. 46
6. 41
7. 10
8. 32
9. 13
10. 3

Lesson Five

Practice (pp. 90-93)

1. 15%
2. $15; $30
3. $100; $200
4. 100
5. $20; $120
6. $7.50; less
7. $15; less
8. $22.50; more
9. $18.75
10. $18
11. 1.2; 1.2
12. 1.2; 1.2; 120
13. Answers will vary.
14. Answers will vary.

Keys

Practice (pp. 94-95)

1. 40
2. 40; 4; 2
3.

Number in Orchestra	Number Playing Strings	Number Not Playing Strings
100	60	40
105	63	42
110	66	44
115	69	46
120	72	48
125	75	50
130	78	52

4. Illustrations will vary.

Practice (pp. 96-97)

1. 1.3
2. 78
3. 78; 78
4. 1.3
5. 52
6. 52
7. 26
8. 52
9. 52
10. 52
11. Answers will vary.
12. Answers will vary.

Practice (p. 98)

1. $27

Practice (pp. 99-100)

1. 1,750
2. 250; 1,750
3. 1,250
4. 250; 1,250
5. 500
6. 500
7. 500
8. $500

Practice (pp. 101-102)

1. 11; 18
2. $\frac{11}{18}$
3. .61
4. 61%
5. 61
6. 5.55 or $5.\overline{5}$
7. 61.11 or $61.\overline{1}$
8. 61; 61%
9. 61%

Practice (pp. 103-105)

1. 4; 6
2. 3,750
3. 3,750; 37,500
4. Answers will vary.
5. Yes
6. 3,750; 37,500
7. 37,500
8. 37,500
9. 22,500

Practice (pp. 106-107)

1. 10; 12
2. Illustrations will vary.
3. 108
4. Answers will vary.

Practice (pp. 108-109)

1. $30,000,000
2. 30,000,000
3. 20,000,000
4. $20 million or $20,000,000

Practice (p. 110)

1. 138,960,000

Practice (p. 111)

1. L
2. I
3. G
4. F
5. B
6. E
7. K
8. C
9. J

Keys

10. A
11. D
12. H

Unit Assessment (pp. 19-23TG)

1. C
2. F
3. E
4. G
5. J
6. A
7. I
8. B
9. H
10. D
11.

$\frac{8}{8}$
$\frac{4}{4}$
$\frac{6}{6}$

12. $\frac{1}{4}$
13. $\frac{1}{6}$
14. $\frac{1}{8}$
15. >
16. >
17. >
18. =
19. <
20. >
21. Students must choose to answer three of the following six.

Problem One

93.36 quadrillion - 72.58 quadrillion = 20.78 quadrillion = 2.078×10^{16} or 93,360,000,000,000,000 - 72,580,000,000,000,000 = 20,780,000,000,000,000 = 2.078×10^{16}. An estimate will likely be based on 90 quadrillion minus 20 quadrillion and may precede work or be used to judge whether the answer is reasonable.

Problem Two

25% of 2,500 rooms is 625; 625 rooms x 1 bed with 2 sheets = 1,250 sheets; 2,500 - 625 = 1,875 rooms x 2 beds with 2 sheets each = 7,500 sheets; 1,250 sheets + 7,500 sheets = 8,750 sheets needed. An estimate might be based on 500 rooms with 1 bed and 2,000 rooms with 2 beds for a total of 9,000 sheets. Other possibilities exist. (Students may think of 25% as $\frac{1}{4}$. They may get rooms with two beds by using 75% of total. There are a number of ways to solve the problem.)

Problem Three

$\frac{1.00}{1.50} = \frac{?}{30}$; ? = \$20

Students may reason that our dollar is worth $1\frac{1}{2}$ of theirs or that theirs is worth $\frac{2}{3}$ of ours. Two thirds of \$30 is \$20. They may divide \$30 by 1.50 and get \$20. An estimate here is likely to be based on paying one-third less.

Problem Four

90 million = 90,000,000

To determine how many dozen: 90,000,000 divided by 12 = 7,500,000. To find average of minimum and maximum values: 1.50 + 4.00 = 5.50; 5.50 divided by 2 is 2.75; 7,500,000 dozen pencils @ 2.75 per dozen = \$20,625,000. An estimate is likely to be based on 9,000,000 dozen at \$2 or \$3 per dozen for an estimate of 18,000,000 to 27,000,000. Other possibilities exist.

Keys

Problem Five

28 theaters out of 150

$\frac{28}{150} = .18666$ or 18.6%, rounded is 19% or $\frac{28}{150} = \frac{?}{100}$; ? = 18.6

An estimate is likely to be based on a comparison of 30 with 150 which would be $\frac{1}{5}$ or 20%.

Problem Six

Since number is even, units digit must be 0, 2, 4, 6, or 8. Since units digit is a prime number, it must be 2, 3, 5, or 7. Two is the only number in both lists so the units digit is 2. The clue about 12 factors is one to revisit later. The tens digit is a prime number so it must be 2, 3, 5, or 7. The tens digit is 1 more than 3 times the units digit. The units digit is 2 and 1 more than 3 times 2 would be 7. The tens digit is 7. The number is 72. Let's check to see if it has 12 factors: 1 x 72; 2 x 36; 3 x 24; 4 x 18; 6 x 12; 8 x 9. Yes, it does. I know my number is the solution.

22. True
23. True
24. False
25. False
26. True
27. 28
28. 0
29. 45
30. 2
31. 36

Scoring Recommendations for Unit Assessment

- For the first matching activity of items 1 to 10, the recommendation is 2 points each.
- For the 3 sketches and 9 statements of items 11 to 20, the recommendation is 1 point each.
- For the problem solving section of item 21, problems one to six, students are asked to choose **three** problems to solve. The recommendation is 16 points each with 4 points for demonstration that the problem was understood, 4 points for choosing appropriate operation(s) to solve and setting them up correctly, 4 points for correct calculations and solution, and 4 points for the estimate given before solving or explanation for why solution is reasonable given after solving.
- For the five true/false statements of items 22 to 26, the recommendation is 2 points each.
- For the last five problems of items 27 to 31, the recommendation is 2 points each.

Item Numbers	Assigned Points	Total Points
1 to 10	2	20 points
11 to 20	1	12 points
21 (3 out of 6)	16	48 points
22 to 26	2	10 points
27 to 31	2	10 points
		Total = 100 points

Benchmark Correlations for Unit Assessment

Benchmark	Addressed in Items
A.1.3.1	1-15
A.1.3.2	14-21 (problem six), 25, 26
A.1.3.3	11-20
A.1.3.4	1-10, 21 (problems one to four)
A.2.3.1	1, 2, 5, 7, 8, 21 (problem one)
A.2.3.2	2
A.3.3.1	22-26
A.3.3.2	21 (all problems), 27-31
A.3.3.3	21, (all problems)
A.4.3.1	21, (all problems)
A.5.3.1	21, (all problems)

Unit 2: Measurement

This unit emphasizes how estimation and measuring are used to solve problems.

Unit Focus

Number Sense, Concepts, and Operations

- Add, subtract, multiply, and divide whole numbers, decimals, and fractions to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculate. (A.3.3.3)

Measurement

- Use concrete and graphic models to derive formulas for finding perimeter, area, surface area, circumference, and volume of two- and three-dimensional shapes, including rectangular solids and cylinders. (B.1.3.1)
- Construct, interpret, and use scale drawings such as those based on the number lines and maps to solve real-word problems. (B.1.3.4)
- Use direct (measured) and indirect (not measured) measures to compare a given characteristic in either metric or customary units. (B.2.3.1)
- Solve problems involving units of measure and convert answers to larger or smaller unit within either the metric or customary system. (B.2.3.2)
- Solve real-world and mathematical problems involving estimates of length, areas, and volume in either customary or metric system. (B.3.3.1)
- Select appropriate units of measurement. (B.4.3.1)
- Select and use appropriate instruments and techniques to measure quantities in order to achieve specified degrees of accuracy in a problem situation. (B.4.3.2)

Algebraic Thinking

- Describe a wide variety of patterns, relationships, and functions through models, such as manipulatives, tables, expressions, and equations. (D.1.3.1)

Lesson Purpose

Lesson One Purpose

- Use direct (measured) and indirect (not measured) measures to compare a given characteristic in either metric or customary units. (B.2.3.1)
- Solve problems involving units of measure and convert answers to a larger or smaller unit within either the metric or customary system. (B.2.3.2)
- Use concrete and graphic models to derive formulas for finding area and volume. (B.1.3.1)
- Solve real-world and mathematical problems involving estimates of measurements including length, area, and volume. (B.3.3.1)

Lesson Two Purpose

- Use concrete and graphic models to derive formulas for finding perimeter, area, surface area, circumference, and volume of two-and three-dimensional shapes, including rectangular solids and cylinders. (B.1.3.1)

Lesson Three Purpose

- Construct, interpret, and use scale drawings such as those based on number lines and maps to solve real-world problems. (B.1.3.4)
- Solve real-world and mathematical problems involving estimates of measurements including length, perimeter, and area in either customary or metric units. (B.3.3.1)

- Select and use appropriate instruments and techniques to measure quantities in order to achieve specified degrees of accuracy in a problem situation. (B.4.3.2)
- Select appropriate units of measurement. (B.4.3.1)
- Create and interpret tables, equations, and verbal descriptions to explain cause-and effect relationships. (D.1.3.2)

Lesson Four Purpose

- Add, subtract, multiply, and divide whole numbers, decimals, and fractions to solve real-world problems, using appropriate methods of computing such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use concrete and graphic models to derive formulas for finding perimeter, area, surface area, circumference, and volume of two-and three-dimensional shapes, including rectangular solids and cylinders. (B.1.3.1)
- Solve real-world and mathematical problems involving estimates of measurements including length, perimeter, and area in either customary or metric units. (B.3.3.1)
- Describe a wide variety of patterns, relationships, and functions through models, such as manipulatives, tables, expressions, and equations. (D.1.3.1.)

Suggestions for Enrichment

1. Have students play Bingo with math vocabulary words. Make a transparency master of a large square divided into 25 equal squares. Give each student a copy for a blank game board. Put the vocabulary terms on the chalkboard or transparency. Ask students to fill in the game board writing one term per square in any order. Play a Bingo game by calling out the definitions or asking questions for which the terms are answers. Ask students to put markers on the terms that are the correct answers. Answers can be verified and discussed after each definition or question. When a student gets five markers in a row, have the student shout out "Payday" or some mathematical reward term. Keep a record of the terms used and continue to play another round.

2. Have groups use a metric ruler to measure and record the length of each person's smile (or hair length) in their group. Record the measurements on the board. Have students order all the measurements from least to greatest and graph the results. Then have students find the sum of the length all the smiles and create one smile out of construction paper that is the length of all the smiles in the room.

3. Have students use the Internet and choose six cities (in logical order for visitation), one on each of the six major continents. Have students find distances for each segment of their journey and then convert the distance to a percentage of the total journey. They must call home at noon from each of the six cities and determine what the local time in each of their cities will be when they place the calls. They will have $30 to spend on souvenirs in each city and need to convert the amount to the local currency of the day. (Optional: Do the same activity and have the student come as close as possible to a specific total distance (such as 30,000 miles).

4. Have students work in pairs to measure the trunk, crown, and height of a tree using vertical and horizontal measurements and graph their findings. Trunk: Ask students to measure 4½ feet high on the trunk. At that height, use a string to measure the trunk's circumference and then measure the length of the string. Round to the nearest inch and record. Crown: Ask students to find the tree's five longest branches and mark the ground beneath the tip of the longest branch. Find the branch opposite the longest branch and mark its tip on the ground. Measure along the ground between the two markers. Record the number. Height: Ask students to have their partner stand at the base of the tree. Have the first student back away from the tree holding a 12" ruler straight out in front of him or her in a vertical position. Tell the student to stop when the tree and ruler appear to be the same size. (Closing one eye helps to line up the tree and ruler.) Next ask the student to turn one wrist until the ruler looks level to the ground and is in a horizontal position, keeping that arm straight. Keeping sight of the base of the ruler at the base of the tree, students will ask their partners to walk to the spot that they see as the top of the tree. Measure how many feet the partner walked to determine the tree's height. Round to the nearest foot. Have groups compare answers and remeasure as necessary. Ask students to make bar graphs with their information. Have students locate the biggest tree and smallest tree of the same species.

5. Have students use weights, measures, and distances they find on the sports page to make interesting comparisons (e.g., instead of just saying a family uses 5,000 gallons of water a month, compare that amount to filling an Olympic-size swimming pool or to a certain number of two-minute showers).

6. Have students use the sports section of the newspaper to list all the units of measurement they can find and indicate which ones are metric.

7. Divide class into groups, and give each group an inexpensive kickball and newspaper. Ask students to cut several strips of paper into 1" x 4" strips and cover the kickball, pasting the strips down and counting the number of strips used. By multiplying the number of strips by the area of each strip, students can approximate the surface area of a sphere.

8. Next have students find the radius of the kickball by rolling the ball on the floor and measuring the distance from the starting point to the ending point of one complete revolution. Discuss diameter and radius, and help students develop formulas for finding the surface area of spheres.

9. Have students use the Internet to find data in both kilometers and miles for two planets. Have students list the names of the two planets and the following information about each planet: diameter, minimum and maximum distance from the sun and from Earth, length of day and year, temperature, and number of satellites. Next have students answer the following.

   - What is the diameter of each planet in yards?
   - Which planet is bigger? How much bigger?
   - Which planet is closer to the sun? How much closer?
   - Which planet has a shorter year? How much shorter?
   - Which planet has a longer day? How much longer?
   - If the space shuttle must reach a speed of 25,000 mph in order to escape Earth's gravitational field

and if it were to maintain that speed, how long would it take to travel from Earth to each of your planets?

- List any other differences between your planets.

10. Give groups various sizes of rectangular boxes to measure the dimensions to the nearest millimeter. Have students create a table or chart with columns for length, width, height, total surface area, and volume to record the original box and a "mini-box" scale model they will create. Ask students to measure and record the length, width, and height in millimeters of the original box. Have students create a scale drawing on graph paper of the original box. Ask students to record the measurement for the mini-box in millimeters, then cut out, fold, and tape the scale drawing together. Then have students calculate the total surface area and volume for the two rectangular boxes. Ask students to find scale factors for length, surface area, and volume and write conclusions about the findings. Discuss the findings about scale factors for similar objects with regard to the length, area, and volume. Discuss careers in which scale drawings are used.

11. Choose a concept such as area, volume, or geometric figures and develop an information sheet for students. Then have students make up word problems on the concept chosen.

12. See Appendices A, B, and C for other instructional strategies, teaching suggestions, and accommodation/modifications.

Unit Assessment

Circle the letter of the correct answer.

1. The area in square inches of a square foot is __________ .
   a. 12 square inches
   b. 24 square inches
   c. 144 square inches

2. The volume in cubic inches of a cubic foot is __________ .
   a. 12 cubic inches
   b. 36 cubic inches
   c. 1,728 cubic inches

3. A reasonable estimate for the length of a typical new pencil is approximately __________ .
   a. 8 inches
   b. 8 centimeters
   c. 12 inches

4. A reasonable estimate for the diameter of a quarter is approximately
   __________ .
   a. 2 millimeters
   b. 2 centimeters
   c. 2 inches

5. The number of customary units or standard measuring cups of milk in one gallon is __________ .
   a. 4
   b. 16
   c. 24

6. Boxes of cereal usually have weight shown in customary units of ounces or in metric units of __________ .
   a. meters
   b. liters
   c. grams

7. The area of a state would likely be expressed in square miles while the area of an apartment would likely be expressed in square ______________ .

   a. centimeters
   b. inches
   c. feet

Answer the following.

8. Illustrate or explain in words why the formula for the area of a triangle is $\frac{1}{2}bh$.

____

____

____

____

____

____

____

____

____

____

____

____

____

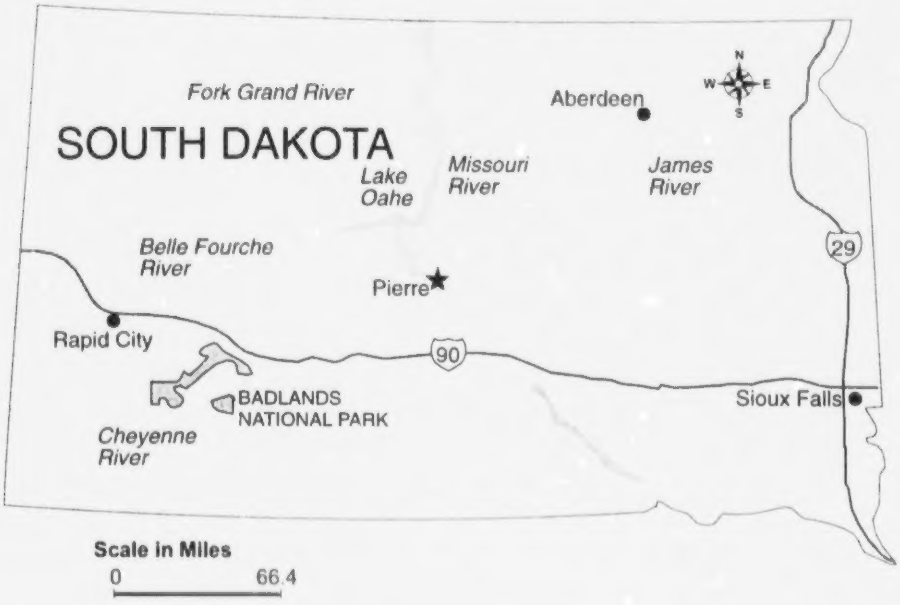
9. On the grid paper below, sketch three different rectangles having an area of 30 square units.

10. On the grid paper below, sketch two different rectangles having a perimeter of 12.

11. On the grid paper below, sketch a triangle having an area of 10 square units.

12. On the grid paper below, sketch a non-rectangular parallelogram having an area of 18 square units.

13. Use the map of South Dakota with the scale shown to find the *approximate* length in miles of its western border. ____________ miles Show your work.


Fork Grand River
Aberdeen
N
W
E
S
SOUTH DAKOTA
Lake Oahe
Missouri River
James River
29
Belle Fourche River
Pierre
Rapid City
90
Sioux Falls
BADLANDS NATIONAL PARK
Cheyenne River
Scale in Miles
0
66.4

14. In the space below, draw a rectangle with actual length of $3\frac{1}{2}$ inches and actual width of 2 inches.

15. In the space below, provide a scale drawing of the floor of a room that is 9 feet long and 12 feet wide. Be sure to show the scale used. Find the number of square yards of carpet needed to carpet the room and show our work.

Keys

Lesson One

Practice (p. 123)

1. Drawing of a line segment 6 inches long with marks at ½ inch, 1 inch, 1½ inches, up to 6 inches.
2. Drawing of a line segment 6 centimeters in length with marks as 1 centimeter, 2 centimeters, up to 6 centimeters.

Practice (pp. 124-125)

Answers will vary.

Practice (p. 126)

1. One square foot of paper and 1 square inch of paper cut from a large sheet of paper or from a paper bag.
2. 144
3. Answer will vary depending on size of desktop.
4. Answer will vary depending on size of classroom.
5. Answer will vary depending on size of palm of hand.
6. 9
7. Answer will vary depending on size of classroom. Students need to remember that 1 square yard = 9 square feet.

Practice (pp. 127-128)

1. Paper model for 1 cubic inch.
2. 1,728
3. Answer will vary depending on size of classroom.
4. Answer will vary depending on size of refrigerator or freezer.

Practice (p. 129)

1. Answer will vary depending on size of glass.
2. 8
3. 16
4. 1
5. $\frac{1}{2}$

Practice (p. 130)

1. Answers will vary. (e.g., Wheaties 18 ounces or 510 g.)
2. Answers will vary. (e.g., flour 5 pounds or 2.26 kg.)

Practice (pp. 131-132)

1. metric units
2. customary units
3. line segment
4. length
5. area
6. diameter
7. square units
8. volume
9. cubic units
10. capacity
11. weight
12. estimation
13. formula
14. cube
15. table

Lesson Two

Practice (pp. 134-135)

1. Tiles arranged as directed.
2. Sketches on grid paper of rectangles 1 x 24, 2 x 12, 3 x 8, and 4 x 6.
3.

Width of Rectangle in Units	Length of Rectangle in Units	Area of Rectangle in Square Units	Perimeter of Rectangle in Units
1	24	24	50
2	12	24	28
3	8	24	22
4	6	24	20

4. The area is the product of the dimensions.

Keys

Practice (pp. 136-137)

1. Tiles arranged as directed.
2. Sketches of rectangles on grid paper 1 x 11, 2 x 10, 3 x 9, 4 x 8, 5 x 7, and 6 x 6.
3.

Width of Rectangle in Units	Length of Rectangle in Units	Area of Rectangle in Square Units	Perimeter of Rectangle in Units
1	11	11	24
2	10	20	24
3	9	27	24
4	8	32	24
5	7	35	24
6	6	36	24

4. The sum of the dimensions is one-half the perimeter.

Practice (p. 138)

1. Tracing of rectangle as directed.
2. Cut-out of triangle as directed; placement of triangle on opposite end of rectangle as directed.
3. The length and width of the original rectangle are the base and height of the new parallelogram.

Practice (pp. 139-144)

1. The two triangles can be placed together to form a parallelogram so the area of the triangle is one-half the area of the parallelogram.
2. Sketches on grid paper will vary. The formula $A = lw$ provides the area of a rectangle because the length represents how many square units are in a row and the width represents how many rows. A rectangle with a length of 7 units would have 7 square units in the first row and if its width was 3, there would be 7 rows of 3 square units or 21 square units in all.
3. Sketches on grid paper will vary. The formula $A = bh$ provides the area of a parallelogram because a rectangle is a parallelogram where the b is the same as the l and the h is the same as the w. If the parallelogram is not a rectangle, it can become one by removing a triangular section from one end and moving it to the other.
4. Sketches on grid paper will vary. Two congruent triangles can be placed together to form a parallelogram. The base and height of triangles becomes the base and height of the parallelogram. Since the two triangles are congruent, the area of one triangle will be $\frac{1}{2}bh$.

Practice (pp. 145-146)

1. Cubes arranged as directed.
2.

Length of Prism in Units	Width of Prism in Units	Height of Prism in Units	Volume of Prism in Cubic Units	Surface Area of Prism in Square Units
1	24	1	24	98
1	12	2	24	76
1	8	3	24	70
1	6	4	24	68
2	6	2	24	56
2	4	3	24	52

3. The product of the length, width and height gives the volume.
4. The product of the length and width tells how many unit cubes are required for the first layer. The height tells how many of these layers are needed.
5. If the rectangular prism is a cube, we can find the area of one face and multiply by 6 to get the area of all 6 faces. If the rectangular prism is not a cube, we can find the area of the top and double it to include the bottom. We can do likewise for the front, back, and the two ends.
6. The $2lw$ in the formula represents the areas of the rectangular surfaces on the top and bottom of the prism. The area of each is

Keys

found by multiplying length by width. The 2 *lh* represents the areas of the rectangular surfaces on the front and back of the prism. The 2 *wh* represents the rectangular surfaces on the ends of the prism.

Practice (p. 147)

1. J
2. F
3. L
4. H
5. K
6. A
7. E
8. B
9. I
10. C
11. D
12. G
13. M

Lesson Three

Practice (pp. 150-151)

1. 2; 4; 2
2. 12.5 ft. by 22.5 ft.; 12.5 ft. by 10 ft.
3. 281.25; 125; 125
4. 875 square feet

Practice (p. 152)

Scale drawing of the classroom with scale clearly shown.

Practice (pp. 153-158)

1. greater
2. $\frac{1}{2}$ or .5
3. 4.5 or $4\frac{1}{2}$; 9; 255 (254.7)
4. 270
5. 4
6. 9.4; 266
7. 3.5
8. 198
9. 52,668
10. is
11. 78
12. Work should be based on length of about $5\frac{1}{6}$ inches or 402 miles; width of about 2.5 inches or 195 miles; area of about 78,390 less 6,654 for the corner Colorado occupies or about 71,736 square miles with 68,000-75,000 square miles fairly accurate.
13. Answers will vary.
14. estimate of 117 miles
15. estimate of 141 miles to 142 miles

Lesson Four

Practice (p. 160)

1. $A = 10$ square units; $P = 15$ units
2. $A = 4$ square units; $P = 10$ units
3. $A = 4$ square units; $P = 8$ units
4. $A = 21$ square units; $P = 20$ units
5. $A = 13.5$ units; $P = 19$ units

Practice (pp. 161-165)

1. Sketches on grid paper of any three rectangles of the following dimensions: 1 x 30, 2 x 15, 3 x 10, or 5 x 6.
2. Sketches on grid paper with dimensions shown of any whole and decimal number or mixed number with a product of 30 (e.g., 12 x 2.5).
3. Sketch on grid paper of a 8 x 16 rectangle.
4. 4 hours
5. Sketches on grid paper of a 10 by 10 square with two 5 by 10 rectangles; 60 units.
6. If $V = 125$, then dimensions are 5 x 5 x 5 and surface area is 150 square units.

Keys

7. Sketches on grid paper of two non-rectangular parallelograms each with an area of 20 units and each with whole number dimensions (i.e., $b = 20, h = 1; b = 2, h = 10; b = 4, h = 5$).
8. Sketches on grid paper of two right triangles each with an area of 12 square units and each having base and height equal to whole numbers.
9. The rectangles would be 8 x 9 and $P = 34$ units, $A = 72$ square units.
10. A car is likely to need at least 8 feet for width. The 16 x 25 would be snug and the 20 by 20 the best fit. The 10 by 40 would work if one car parked behind the other.

Practice (p. 166)

1. E
2. F
3. B
4. C
5. A
6. D

Practice (pp. 167-168)

1. estimation
2. cubic units
3. perimeter
4. square units
5. rectangle
6. triangle
7. formula
8. metric units
9. customary units
10. weights
11. diameter
12. length
13. volume
14. area

Unit Assessment (pp. 41-48TG)

1. c
2. c
3. a
4. b
5. b
6. c
7. c
8. Answers will vary but may include the following: The formula $A = bh$ provides the area of a parallelogram. Since any two congruent triangles can be placed together forming a parallelogram with the base and height of the original triangle, the area of the triangle will be one-half the area of the parallelogram.
9. Sketches on grid paper of any three rectangles of the following dimensions: 1 x 30, 2 x 15, 3 x 10, 5 x 6, or any rectangle with an area of 30 square units.
10. Sketches on grid paper of any two rectangles of the following dimensions: 1 x 5, 2 x 4, 3 x 3, or any rectangle with a perimeter of 12 units.
11. Sketch on grid paper of a triangle with an area of 10.
12. Sketch on grid paper of a non-rectangular parallelogram with an area of 18 square units.
13. 199.2 miles or other reasonable estimate such as 200 miles
14. Sketch of a rectangle with actual length of 3.5 inches and actual width of 2 inches.
15. Scale drawing of a rectangle with dimensions of 9 feet by 12 feet with scale shown. The amount of carpet needed in square yards is 12.

Keys

Scoring Recommendations for Unit Assessment

Item Number	Assigned Points	Total Points
1 to 7	4	28 points
8, 10, 14	8	24 points
9, 13, 15	12	36 points
11, 12	6	12 points
		Total = 100 points

Benchmark Correlations for Unit Assessment

Benchmark	Addressed in Items
B.1.3.1	8 - 12, 14, 15
B.1.3.4	13, 15
B.2.3.1	14
B.2.3.2	1, 2, 5, 15
B.3.3.1	3, 4
B.4.3.1	6, 7
B.4.3.2	13

Unit 3: Geometry

This unit emphasizes how models are used to sort, classify, make conjectures, and test geometric properties and relationships to solve problems.

Unit Focus

Number Sense, Concepts, and Operations

- Select the appropriate operation to solve problems involving ratios and proportions. (A.3.3.2)
- Add, subtract, multiply, and divide whole numbers and decimals, to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to predict results and to check the reasonableness of results. (A.4.3.1)

Measurement

- Use concrete and graphic models to derive formulas for finding perimeter, area, and volume. (B.1.3.1)
- Use concrete and graphic models to derive formulas for finding angle measures. (B.1.3.2)
- Understand and describe how the change of a figure in such dimensions as length, width, or radius affects its other measurements such as perimeter and area. (B.1.3.3)
- Use direct (measured) and indirect (not measured) to compare a given characteristic in customary units. (B.2.3.1)

Geometry and Spatial Relations

- Understand the basic properties of, and relationships pertaining to, geometric shapes in two dimensions. (C.1.3.1)
- Understand the geometric concepts of symmetry reflections, congruency, similarity, perpendicularity, parallelism, and transformations, including flips, slides, turns, and enlargements. (C.2.3.1)
- Represent and apply geometric properties and relationships to solve real-world and mathematical problems. (C.3.3.1)
- Identify and plot ordered pairs of a rectangular coordinate system (graph). (C.3.3.2)

Algebraic Thinking

- Describe relationships through expressions and equations. (D.1.3.1)
- Create and interpret tables, equations, and verbal descriptions to explain cause-and-effect relationships. (D.1.3.2)
- Represent problems with algebraic expressions and equations. (D.2.3.1)

Lesson Purpose

Lesson One Purpose

- Add and multiply whole numbers and decimals to solve problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Represent problems with algebraic expressions and equations. (D.2.3.1)
- Use concrete and graphic models to derive formulas for finding area, perimeter, and volume. (B.1.3.1)

- Understand the basic properties of, and relationships pertaining, to geometric shapes. (C.1.3.1)
- Understand the geometric concepts of reflections, congruency, perpendicularity, parallelism, and symmetry. (C.2.3.1)
- Identify and plot ordered pairs in a rectangular coordinate system graph. (C.3.3.2)

Lesson Two Purpose

- Add, subtract, and multiply whole numbers to solve problems using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to predict results and to check the reasonableness of results. (A.4.3.1)
- Use graphic models to derive formulas for finding angle measures. (B.1.3.2)
- Describe relationships through expressions and equations. (D.1.3.1)
- Understand the basic properties of, and relationships pertaining to, geometric shapes. (C.1.3.1)
- Identify and plot ordered pairs in a rectangular coordinate system graph. (C.3.3.2)

Lesson Three Purpose

- Add, subtract, multiply, and divide whole numbers and decimals to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to predict results. (A.4.3.1)
- Use direct (measured) and indirect measures (not measured) to compare a given characteristic in customary units. (B.2.3.1)

- Use graphic models to derive formulas for finding perimeter and area. (B.1.3.1)
- Understand and describe how the change of a figure in such dimensions as length, width, or radius affects its other measurements such as perimeter and area. (B.1.3.3)
- Create and interpret tables, equations, and verbal descriptions to explain cause-and-effect relationships. (D.1.3.2)
- Understand the basic properties of, and relationships pertaining to, geometric shapes in two dimensions. (C.1.3.1)
- Represent and apply geometric relationships to solve real-world problems. (C.3.3.1)
- Understand the geometric concept of enlargements. (C.2.3.1)

Lesson Four Purpose

- Understand the geometric concepts of congruency and similarity. (C.2.3.1)
- Represent and apply geometric properties and relationships to solve real-world and mathematical problems. (C.3.3.1)
- Select the appropriate operation to solve problems involving ratios and proportions. (A.3.3.2)
- Multiply and divide whole number to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)

Lesson Five Purpose

- Understand the geometric concepts of symmetry and related transformations. (C.2.3.1)
- Use concrete models to derive formulas for finding angle measures. (B.1.3.2)

Suggestions for Enrichment

1. Ask students to look for two-and three-dimensional figures as they walk, ride a bike, or travel by car or bus. Have students make a quick sketch of the figures and note their use. Example: The octagon is used as a stop sign. The cylinder is used as a trash can. Ask students to find at least 15 figures.

octagon

cylinder

2. Have students break pieces of uncooked spaghetti in lengths of 1", 2", 3", 4", 5", 6", 7", 8", 9", and 10" and make at least 2 of each length. Have students try to make triangles using different combinations of spaghetti sides. Have them find at least two combinations that will and will not form triangles. Ask students to use glue or tape to make a display of their work. Have students record results. Then write a short summary explaining why some combinations work and some do not.

work	won't work

3. Ask students to develop a creative way to demonstrate why a square is also a rhombus, a rectangle, a parallelogram, and a quadrilateral.

4. Allow students access to a computer and software with sketching capability and have them create a figure and record the set of commands required to create the figure. Have student share the results with the class. (Logo is an example of the software needed.)

5. To emphasize the properties of the sides and the angles of a square, rectangle, parallelogram, rhombus, and trapezoid, cut out the bottom of a rectangular shoe box and a square box, so the boxes can now bend at the corners. Hold the rectangle box so as to view the open face and bend the quadrilateral into a parallelogram. Bending the shoe box illustrates the change in angles and the fact the length of the sides have not changed. Bend the square box, and the quadrilateral becomes a rhombus.

6. Bring in various foods shaped like polygons having three or more sides (e.g., triangular-shaped chips, square-shaped crackers). Give students one of each type of food and have them measure length of sides and calculate perimeter. Ask students to name and classify the polygons based on the number of sides and length of sides. Classify the three-sided polygon according to the measure of its angles, list the characteristics used to classify the four-sized polygon, and name the food represented by the polygon. (Optional: Bring in soda and paper cups to talk about volume and figure out the amount of liquid that each cup holds. Talk about capacity.)

7. Have students use math vocabulary and definitions to create crossword puzzles to trade with other students and solve each others' puzzles.

8. Have students create a simple design on grid paper. Ask students to enlarge the design so that the perimeter of the enlargement is twice the perimeter of the original. Ask students to be sure that corresponding angles are congruent and that ratios of corresponding sides are equal.

9. Have students cut out a photograph from a magazine and trim it to whole-number dimensions in centimeters (e.g., 10 centimeters by 15 centimeters or 8 centimeters by 13 centimeters) to create realistic "blowup" or enlargement drawings of the photograph. Have students mark off centimeters on all four sides of their photographs and connect the line in ink to form a grid. Have students mark off 10-centimeter increments on their blowup frame on a plain sheet of paper and draw a grid in pencil. Ask students to find a point on a feature in the magazine photograph. Measure the distance from the square's top and side to that point. Multiply these two measurements by 10. Find the corresponding point on the corresponding blowup square and mark it. Continue to mark several points and draw a feature by connecting the dots. After drawing the feature, color the picture and erase the pencil grid marks. Display the blowup drawings next to photographs. (Optional: Have students measure something big and make a scale model of it.)

10. Have students explore shapes and patterns using tangram pieces. Ask students to fold and cut a square piece of paper following these directions.

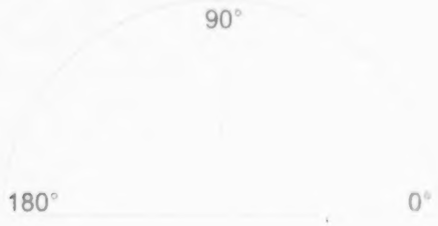
   - Fold the square sheet in half along the diagonal, unfold, and cut along the crease. What observations can be made and supported about the two pieces?
   - Take one of the halves, fold it in half, and cut along the crease. What observations can be made and supported?
   - Take the remaining half and lightly crease it to find the midpoint of the longest side. Fold it so that the vertex of the right angle touches that midpoint and cut along the crease. What observations can be made and supported? Discuss congruent and similar triangles and trapezoids.
   - Take the trapezoid, fold it in half, and cut along the crease. What shapes are formed? (trapezoids) What relationships do the pieces cut have? Can you determine the measure of any of the angles?
   - Fold the acute base angle of one of the trapezoids to the adjacent right base and cut on the crease. What shapes are formed? How are these pieces related to the other pieces?
   - Fold the right base angle of the other trapezoid to the opposite obtuse angle and cut on the crease. (Students should now have seven tangram pieces.) What other observation can be made?

   Have students put the pieces together to form the square they originally started with. Have students order the pieces from smallest to largest based on area, using the small triangle as the basic unit of area. Ask students what the areas of each of the pieces are in triangular units.

Have students create squares using different combinations of tangram pieces and find the area of squares in triangle units. (e.g., one square with one tangram piece: two triangle units in area; two tangram pieces of the two small triangles: two triangular units; or the two large triangles: eight triangular units in area.)

Have students try to form squares with three pieces, four pieces, five pieces, six pieces, and all seven pieces. Ask students: Are there multiple solutions for any? Are there no solutions for any? Do you notice any patterns? (Possible questions from tangram folding to ask: If the length of a side of the original square is two, what are the lengths of the sides of each of the tangram pieces cut? Possible conjectures based on finding from square making activity: Areas of the squares appear to be powers of two; they are unable to make a six-piece square; when all combinations six-pieces are considered, the possible areas are not powers of two.)

11. Ask students to cut a semicircle from a piece of stiff paper. Then have students fold the semicircle in half to make a quarter circle and mark the center. Have students mark a 0°, 90°, and 180° as shown. This is a quick way to make a protractor to help estimate the measure of angles.



12. Have students determine the optimum angle to achieve the greatest distance. Attach a hose to an outside tap and adjust the flow of water to a constant pressure. Start at an angle of zero degrees to the ground. Measure and record the distance the streams travels in a horizontal direction along the ground. Repeat this process at 20, 30, 45, 60, and 75 degrees. Ask students the following: Which angle allowed you to achieve the maximum distance? Describe a method to determine the maximum height the water achieved at the

optimum angle. Draw the approximate path the water followed in its flight. What is the shape of the path? If the pressure on the hose increased, what effect would it have on the angle you would use to achieve maximum distance at the new pressure? Do you think that a shot put or a javelin would need to be thrown at a different angle to achieve its maximum distance?

13. Draw a very large circle (on a rug, use yarn, or on concrete, use chalk). To draw a perfect circle, use a marker tied to a piece of string taped to the floor. Draw the diameter, a radius, and the chord of the circle to help teach pi, area, and circumference. Define each part with the students. Then have students count their steps as they pace off each part and record their data on a chart. Ask students to note that their walk around the circle took about three times as many steps as the walk across the circle to help them remember what pi means.

14. Have groups make a table or chart with columns for names or number of objects, circumference, diameter, and a column with a question mark (?). Give students round objects such as jars and lids to measure. Have students measure and record each object's circumference and diameter, then divide the circumference by the diameter and record the result in the ? column. Ask students to find the average for the ? column. Record the group's averages on the board, and have students find the average number for the class. Explain to students they have just discovered pi (π). Then have students come up with a formula to find the circumference of an object knowing only the diameter of that object and the number that represents pi. Ask students to verify that their formula works by demonstrating and by measuring to check their results. Have students write conclusions for the activities they have just performed. Give students three problems listing only the diameter of each object and have them find the circumference.

15. Design a large *Jeopardy* board with six categories going across the top and values of 10, 20, 30, 40, and 50 going down the left side. Write the skill to be reviewed under each of the six categories and then fill in your master game sheet so that under each category, problems range from easiest to hardest. Divide so that each team represents varying abilities and distribute a blank *Jeopardy* game to each student to be collected later for assessment. One student is

selected to be the group scorekeeper and write point amounts for their team on the board and to write problems in the correct category box on the board.

JEOPARDY

Value	Category #1 Estimation	Category #2 Proportions	Category #3 Subtraction	Category #4 Geometry	Category #5 Division	Category #6 Equations
10			Problem from easiest to hardest levels of difficulty			
20			easiest			
30						
40						
50			↓ hardest			

One student from the first team goes to the board and selects a category and a point amount. All students in the class then write the problem on their game sheet and solve it showing all work. The teacher calls on a student from the team whose turn it is to answer. If correct, the student must explain the steps necessary to solve the problem. The team's scorekeeper writes the correct problem on the class-size game board and places an "x" through the problem. The game continues until all students on all the teams have a turn. You may secretly place double *Jeopardy* points in selected boxes to be revealed only upon being chosen. Tally team points to determine a winner.

16. Have students research M. C. Escher and his art. Ask students to create their own Escher-like tessellation. Have students use shape sets to realize significant properties of polygons, then take photographs of tessellations around their school.

17. Have students use pattern blocks to discover how many ways these blocks can be put together to make different polygons (e.g., a rhombus and equilateral triangle can form a trapezoid). Then have students create their own tessellates, or forms repeating shapes that fill a plane without gaps or overlapping, and color them. (Optional: You could show representations of tessellations in tile patterns and wallpaper.)

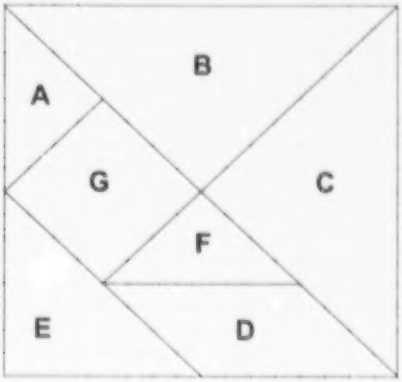
18. See Appendices A, B, and C for other instructional strategies, teaching suggestions, and accommodations/modifications.

Unit Assessment

Circle the letter of the correct answer.

Figure 1, below, is a **square** that has been subdivided into various shapes. It is a Chinese puzzle called a **tangram**. You will refer to it to answer questions 1-5.

figure 1



1. Figure A is not _____________ .
   - a. an isosceles triangle
   - b. a right angle
   - c. an acute triangle
2. Figure G is not _____________ .
   - a. quadrilateral
   - b. a square
   - c. a rectangle
   - d. a pentagon
3. Figure D is not _____________ .
   - a. a rectangle
   - b. a parallelogram
   - c. a quadrilateral
   - d. a polygon

4. Which of the following pairs of figures are not congruent?

   a. figures A and F
   b. figures B and C
   c. figures A and E

5. The sum of the angle measures in figure C is ________ .

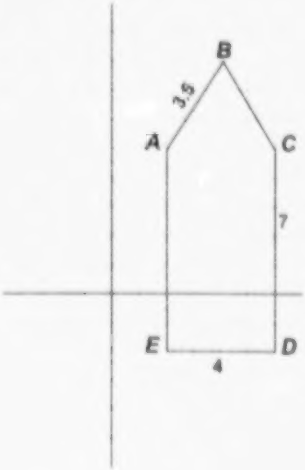
   a. 90 degrees
   b. 180 degrees
   c. 270 degrees
   d. 360 degrees

6. The sum of the angle measures in figure D is ________ .

   a. 180 degrees
   b. 270 degrees
   c. 360 degrees
   e. 540 degrees

Refer to **figure 2** below to answer the questions 7-17.

figure 2



7. Figure 2 is ________ .

   a. a rectangle
   b. a hexagon
   c. a quadrilateral
   d. a pentagon

8. Angle *A* is ____________ .

    a. a right angle
    b. an obtuse angle
    c. an acute angle
    d. a straight angle

9. Angle *B* is ____________ .

    a. a right angle
    b. an obtuse angle
    c. an acute angle
    d. a straight angle

10. Line segments perpendicular to each other include ____________ .

    a. line segments *EA* and *AB*
    b. line segments *AB* and *BC*
    c. line segments *CD* and *DE*
    d. line segments *AE* and *CD*

11. Line segments parallel to each other include ____________ .

    a. line segments *AE* and *CD*
    b. line segments *AE* and *ED*
    c. line segments *AB* and *BC*
    d. line segments *AE* and *AB*

12. The measure of angle *A* is ____________ .

    a. 38 degrees
    b. 142 degrees
    c. 127 degrees
    d. 53 degrees

13. The coordinates of point *D* are ____________ .

    a. (2, -2)
    b. (-2, 2)
    c. (6, -2)
    d. (-2, 6)

14. The perimeter of figure 2 is ________ .

a. 25 units
b. 29 units
c. 14.5 units
d. 34 units

15. The area of figure 2 is approximately ________ .

a. 28 square units
b. 34 square units
c. 42 square units
d. 25 square units

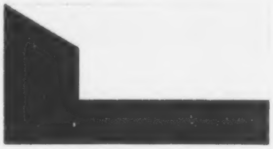
16. If the length of each side of figure 2 were doubled, the perimeter would ________ .

a. remain the same
b. increase by 2 units
c. double
d. be 4 times as great

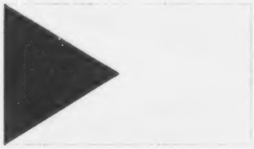
17. If the length of each side of figure 2 were tripled, the perimeter would ________ .

a. remain the same
b. increase by 3 units
c. triple
d. be 9 times as great

Use the flags above each section to choose the flag that **meets all criteria**. *Write the name of country's flag on the line provided and then* **illustrate** *or* **explain how all the criteria were met**.



Kuwait



Guyana

18. Their flag's pattern is divided into an isosceles triangle, a concave quadrilateral, and two right triangles. ______________________

____

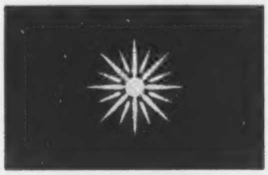
____

____

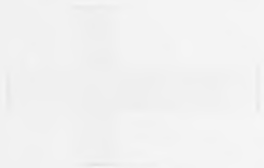
____

____

____

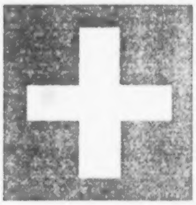


Macedonia

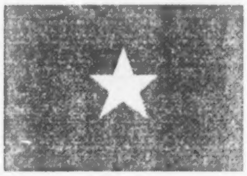


Finland

19. Exactly two lines of reflectional symmetry can be drawn in the flag of this country. ______

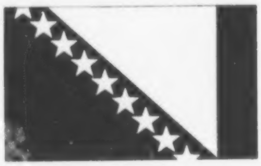


Switzerland

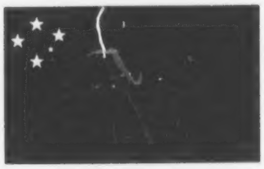


Vietnam

20. Rotational symmetry is seen in this flag. ____________________



Bosnia/Herzegovina



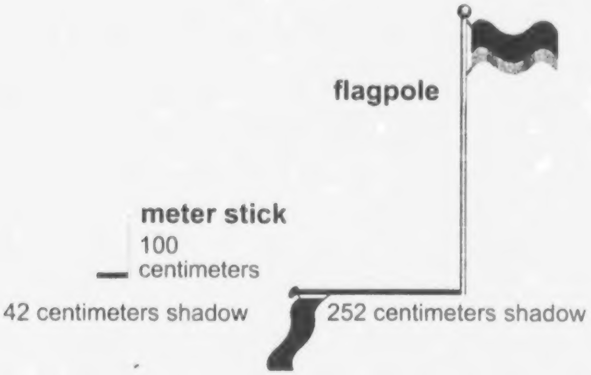
Samoa

21. The stars in this flag represent translational symmetry. ________

Answer the following.

22. The area of a round pizza with a diameter of 16 inches is __________ times the area of a round pizza with a diameter of 8 inches.

23. Using the shadow method to estimate the height of the flagpole shown, the following measurements were recorded. Use them to find the approximate height of the flagpole.


flagpole
meter stick
100 centimeters
42 centimeters shadow
252 centimeters shadow

The approximate height of the flagpole is ____ centimeters.

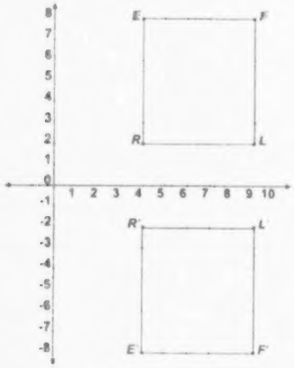
Keys

Lesson One

Practice (pp. 185-190)

1. True
2. False (Opposite sides are congruent and parallel.
3. True
4. True
5. True
6. False (the line segment is 5 units long.)
7. True
8. True
9. False (Vertices should be named clockwise or counter-clockwise.)
10. True
11. True
12. True

13 True
14. True
15. True
16. True
17. False (4 by 3.5 has area of 14 square units, 9 by 1.5 has area of 13.5 square units)
18. True
19. True



20. False (Paper must be folded along the *x*-axis.)
21. True

Practice (p. 191)

1. quadrilaterals; parallelograms
2. squares
3. right
4. perpendicular
5. parallel; congruent
6. area
7. perimeter
8. square units
9. units

Practice (pp. 192-193)

1. *x*-axis
2. origin
3. line
4. coordinates (ordered pairs)
5. point
6. *y*-coordinate
7. axes
8. quadrant
9. parallel
10. *x*-coordinate
11. ordered pairs (or coordinates)
12. *y*-axis
13. coordinate grid or system
14. number line
15. intersection
16. line of symmetry

Practice (p. 194)

1. D
2. E
3. A
4. C
5. F
6. B

Practice (p. 195)

1. C or B
2. B

Keys

3. A
4. E or B or C
5. D
6. F or B

Practice (p. 196)

1. degree
2. angle
3. length
4. perpendicular
5. congruent
6. units
7. line segment
8. side
9. polygon
10. width
11. vertex
12. grid

Practice (p. 197)

1. K
2. E
3. A
4. F
5. B
6. D
7. J
8. C
9. I
10. G
11. H

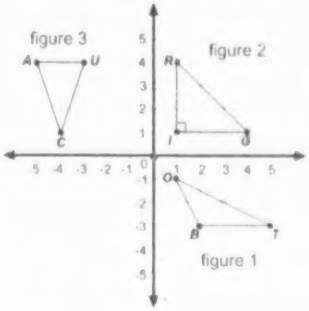
Lesson Two

Practice (p. 201)

Drawings should be of a right triangle, an obtuse triangle, and an acute triangle, with measurements provided.

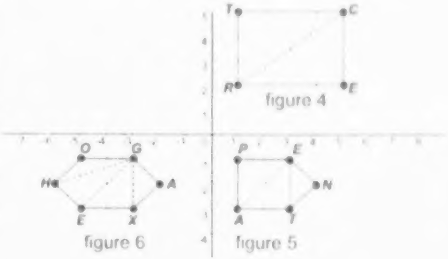
Practice (p. 202)

1. Figure one should be a right triangle, figure two should be an obtuse triangle, and figure three should be an acute triangle. Vertices should be at specified locations (see grid sheet).



Practice (pp. 203-209)

1. right; 90; right; right
2. obtuse; greater; obtuse; obtuse
3. acute; less; acute; acute
4. scalene
5. isosceles
6. equilateral
7. 70; 70; 40; 180
8. 45; 90; 45; 180
9. 22; 120; 38; 180
10. Triangles and angle measures will vary; 180
11. Figure four should be a rectangle, figure five should be a pentagon, and figure six should be a hexagon. Vertices should be at specified locations (see grid sheet).



12. rectangle; right
13. pentagon; obtuse; acute
14. hexagon; obtuse; acute
15. triangles; 180; 360
16. triangles; 180; 540
17. triangles; 180; 720

18. triangles; triangles; triangles; five; 900
19. 180
20. Drawing of an octagon with 6 triangles within; triangles; 180; 1,080

Practice (pp. 210-214)

1. Drawings of an acute angle, obtuse angle, and right angle with an explanation/definition for each.
2. Drawings of an acute triangle, obtuse triangle, and right triangles with an explanation/definition for each.
3. Drawings of a scalene triangle, an isosceles, triangle, and an equilateral triangle with an explanation/definition for each.
4. One or more drawings of triangles with angle measures shown and/or explanation of sum of measures always being 180 degrees.
5. Drawings of polygons with appropriate triangles within and explanation of total sum of angle measures.

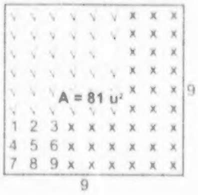
Practice (p. 215)

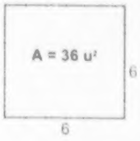
1. D
2. A
3. C
4. E
5. B
6. A
7. B
8. C
9. C
10. B
11. A

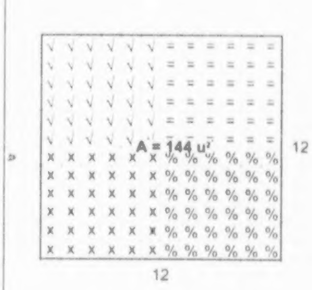
Lesson Three

Practice (p. 218)

1. Three squares should be drawn on grid paper measuring 6 by 6, 9 by 9 and 12 by 12; A = 36 square units;
A = 81 square units; A = 144 square units.







Practice (pp. 219-221)

1. 2
2. 2
3. 4
4. 1.5
5. 1.5 or $1\frac{1}{2}$
6. 2.25 or $2\frac{1}{4}$
7. 2
8. 1.5 or $1\frac{1}{2}$
9. 4
10. 2.25 or $2\frac{1}{4}$
11. On the 9 by 9 square, there should be check (√) marks on 36 of the unit squares, *x* marks on 36, and those remaining should be numbered 1-9. This shows that the 9 by 9 is 2 times as large as the 6 by 6 with 9 square units left over so it is actually 2.25 or $2\frac{1}{4}$ times as large.

Keys

12. On the 12 by 12 square, there should be check (√) marks on 36 unit squares, *x* marks on 36, percent signs (%) marks on 36, and equal (=) marks on 36 with no squares left over. This shows that the 12 by 12 is exactly 4 times as large as the 6 by 6.
13. 2.25; 1.5; The 9 by 9 would be the better buy.
14. 4; 2; The 12 by 12 would be the better buy.
15. 1.777; 1.333; The 12 by 12 would be the better buy.
16. $0.166 or $.17
17. $0.111 or $.11
18. $0.083 or $.08
19. The cost per square unit of pizza is less for the 12 by 12 than for the others. The cost per square unit of pizza is less for the 9 by 9 than for the 6 by 6.

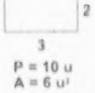
Practice (pp. 222-223)

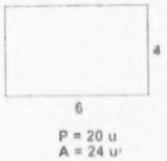
Statements provided in the problem should be copied.

Practice (pp. 224-226)

1. 10
2. 6
3. 20
4. 24

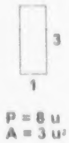
Drawings should be of a 2 by 3 rectangle and a 4 by 6 rectangle, with the growth factor of 2 resulting in the perimeter being 2 times as great and the area being 4 times as much.

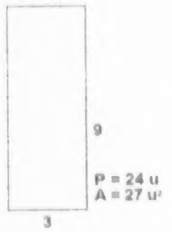




5. 8
6. 3
7. 24
8. 27

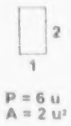
Drawings should be of a 1 by 3 rectangle and a 3 by 9 rectangle, with the growth factor of 3 resulting in the perimeter being 3 times as great and the area being 9 times as great.

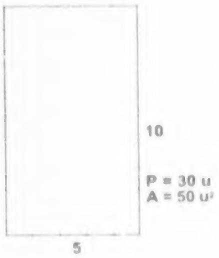




9. 6
10. 2
11. 30
12. 50

Drawings should be of a 1 by 2 rectangle and a 5 by 10 rectangle, with a growth factor of 5 resulting in the perimeter being 5 times as great and the area being 25 times as great.





Keys

Practice (p. 227)

Answers will vary but should be in reasonable range of the difference in area column in following practice:

1. 8
2. 17-18
3. 30-32

Practice (p. 228)

Square Pizza Side Measure	Area of Square	Diameter of Circle	Radius of Circle	Area of Circle	Difference in Areas
6 units	36 square units	6 units	3 units	28.26 square units	7.74 square units
9 units	81 square units	9 units	4.5 units or $4\frac{1}{2}$ units	63.585 square units	17.415 square units
12 units	144 square units	12 units	6 units	113.04 square units	30.96 square units

Practice (p. 229)

1. \$0.177 or \$.18
2. \$0.126 or \$.13
3. \$0.097 or \$ 10
4. 4
5. 2.25 or $2\frac{1}{4}$

6. Tables will vary but correct cost per units of square and round pizzas will be determined by the teacher. Square pizzas are the better buy than round pizzas of comparable measures.

Practice (pp. 230-231)

1. Answers will vary but should summarize work from this lesson.
2. Answers will vary but should summarize work from this lesson.
3. Answers will vary. If the 6" square remains priced at \$6, a suggested price for the 9" may approximate \$12 since the area of the 9" is 2.25 times as great as the 6". A suggested price for the 12" could be up to \$24 since it is 4 times as great in area as the 6". The student may reason that \$24 is too much and recommend more than the current \$12 but less than \$24. The student may also launch into an argument to reduce the price of the 6" and to base new prices of the 9" and the 12" pizzas' increase on area. (Push for thoughtful reasoning.)

Lesson 4

Practice (pp. 233-237)

Cut out figures of tracings should look like the figures on page 233 of the student book.

1. yes
2. no

Keys

3. yes
4. no
5. right; right
6. 180
7. 2; 4
8. 3; 5
9. 2; similar
10. 4; 3
11. 2
12. 2
13. right
14. right; right
15. right
16. 4; 4; 144
17. 10

Practice (p. 238)

Answers will vary.

Practice (p. 239)

1. F
2. C
3. K
4. J
5. E
6. H
7. I
8. D
9. G
10. B
11. A
12. L

Lesson Five

Practice (p. 242)

1. no
2. yes
3. yes
4. yes
5. yes
6. no

Practice (p. 245)

1. yes; 45 degrees
2. yes; 120 degrees
3. no
4. yes; 90 degrees
5. no
6. yes; 180 degrees

Practice (p. 246)

Answers will vary but should include three illustrations of rotational symmetry and their angle of rotation.

Practice (p. 248)

1. yes
2. no
3. yes

Practice (p. 249)

Answers will vary but should include three illustrations of translational symmetry.

Practice (p. 250)

Lessons on symmetry will vary but should include the three types of symmetry, their definitions, real-world illustrations, and explanations of each illustration.

Practice (p. 251)

1. A
2. F
3. D
4. G
5. E
6. B
7. C
8. B
9. C
10. A

Keys

Practice (p. 252)

1. C
2. A
3. B
4. A
5. C
6. B
7. A
8. B
9. C

Practice (p. 253)

1. length
2. coordinates
3. square units
4. angle
5. area
6. perimeter
7. degree
8. sum
9. congruent

Practice (p. 254)

1. B
2. A
3. C
4. D

Unit Assessment (pp. 67-75TG)

1. c
2. d
3. a
4. c
5. b
6. c
7. d
8. b
9. c
10. c
11. a
12. b
13. c
14. a
15. b
16. c
17. c
18. The flag of Guyana meets the criteria. The dark-colored shape is an isosceles triangle. The light-colored shape is a concave quadrilateral. The two medium-colored shapes are right triangles.
19. The flag of Macedonia meets the criteria. A vertical line of symmetry is present as well as a horizontal line. Each one passes through the center of the flag.
20. The flag of Switzerland meets this criteria. When turned 90 degrees, 180 degrees, and 270 degrees, it appears the same.
21. The flag of Bosnia and Herzegovina meets this criteria. The distance between stars is uniform and they lie in a straight line.
22. 4
23. 600 centimeters

Scoring Recommendations for Unit Assessment

Item Number	Assigned Points	Total Points
1 to 15	4	60 points
16 to 23	5	40 points
		Total = 100 points

Keys

Benchmark Correlations for Unit Assessment

Benchmark	Addressed in Items
C.1.3.1	1, 2, 3, 5, 6, 7, 8, 9, 12, 18
C.2.3.1	4, 10, 11, 19, 20, 21
C.3.3.1	22, 23
C.3.3.2	13
B.1.3.1	14, 15
B.1.3.3	16, 17, 22, 23
B.1.3.2	5, 6, 12
B.2.3.1	23

Unit 4: Creating and Interpreting Patterns and Relationships

This unit emphasizes how patterns of change and relationships are used to describe, estimate reasonableness, and summarize information with algebraic expressions or equations to solve problems.

Unit Focus

Number Sense, Concepts, and Operations

- Use exponential and scientific notation. (A.2.3.1)
- Add, subtract, multiply, and divide whole numbers and decimals to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculators. (A.3.3.3)
- Use estimation strategies to predict results and to check for reasonableness of results. (A.4.3.1)
- Use concepts about numbers to build number sequences. (A.5.3.1)

Measurement

- Use a model to derive the formula for circumference. (B.1.3.1)
- Derive and use formulas for finding rate, distance, and time. (B.1.3.2)

Geometry and Spatial Relations

- Understand the basic properties of, and relationships to, geometric shapes in two dimensions. (C.1.3.1)
- Identify and plot ordered pairs in a rectangular coordinate system. (C.3.3.2)

Algebraic Thinking

- Describe a wide variety of patterns and relationships through tables and graphs. (D.1.3.1)
- Create and interpret tables, graphs, and verbal expressions to explain cause-and-effect relationships. (D.1.3.2)
- Represent and solve real-world problems graphically and with algebraic expressions and equations. (D.2.3.1)
- Use algebraic problem-solving strategies to solve real-world problems. (D.2.3.2)

Lesson Purpose

Lesson One Purpose

- Create and interpret tables to explain cause-and-effect relationships. (D.1.3.2)
- Describe patterns and relationships through tables. (D.1.3.1)
- Derive formulas for finding rates, distance, and time. (B.1.3.2)
- Add, subtract, multiply, and divide whole numbers to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)

Lesson Two Purpose

- Create and interpret graphs to explain cause-and-effect relationships. (D.1.3.2)
- Describe patterns and relationships through graphs. (D.1.3.1)

- Identify and plot ordered pairs in a rectangular coordinate system (graph). (C.3.3.2)
- Use estimation strategies to predict results and to check for reasonableness of results. (A.4.3.1)

Lesson Three Purpose

- Describe a wide variety of patterns. (D.1.3.1)
- Use exponential and scientific notation. (A.2.3.1)
- Use concepts about numbers to build number sequences. (A.5.3.1)
- Understand the basic properties of, and relationships pertaining to, geometry shapes in two dimensions. (C.1.3.1)
- Use a model to derive the formula for circumference. (B.1.3.1)
- Add, subtract, multiply, and divide whole numbers and decimals to solve problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)

Lesson Four Purpose

- Create and interpret tables and verbal descriptions. (D.1.3.2)
- Represent problems with algebraic expressions. (D.2.3.1)
- Use exponential and scientific notation. (A.2.3.1)
- Add, subtract, multiply, and divide whole numbers and decimals to solve problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculators. (A.3.3.3)

Lesson Five Purpose

- Represent and solve real-world problems graphically and with algebraic expressions and equations. (D.2.3.1)
- Create and interpret tables. (D.1.3.2)
- Use algebraic problem-solving situations to solve real-world problems. (D.2.3.2)
- Add, subtract, multiply, and divide whole numbers and decimals to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use the formulas for finding rate, time, and distance. (B.1.3.2)

Suggestions for Enrichment

1. Have students access the Internet to find information on the past baseball season's American and National leagues' attendance and win statistics. Have students calculate an attendance-to-win ratio (attendance/win rounded to the nearest whole number) for each of the 28 major league teams and determine if winning always leads to good attendance. Then have students plot points for wins on the horizontal axis and attendance on the vertical axis. Do more wins result in greater attendance and why?
2. Have students choose a city and use the Internet to find the five-day forecast of that city's temperature and graph the information.

Five-day Forecast

3. Have students access the Internet to find the statistics on all the roller coasters at Six Flags in Georgia. Ask students to calculate which is the highest and fastest roller coaster at Six Flags by computing the average speed rate = distance/time ($r = d/t$) and converting feet/minute to miles/hour. Have students record other interesting facts.

Have students display data in a summary table as follows.

	Six Flags Coaster Computation			
	ride name	ride name	ride name	ride name
height				
length				
distance				
feet/minute				
miles/hour				
extra information				

Ask students to answer the following: What coaster has the highest drop? What coaster has the highest top speed? What coaster has the slowest average speed? What is the difference between the highest and lowest average speeds? What is the highest average speed of the coasters at Six Flags? What is your favorite coaster? Why (in terms of this activity)?

4. Have students use the Internet to compare records of Olympic gold medalists in one event for the last 100 years or provide them with a set of medalists. Ask students to plot the event on a two-dimensional graph with years on the horizontal axis and statistics on the vertical axis. Then have students answer the following: What trends or patterns did you notice? Were there any years that did not fit the overall picture? Did your trends match other students' trends? Did they have data that did not fit their pattern? Explain your pattern and why you think it happened. Construct an equation that describes the pattern of your data.

Olympic Gold Medal Winners

statistics

years

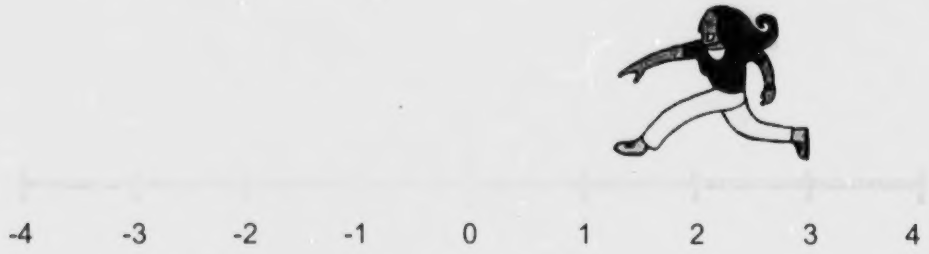
5. Show students a unit cube and ask them to describe the cube (e.g., eight corners, six faces, 12 edges). Have students build a second cube around the first cube so that first cube is encased by the second and then describe it in writing. Ask students how many unit cubes it will take to build a third cube around the second cube, a fourth cube around the third, and so on up to a tenth cube.

To extend the activity: Ask students to imagine that the entire outside of the tenth cube has been painted. If the cube is taken apart into unit cubes, how many faces of cubes are painted on three faces, two faces, one face, no faces? Have students chart their finding for each cube, first through tenth, and look for patterns.

Have students write exponents for the number of cubes needed and painted on three faces, two faces, one face, or no faces. Then have students graph their findings for each dimension of cube, first through tenth, and look for graph patterns.

6. Spray paint lima beans red on one side only. Give each student 10 painted lima beans and a cup for storing and tossing the beans. Have students decide which color will stand for negative and which positive. Working in pairs, one student will toss the beans and the other will record the score. The score is obtained by pairing the white and red beans to make zeros and counting what is left (For example, if the white side is positive, and there is a toss of seven whites and three reds, the three reds pair up with the three whites to make zeros, with four whites left. The score is +4 because +7 + -3 = +4). Have students play at least 15 rounds in order to see some patterns and arrive at a set of rules that can be used with all examples. This will enable them to predict outcomes correctly (e.g., you can find the difference between the units and use the sign of the greater number of units). Discuss. Record on the board several examples written as equations. (Optional: Extend the activity. Subtract, multiply, and divide using lima beans.)

7. Create a number line on the classroom floor using masking tape. Indicate zero (0), the directions of positive (+) and negative (-), and mark integers at intervals of about two feet. Have students take forward steps for positive and backward steps for negative on the line to solve addition of integers. Record movement on number lines on paper.



8. Tell students to imagine that they have been asked to choose between two salary options.

   - One cent on the first day, two cents on the second day, and double their salary every day thereafter for thirty days; or
   - $1,000,000 after 30 days.

   After choosing an option, ask students to complete a table for the first option with columns for day number, pay for the day, and total pay in dollars. (In 30 days this option increases from one penny to over 10 million dollars!)

Option #1 of Pay

Day Number	Pay for That Day	Total Pay (in dollars)
1	.01	.01
2	.02	.03
3	.04	.07
4	.08	.15
etc.	etc.	etc.

9. Ask students which they would choose to receive.

   - $4.50 per day for 30 days; or
   - one penny the first day, two the second day, four the third day, with the amount doubling every day for 30 days.

   Ask the students to compare the two methods on a spreadsheet and graph the results.

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10. Pose the following question to students.

    - You have taken a sip from a friend's soda and picked up a bacterium from your friend. If the bacterium divides once every 20 minutes how many potential bacteria could you host in 24 hours? In 48 hours?

    Have students use spreadsheets to explore exponential growth. Have students research growth rates for different species such as bacteria, flies, cats, dogs, or people and transfer the information to a spreadsheet and then graph it. For example, census data can be found in the *World Almanac* or from the United States Bureau of Census (http://www.census.gov). The population data can be used to make projections and then compared to professional projections.

11. See Appendices A, B, and C for other instructional strategies, teaching suggestions, and accommodations/modifications.

Unit Assessment

Circle the letter of the correct answer.

1. Venice and Gloria were jumping rope, and Venice jumped one-third as many times as Gloria jumped. Let *j* represent the number of jumps Venice made. Which expression could be used to determine the number of jumps for Gloria?

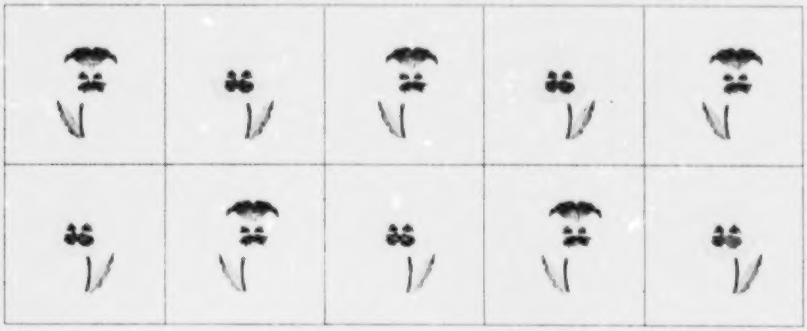
   a. $3 + j$
   b. $j + 3$
   c. $3j$
   d. $j - 3$

2. In Mr. Faulkner's class, 10 marbles are placed in a jar if every student is seated when the bell rings for class to begin, 5 marbles if every student is in classroom but some are not seated, and no marbles if one or more students are late. When the jar has 500 marbles in it, Mr. Faulkner provides refreshments for his class. The jar now has 420 marbles.

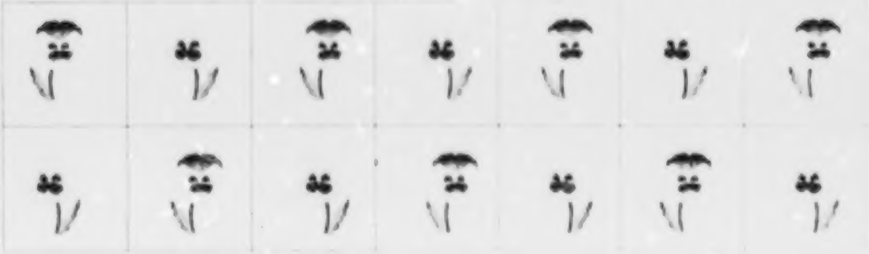
   Tardies have occurred 4 times, and all students have been seated 36 times. Which equation could be used to determine how many days all students were in the classroom on time but some were not seated if *n* represents this number of days?

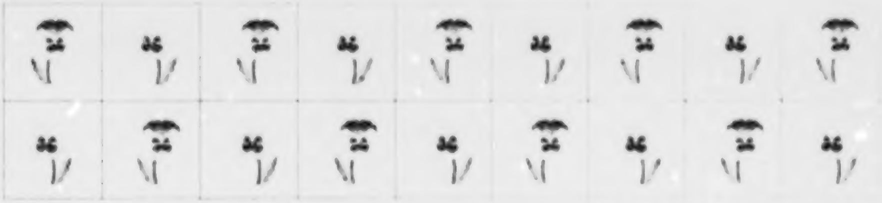
   a. $4(0) + 5n = 420$
   b. $4(0) + 36n = 420$
   c. $4(0) + 5n + 36(10) = 420$
   d. $4(0) + 36n + 5(10) = 420$

3. There are 5 pansy plants in each of two rows in the yard of the 1st house on the street.



At the second house there are 7 plants in each two rows, and at the 3rd house, there are 9 in each two rows. If this pattern continues, which house will have 30 plants total?





- a. 4th house
- b. 5th house
- c. 6th house
- d. 7th house

4. What is the 6th number in this sequence? 1, 4, 9, 16, ____, ____ .

   - a. 23
   - b. 30
   - c. 32
   - d. 36

Complete the following.

5. What is the next number in this sequence?

   0.25, 0.75, 2.25, 6.75, ____ .

6. The following table shows the amount of pay Leroy earns depending on how many hours he works.

Leroy's Earnings

Number of Hours Worked	Payment of Services
0	0
1	6.25
2	12.50
3	18.75
4	25.00
5	31.25
6	________
7	________

Determine his pay for 6 hours. ______________

Determine his pay for 7 hours. ______________

Explain in words or write an algebraic expression of how his pay

for any number of hours could be determined. ________________

____

____

____

7. The Jackson family plans a trip of 800 miles and will drive 415 miles the first day. If they average 55 miles per hour on the second day, how long will they travel the second day to reach their destination?

____

8. Mike's Pizza Place charges $7.00 for a large cheese pizza and $1.00 for each topping. Arlene's Pizza Palace charges $9.00 for a large cheese pizza and $0.75 for each topping. Write an algebraic equation for each company to determine cost for pizza. Let *c* represent the cost and *t* represent the number of toppings.

Mike's Pizza Place $c =$ ____________

Arlene's Pizza Palace $c =$ ____________

Use the two equations to complete the following table.

Cost of Pizzas

Number of Toppings	Cost at Mike's Pizza Place	Cost at Arlene's Pizza Palace
0	$7.00	$9.00
1		
2		
3		
4		
5		
6		
7		
8		
9		

9. A person wanting 3 toppings would find ____________ (Mike's or Arlene's) a better buy.

10. A person wanting 9 toppings would find ____________ (Mike's or Arlene's) a better buy.

11. Make a graph of the data and circle the point on the graph where the charge is the same at both companies. Be sure to title your graph, label your axes, and include a key.

12. Attach to this test the task assigned at the beginning of the unit on page 264. Be sure that the pattern you found interesting is described, a table and graph are provided, and conclusions and projections are written.

Keys

Lesson One

Practice (pp. 266-268)

1. True
2. True
3. True
4. True
5. True
6. True
7. True
8. False; all entries are not divisible by 100
9. True
10. True
11. True
12. True
13. False; the distance traveled in 18 hours would be 900 miles
14. True
15. True

Practice (p. 269)

See table below.

Time in Hours	Distance Traveled at 50 mph	Distance Traveled at 65 mph
0	0	0
1	50	65
2	100	130
3	150	195
4	200	260
5	250	325
6	300	390
7	350	455
8	400	520
9	450	585
10	500	650
11	550	715
12	600	780
13	650	845*
14	700	910*
15	750	975*

*Students may not have these entries since original problem was based on maximum distance of 750 miles.

Practice (p. 270)

Answers will vary.

Practice (pp. 271)

1. 9.7 to 10; 7.5
2. 13.5; 10 to 10.4
3. 5.5 to 6; 4 to 4.5

Practice (p. 273)

1. Yes; the conjecture should be found true with perhaps statements and work shown to illustrate
2. Yes; formulas for distance, time and rate should be written in words and symbols as directed

Practice (pp. 274-275)

1. Answers will vary but may include the following: The table would be helpful if you were using it to answer many questions and could simply refer to the table for your answers.
2. Answers will vary but may include the following: Dividing the distance by the rate to determine the travel time would be helpful if you were finding travel time for one or maybe a few questions.
3. Decreasing speed increases travel time.
4. Answers will vary.
5. Answers will vary.

Practice (p. 276)

1. B
2. D
3. E
4. C

Keys

5. F
6. A

Practice (p. 277)

1. B
2. A
3. C
4. E
5. D

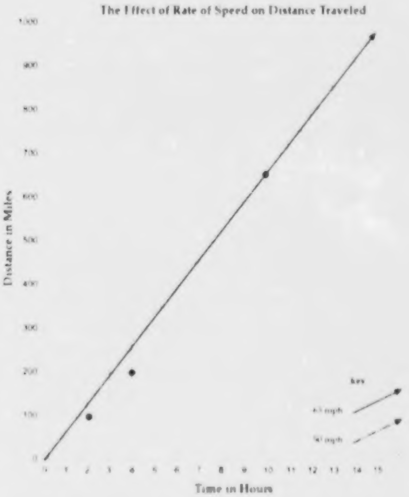
Lesson Two

Practice (pp. 282-283)

1. True
2. True
3. True
4. True
5. True
6. True
7. False; the distance for 7 hours at 65 mph is 455 or about 450, not 550
8. False
9. True
10. False; a key is necessary
11. True
12. True

Practice (p. 284)

See graph below. Fifty miles per hour key symbols will vary.



Practice (pp. 285-286)

1. (0; 0); 0 hours; 0 miles
2. more
3. above
4. 50 mph; 65 mph
5. increases
6. increases
7. 700 miles to 750 miles
8. 5.5 hours
9. somewhat like
10. 150; somewhat like

Keys

Practice (p. 287)

1. Answers will vary but may include the following: If I want specific information, I might use the table. An example might be to find distances traveled at 55 mph at 3, 4, 5, 6, and 7 hours.
2. Answers will vary but may include the following: If I want the "big picture" of the overall effect of rate on distance traveled, the graph is helpful. I can also secure specific information from the graph also but they might be more convenient with the table.
3. Answers will vary but may include the following: Estimation was used when plotting a point that did not lie at an intersection on the grid. It was also used when determining distance traveled for a given time or time for a given distance using the graph in Lesson Two.

Practice (p. 288)

1. G
2. E
3. F
4. A
5. B
6. C
7. D

Lesson Three

Practice (pp. 290-294)

1. 49
2. 29, 31, 37
3. 4, 2, 1
4. 56, 92; square
5.

10	15
X	X
X X	XX
X X X	XXX
X X X X	XXXX
	X X X X X

6. 243, 729; Multiply an entry by 3. (All entries are powers of 3.)
7. ⅞, 8⁄8 or 1; Add ⅛ to entry.
8. 1.75, 2.10; Add 0.35 to an entry to get the next one or multiply 0.35 by 1 for the first entry, by 2 for the second entry, etc.
9. 7½, 8 ¾; Multiply 1¼ by 1 for the first entry, 1¼ by 2 for the second entry, etc. Each entry is 1¼ more than the previous one. Or add 1¼ to entry.
10. 12345654321
11. Let students share their five favorites from page 289 and discuss.

Practice (p. 295)

1. C
2. D
3. H
4. A
5. F
6. B
7. E
8. G

Practice (pp. 296-304)

1. 900; 1,080; The sum of interior angle measures is 180 times the difference in the number of sides and 2. (Students may say to add 180 to the previous entry. Help guide thinking to enable response for 10-sided polygon without building table.)
2. 15.7; 18.84; The circumference of a circle is the diameter multiplied by 3.14. (Students may say to add 3.14 to the previous entry. Guidance is needed as in #1.)
3. 16; 1+2+4+8=15;
   32; 1+2+4+8+16=31;
   64; 1+2+4+8=+16+32= 63;
   For powers of two, the sum of the proper factors is one less than the

Keys

value of the exponential expression. (Students may say to add value of previous power of two to previous sum of proper factors. Guidance is needed as in #1.)

4. 3.5 square units; 4 square units; The area of the triangle is one-half the area of the "related parallelogram."
5. 10 cubic units; 12 cubic units; 2/3
6. 28 square units; 32 square units; 4; Sketches will vary.
7. 63 square units; 72 square units; 9; Sketches will vary.
8. 56 cubic units; 64 cubic units; 8
9. Answers will vary.
10. Answers will vary.

Practice (p. 305)

1. L
2. A
3. E
4. I
5. D
6. J
7. H
8. K
9. F
10. G
11. B
12. C

Lesson Four

Practice (pp. 307-312)

1. 3; 5; 7; 9; 11
2. 1; 3; 5; 7; 9; $2n - 1$
3. 1; 16; 81; 256; 625
4. 3; 6; 9; 12; 15; $3n$
5. 95; 195; 295; 395; 495
6. $\frac{1}{4}$ or .25; $\frac{2}{4}$ or 0.5; $\frac{3}{4}$ or 0.75; 1; $\frac{5}{4}$ or $1\frac{1}{4}$ or 1.25; $n/4$
7. $\frac{1}{10}$ or 0.1; $\frac{2}{10}$ or 0.2; $\frac{3}{10}$ or 0.3; $\frac{4}{10}$ or 0.4; $\frac{5}{10}$ or 0.5
8. 10; 17; 24; 31; 38; $7n + 3$
9. 28; 35; 42; 49; 56
10. 1; 2; 3; 4; 5; $(n/4)4$

Practice (p. 313)

Answers will vary.

Practice (pp. 315-319)

Answers may vary. Students may provide expressions other than those provided here. If so, check their responses for validity.

1. $n+5$
2. $n-1$
3. $4n$
4. n^2
5. $3n+1$
6. $2n-2$
7. $10n$
8. $n/2$ or $\frac{1}{2}n$
9. n^3

Practice (p. 320)

Answers will vary.

Practice (pp. 321-322)

1. .15; 15%
2. $556.92
3. $1,092.00
4. $7,280.00
5. $5,361.08
6. 22.65%

Lesson Five

Practice (pp. 325-328)

1. Plan A: $15 + 0.50m$
   Plan B: $30 + 0.25m$
2. He should use Plan A. It would cost $20 for 10 minutes while Plan B would cost $32.50.
3. He should use Plan B. The cost for Plan A is $45.50 for 61 minutes, while for Plan B it is $45.25. Another example is that at 70 minutes the cost for Plan A is $50

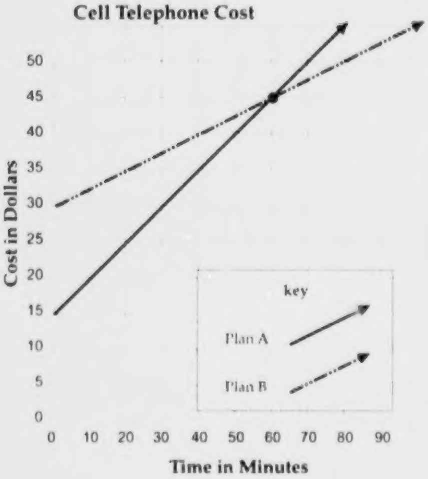
Keys

and for Plan B it is $47.50.

4. See table below.

Minutes of Usage	Plan A	Plan B
0	15	30
10	20	32.50
20	25	35
30	30	37.50
40	35	40
50	40	42.50
60	45	45
70	50	47.50

5. See graph below.



6. *m* (which stands for minutes) can be replaced in the rule (for either plan) with the number of minutes, and cost can be determined. Answers will vary but may include the following: The organization of the data in the table allows quick comparisons of cost; The graph provides a visual representation with one line representing the better plan for 0 to 60 minutes, the other line for more than 60 minutes, while the intersection of the two lines highlights the point at which both plans have the same cost; Answers will vary.

Practice (p. 329)

1. 1368 miles per day
2. 57 mph

Unit Assessment (pp. 95-99TG)

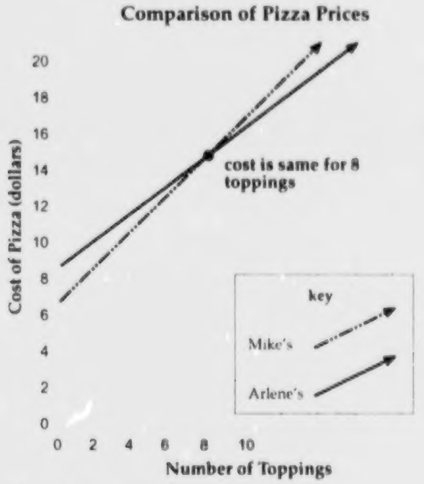
1. c
2. c
3. c
4. d
5. 20.25
6. $37.50; $43.75; The hourly rate of $6.25 can be multiplied by the number of hours worked or 6.25*n* where *n* represents the number of hours worked.
7. 7 hours
8. 7 + 1*t*; 9 + .75*t*; See table below.

Cost of Pizza

Number of Toppings	Mike's Pizza Place	Arlene's Pizza Palace
0	$7.00	$9.00
1	$8.00	$9.75
2	$9.00	$10.50
3	$10.00	$11.25
4	$11.00	$12.00
5	$12.00	$12.75
6	$13.00	$13.50
7	$14.00	$14.25
8	$15.00	$15.00
9	$16.00	$15.75

Keys

9. Mike's
10. Arlene's
11. See graph below. Key symbols will vary.



12. Answers will vary.

Scoring Recommendations for Unit Assessment

Item Number	Assigned Points
1	6 points
2	7 points
3	6 points
4	7 points
5	7 points
6	6 points for table; 6 points for explanation
7	7 points
8	6 points for algebraic expressions
9	6 points for table (if errors are present in table, credit may be given for analysis and graph if based on information in table)
10	6 points for analysis
11	6 points for graph
12	6 points for pattern; 6 points for table; 6 points for graph; 6 points for conclusions/conjectures
	Total = 100 points

Benchmark Correlations for Unit Assessment

Benchmark	Addressed in Items
D.1.3.1	3, 4, 5, 6
D.1.3.2	1, 2
D.2.3.1	8, 9, 10, 11, 12
D.2.3.2	7, 8, 9, 10, 11
A.3.3.3	7, 8
A.5.3.1	4, 5, 6
B.1.3.2	6, 7, 8

Unit 5: Probability and Statistics

This unit emphasizes how statistical methods and probability concepts are used to gather and analyze data to solve problems.

Unit Focus

Numbers Sense, Concepts, and Operations

- Understand the relative size of whole numbers and fractions. (A.1.3.2)
- Understand concrete and symbolic representations of rational numbers in real-world situations. (A.1.3.3)
- Understand and explain the effects of addition and multiplication on whole numbers. (A.3.3.1)
- Add, subtract, multiply, and divide whole numbers, decimals, and fractions to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to predict results and to check for reasonableness of results. (A.4.3.1)

Measurement

- Use concrete and graphic models to derive formulas for finding angle measures. (B.1.3.2)
- Solve problems involving units of measure and convert answers to a larger or smaller unit. (B.2.3.2)

Algebraic Thinking

- Create and interpret tables, graphs, and vertical descriptions to explain cause-and-effect relationships. (D.1.3.2)

Data Analysis and Probability

- Collect, organize, and display data in a variety of forms, including tables, charts, and bar graphs, to determine how different ways of presenting data can lead to different interpretations. (E.1.3.1)
- Understand and apply the concepts of range and central tendency (mean, median, and mode). (E.1.3.2)
- Analyze real-world data by applying appropriate formulas for measures of central tendency and organizing data in a quality display, using appropriate technology, including calculators and computers. (E.1.3.3)
- Compare experimental results with mathematical expectations of probabilities. (E.2.3.1)
- Determine the odds for and the odds against a given situation. (E.2.3.2)

Lesson Purpose

Lesson One Purpose

- Add and subtract whole numbers to solve real-world problems using appropriate methods of computing such as mental mathematics, paper and pencil and calculator. (A.3.3.3)
- Use concrete and graphic models to derive formulas for finding angle measures. (B.1.3.2)
- Understand the relative size of fractions. (A.1.3.2)
- Understand concrete and symbolic representations of rational numbers in real-world situations. (A.1.3.3)
- Collect, organize, and display data in tables. (E.1.3.1)
- Compare experimental results with mathematical expectations of probabilities. (E.2.3.1)

- Determine the odds for and the odds against a given situation. (E.2.3.1)

Lesson Two Purpose

- Add, subtract, multiply, and divide whole numbers to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Understand the relative size of whole numbers. (A.1.3.2)
- Organize and display data in bar graphs. (E.1.3.1)
- Understand and apply the concepts of range and central tendency (mean, median, and mode). (E.1.3.2)
- Analyze real-world data by applying appropriate formulas for measures of central tendency and organizing data in a quality display, using appropriate technology, including calculators and computers. (E.1.3.3)

Lesson Three Purpose

- Create and interpret tables, graphs, and verbal descriptions to explain cause-and-effect relationships. (D.1.3.2)
- Collect, organize, and display data in a variety of forms, including tables, charts, and bar graphs, to determine how different ways of presenting data can lead to different interpretations. (E.1.3.1)
- Understand and apply concepts of range and central tendency (mean, median, and mode). (E.1.3.2)
- Analyze real-world data by applying appropriate formulas for measure of central tendency and organizing data in a quality display, using appropriate technology, including calculators and computers. (E.1.3.3)

- Add, subtract, multiply, and divide whole numbers to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)

Lesson Four Purpose

- Understand and explain the effects of addition and multiplication on whole numbers. (A.3.3.1)
- Add, subtract, multiply, and divide whole numbers and decimals to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to predict results and to check for reasonableness of results. (A.4.3.1)
- Solve problems involving units of measure and convert answers to a larger or smaller unit. (B.2.3.2)
- Collect and organize data in a variety of forms including tables and charts. (E.1.3.1)
- Understand and apply the concepts of range and central tendency (mean, median, and mode). (E.1.3.2)
- Analyze real-world data by applying appropriate formulas for measures of central tendency. (E.1.3.3)
- Determine the odds for and the odds against a given situation or determine the probability of an event occurring. (E.2.3.2)

Suggestions for Enrichment

1. Have pairs of students (player A and player B) play the game *Rock, Scissors, Paper* 18 times, recording the wins for each player. Use a grid on the overhead projector to graph the wins of player A in one color and player B in another color. Have students determine the range, mode, and mean for each set of data and compare results.

Do a tree diagram to determine possible outcomes. Have students answer the following to determine if the game is fair.

- How many outcomes does the game have? (9)
- Label each possible outcome on the tree diagram as to a win for A, B, or tie.
- Count wins for A. (3)
- Find the probability that A will win in any round ($\frac{3}{9} = \frac{1}{3}$). Explain what probability means. (favorable outcomes/possible outcomes)
- Count wins for B. (3)
- Find the probability that B will win in any round. ($\frac{3}{9} = \frac{1}{3}$)
- Is the game fair? Do both players have an equal probability of winning in any round? (yes)

Have students compare the mathematical model with what happened when students played the game.

Now play the game again using three students and the following rules.

- A wins if all three hands are the same.
- B wins if all three hands are different.
- C wins if two hands are the same.

There will be 27 outcomes this time. Three to the third power (3^3) ($3 \times 3 \times 3 = 27$).

2. Discuss different ways you can order a hamburger. Have students choose three different toppings, discuss ways to find all the different combinations that could be ordered (e.g., list, picture, chart), and determine the number of different combinations. Have students estimate and then determine how many different ways a hamburger could be served if there were four toppings from which to choose.

3. Have students individually predict how many total eyelets there are in the students' shoes in class, without looking at other students' shoes. Tell students that there are usually 12 eyelets in one running shoe, about 24 in one hightop or boot, and some shoes do not have any. Have groups discuss their individual predictions. Tell students to count the eyelets in their group's shoes and then predict or estimate how many eyelets are in the class.

   Give each group a different color strip of construction paper. One inch will equal 100 eyelets. Ask the group to discuss their data and cut the strip to the length equivalent to their prediction. Have a member from each group glue their strip to your master graph on a poster board.

   Ask each group to tell you the total of eyelets in their own group. Total the figures to get an actual sum of eyelets in the classroom. Have groups discuss methods used for predictions. Discuss which method seemed to work the best. (Optional: graph types of shoes in the class or how many eyelets there are for each type of shoe.)

4. Give students pizza menus to look at to create a table and a circle graph on the classes' favorite one-topping pizza. Discuss how to efficiently gather the statistics for each student's favorite one-topping pizza and then gather the data. Have students write the data in fraction form and then calculate the percentage. Discuss how to determine this and how to transfer the information into degrees using a protractor. Ask students to create a chart of the fraction, decimal, percent, and degree equivalents for their circle graph. Have students share their graphs.

5. Have students look through newspaper or magazines to cut out examples of charts and graphs (e.g., circle, bar, line graphs). Interpret some of these in class.

6. To have students gain a conceptual understanding of the measure of central tendency, have an odd number of students stand and arrange themselves according to height. The height of the person in the middle is the median height. Repeat this activity with an even number of students. The median will be halfway between the heights of the two students in the middle. Have students define median in their own words.

If there are some students who are the same height, then the height that occurs most frequently is the mode. (It is possible that no two students will be the same height. It is also possible to have more than one mode.) Have students define mode in their own words.

Convert the height of the students to inches and then have students add heights and divide by the number of students in the sample to get the mean. Now have everyone except the tallest and shortest students in the group sit down. Measure the distance from the top of one of their heads to the top of the other person's head and guide students to tell you that subtraction can be used to find the range.

7. Have students estimate the circumference of the Earth and find the range, mean, median, and mode of class estimations.

8. Distribute small bags of M&Ms to each student. Before they open the bags, ask students to write down on a chart (see below) how many M&Ms they predict are in their bag (for a total and how many M&Ms they predict for each color). Have students open the bags, count the actual number, and write the total on the chart and on a sticky note.

Bag of M&Ms		
	Estimation Total	**Actual Total**
colors	**prediction**	**actual amount**
red		
orange		
yellow		
green		
blue		
brown		
total		

Construct a grid for a class line plot on the board (see below) and have students post the sticky notes of their total. Have students find the mode, median, mean, and range. Discuss range and extremes.

Class M&Ms

18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33

total number ____________ mean ____________

mode ____________ range ____________

median ____________

Use the chart for one color at a time. Ask students to count their total number of M&Ms by color, and write the number on a sticky note to post on a class line plot (see below). A separate chart will be used for each color.

Class M&Ms by Color

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

color ____________ median ____________

total of color ____________ mean ____________

mode ____________ range ____________

Have students find the mode, median, mean, and range for each color. Then find the ratio (as a fraction in lowest terms) (e.g., find the ratio of red to total, and so on) and percentage for each color and post answers on a class chart on the board (see below). Create a class circle graph with the percentages.

Class M&Ms per Color			
colors	**number per color**	**ratio per color** (as a fraction in lowest terms)	**percentage per color**
red			
orange			
yellow			
green			
blue			
brown			
total			

Note: Discuss with the students whether the median or mean should be used for determining ratios or percentages.

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Have students make a bar graph (see below) using the original data they collected upon opening their M&M bags.

Student Bags of M&Ms per Color

12						
11						
10						
9						
8						
7						
6						
5						
4						
3						
2						
1						
	red	orange	yellow	green	blue	brown

Ask students to use their bar graph information to convert each color to a ratio (as a fraction in lowest terms) and then a percentage (see below). Have students create circle graphs with the percentages.

Discuss whether the class or student charts should be used for best results.

Student Bags of M&Ms per Color			
colors	**number per color**	**ratio per color** (as a fraction in lowest terms)	**percentage per color**
red			
orange			
yellow			
green			
blue			
brown			
total			

Ask students to note the ratio of each color of M&M and predict the following.

- probability of selecting a particular color at random from a large bag
- number of each color they might find in a handful of 10 M&Ms
- number of each color they might find in a handful of 20 M&Ms

(Optional: Ask students why they think the makers of M&Ms make more of one color than another. Why there were periods of years that no red M&Ms were made? Do the same activity using various holiday candies at other times of the year or small packages of raisins for total number exercise only. Charts can be saved from each activity to make comparisons and predictions.)

9. Give pairs of students one measured tablespoon of uncooked rice to count and report findings to the class. Have each pair record the class data, arrange the data from least to greatest, and then calculate the range, mean, median, and mode.

   Have students use the class mean to compute the number of tablespoons in 1,000,000 grains using an equivalency table to convert tablespoons to a more appropriate measure. Then have students determine a suitable size of container for 1,000,000 grains of rice. Next have students compute the approximate number of grains of rice the average American eats per year if he or she consumes 16½ pounds or 34¼ cups of rice per year (weight and measure is for uncooked rice). (Optional: How would you determine the appropriate container to hold 1,000,000 pieces of popped popcorn?)

10. Have students gather and display data of interest to them (e.g., number of potatoes or onions in different packages of same weight; number of raisins in different brands of cereals with raisins; comparison of a person's foot length to a person's height for a number of people; comparison of a person's arm span and a person's height for a number of people; number of letters in names of students in the class).

11. Have students provide a listing of their own personal data (e.g., height in centimeters; weight in kilograms; age in months; gender; right or left handed; hair color; eye color; number of brothers; number of sisters; number of furry pets; number of other pets; number of cities lived in; monthly allowance; shoe size). Have students brainstorm ways to display data so that each member of the class will receive a complete listing of combined student data.

    Have students determine range, scale, and interval for each of the categories. (Introduce open-ended categories, e.g., such as hair color and gender). Ask students to create frequency tables which include the data intervals, the tally, and the number of frequency. Ask students to create line plots and/or double line plots (male/female) in height, weight, age, and shoe size. Have students determine the mean, median, and mode for all selected categories of data and which measure of central tendency best represents the data as it represents their class. In groups have students determine the best method to represent their statistical averages (e.g., bar graphs, pie graphs, pictograms, composite posters, model of an average student).

12. Have students collect and analyze data about right- and left-handedness. For 30 seconds have students use their right hand to put a check mark in each square of grid paper, as fast as possible. Have them count the number of checks. Then for 30 seconds, have students do the same with their left hand and count the checks.

    Record each student's checks for each hand on a table.

Right Hand / Left Hand		
name of student	**Number of Checks (Right Hand) (x)**	**Number of Checks (Left Hand) (y)**

Then have students graph the data by plotting the points, letting the x-axis represent the number of right-handed checks and the y-axis represent the number of left-handed checks. Draw the ambidextrous line, or $x=y$. What do the points above the line and below the line represent? What is the significance of points near the line? What percentage of the class is left-handed? Research what percentage of the population is left-handed.

(Optional: Count how many times you can jump on one foot before falling. Graph the data as before. Find a correlation between the data sets to evaluate whether right-handed people also tend to be right-footed. What about a correlation between left and right eye dominance?)

13. Have students conduct an experiment and measure the amount of time it takes for a hand squeeze to pass around a circle. Start with two students holding hands. When the time keeper says "now" the first person squeezes the hand of the second who then squeezes the other hand of the first person. The last person says "now" when he or she feels the hand squeeze come back to him or her. Record the data on a table with columns for number of students and number of seconds.

Number of Students	**Number of Seconds**
(x)	(y)
2	
4	
6	
8	
____	
____	
____	
____	

Add two more students to the circle and repeat the process. Continue until everyone has joined the circle. Ask students to try to pass the squeeze as quickly as they feel it. If someone messes up, it's okay to disregard that time and repeat the round.

Have students make a graph of the data by plotting the points, letting the x-axis represent students and the y-axis represent the number of seconds. Ask students the following: Should they connect the points to make a solid graph? Why or why not? Are the points scattered all around the plane or do the points tend to be a certain shape? Based on the data collected, how many seconds would it take to pass the hand squeeze around a circle of 100 people? How many minutes?

14. Have students use the Internet to record the high temperatures during a predetermined time period for selected cities. Have students construct a chart and find the mean, mode, median and range of the data.

15. Discuss the television show *Wheel of Fortune* or the game of Hangman in relation to these questions: Are there some letters that we use more than others? Are there some letters that we hardly use at all? Is there a mathematical rule that could improve chances of winning at these word games?

Have students choose a book or magazine to research the use of letters. Ask students to choose a page and a place at random and count off 300 letters. Tally the letters one at a time (without skipping around) filling out the table with columns for letter (A-Z), tally, total, and percentage. See table on following page.

Letter Tally

letter	tally	total	%
A			
B			
C			
D			
E			
F			
G			
H			
I			
J			
K			
L			
M			
N			
O			
P			
Q			
R			
S			
T			
U			
V			
W			
X			
Y			
Z			
grand total			100%

Add up the totals, which should come to about 300. Calculate (to one or two decimal places) the percentage probability of finding each letter. Check accuracy by adding up percents, which should total between 99 percent and 100 percent (allowing for rounding).

Have students complete the following.

Top 10 letters

1. ______ ______%
2. ______ ______%
3. ______ ______%
4. ______ ______%
5. ______ ______%
6. ______ ______%
7. ______ ______%
8. ______ ______%
9. ______ ______%
10. ______ ______%

Bottom five letters

22. ______ ______%
23. ______ ______%
24. ______ ______%
25. ______ ______%
26. ______ ______%

- How many vowels were in the top 10?
- Which consonants would be most useful in *Wheel of Fortune* or Hangman?

- Which vowel might be the least useful?
- What percentage of letters surveyed were vowels?
- Make up 10 different words using only the top five letters.

16. Have students research to obtain data on the cost of first-class postage and the year in which each price increased (or decreased). Ask students to create a scatter plot with time on the x-axis and price on the y-axis. Have students find the line of best fit taking recent trends into effect. (Mathematical software or a graphing calculator may be used to find curve of best fit.) Ask students to extend the line to the year 2020 and determine a corresponding cost.

17. Have your students choose one of the 50 states to find the population and area in square miles for each state on the Internet. Construct two class graphs, one for population with ranges of 100,000 and the other for areas with ranges of 100 square miles. Have students record their data. Using the class graphs, have students answer the following: Which state has the largest population? Smallest population? Largest area? Smallest area? Does the smallest state have the smallest population? Why or why not? Explain. Does the largest state have the largest population? Why or why not? Explain. Choose two states and determine how many people live in the states per square mile rounded to the nearest whole number (population/area).

18. Are exactly that number of people living in each square mile of the states you chose? Why or why not? Explain. Write two questions for other students to answer using the graphs.

19. Have students compare selected statistics for their county with other Florida counties or their city with other major United States cities.

20. Have students use the Internet to find a sports Web site giving current information about baseball teams and have them gather data on individual players of a particular team (e.g., game, at bats, runs, hits, and bases on balls.) Have students calculate batting

averages: hits/at bats; on base percentage: hits + walks + hits by pitch/at bats + walks; slugging percentage: hit + doubles + 2 (triples) + 3 (home runs)/at bats; winning percentage: wins/wins + losses; earned run average: 9 x earned runs allowed/innings pitched; strikeout to walk ratio: strikeouts/walks allowed. Depending on the season, this activity can be adapted to any sport.

(Optional: Have students select another sport and research how statistics are used in that sport. Make a chart comparing and contrasting the use of statistics in baseball compared to another selected sport.)

21. Have students use information on the Internet about sports league leaders to create a scattergram or scatterplot graph to find the correlations to the following.

    - Do players who weigh more hit more home runs?
    - Do players who are taller get more rebounds?
    - Do older players catch more passes?

    (Optional: Have students collect information on students and create a scatterplot graph to find correlations to the following: hand size/height; hat size/grade point average; time on phone each night/grade point average.)

22. Have students use the Internet to find the dimensions of various sport-playing areas (e.g., football field, tennis court). Have students record the data, compute area of each field, create a scattergram or scatterplot with width on the vertical axis and length on the horizontal axis, measure the dimensions of the field at your school and compute the area, then compare your school's dimensions to regulation dimensions.

    Have students use the same data to compare perimeter and create scale models of playing fields.

23. Show connections in the study of probability and statistics in math and applications in science, social studies, or other courses.

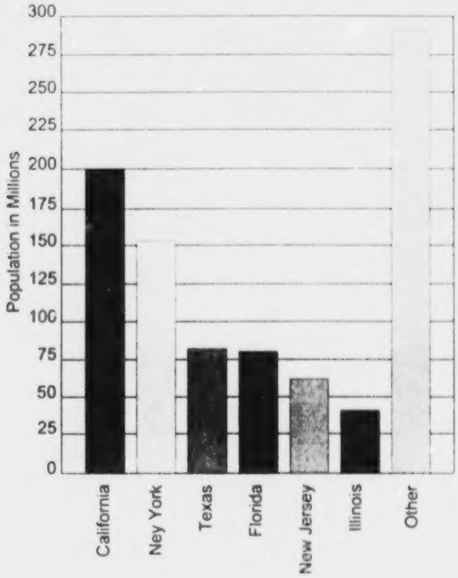
24. To review unit using a *Jeopardy* format, divide topics into five subtopics and students into five groups. Have each group write five questions and the answers with a specific colored marker on index cards and assign point values from easiest (100) to hardest (500). Ask students to tape cards on the board under their subtopic. The first group to finish taping cards goes first. Then go clockwise from group to group. When a subtopic and point value is chosen by the group, read the question. If correct, assign points; if incorrect, subtract points and put card back on the board. (Students may not pick any questions submitted by their group.)

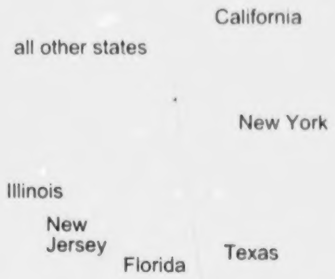
25. See Appendices for A, B and C for other instructional strategies, teaching suggestions, and accommodations/modifications.

Unit Assessment

Use the **bar graph** *and the* **circle graph** *below to answer the following.*

Places Immigrants Settled in United States in 1996





In 1996 the number of immigrations from all countries to the United States was reported to be 915,900. The six states in which most immigrants settled were California, New York, Texas, Florida, New Jersey, and Illinois. Data is displayed on the bar graph and circle graph above.

1. Which graph shows most clearly that more than one-half of the immigrants settled in the states of Florida, Texas, California, and New York? Explain why you chose the graph you did.

____

____

____

2. If you wanted to illustrate that around one-third of the immigrants settled in 44 states while about two-thirds of the immigrants settled in six states, which graph would you use? Why?

____

____

____

3. If the six states petitioned Congress for additional funds to assist immigrants and you represented one of the 44 states, which graph might you use to show the other 44 states deserve funding also? Why?

____

____

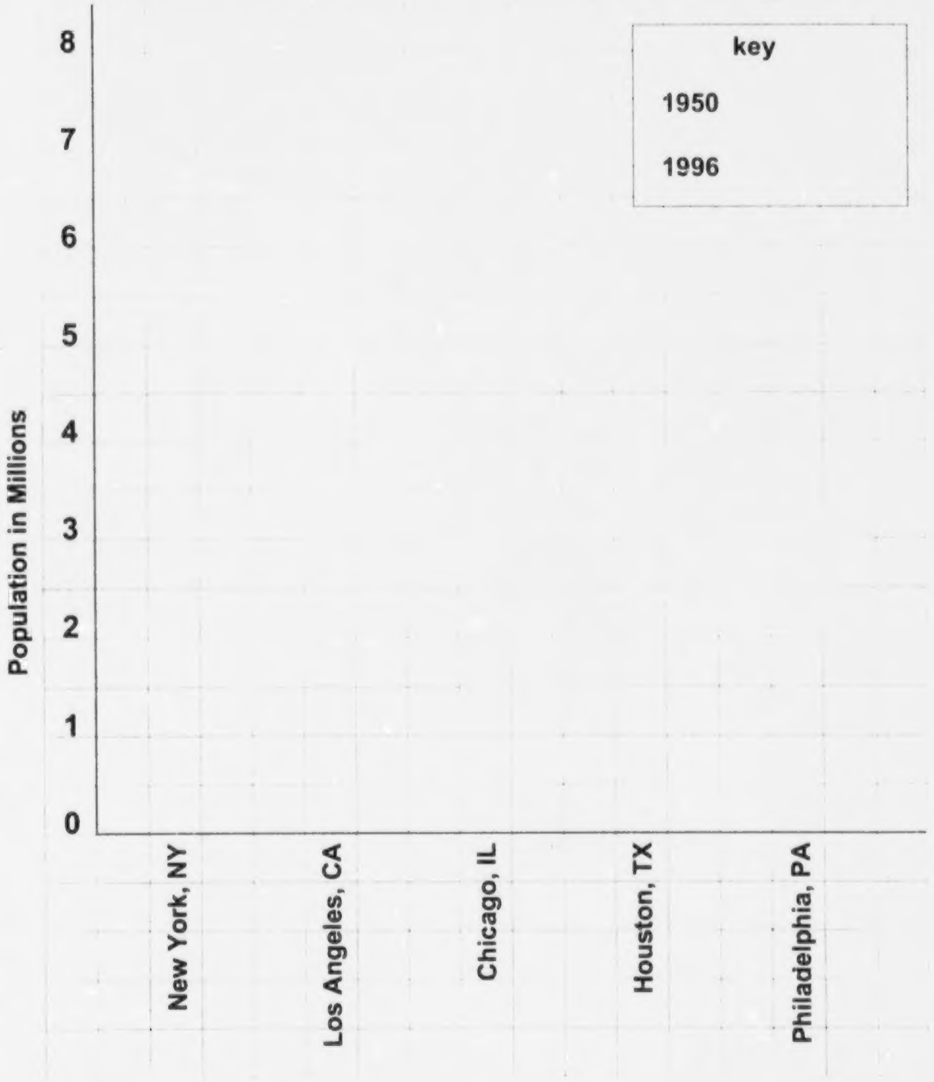
____

Use the directions below to answer the following.

The chart below shows the 1950 and 1996 populations of the five cities in the United States with the largest populations as of 1996.

Rank and City		Population in 1950	Population in 1996
1st	New York, NY	7,891,957	7,380,906
2nd	Los Angeles, CA	1,970,358	3,553,638
3rd	Chicago, IL	3,620,962	2,721,547
4th	Houston, TX	596,163	1,744,058
5th	Philadelphia, PA	2,071,605	1,478,002

4. A grid has been set up for you below to use to display the data from the previous page in a double bar graph illustrating the changes in each city's population.


Population Changes in Five Largest Cities in United States

5. The population of three cities decreased between 1950 and 1996.

Name them. ____

____

____

6. There was a population increase in two of the cities. Name the city

with the greater increase. ______________________________

7. The answers to questions 5 and 6 could be obtained from the chart

or the graph. Tell which you used and why. ______________

____

____

____

____

____

In the first lesson of this unit, you worked with a situation in which the sum of the dice might be used for choosing a member of the Chance family to take out the garbage.

8. Complete the table below of possible sums when two dice are rolled to use in answering the next set of questions.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4				
3						
4						
5						
6						

9. Determine the *theoretical probability* of getting each of the following sums:

a. P(sum of 2) = ______________________

b. P(sum of 3) = ______________________

c. P(sum of 4) = ______________________

d. P(sum of 5) = ______________________

e. P(sum of 6) = ______________________

f. P(sum of 7) = ______________________

g. P(sum of 8) = ______________________

h. P(sum of 9) = ______________________

i. P(sum of 10) = ____________________

j. P(sum of 11) = ____________________

k. P(sum of 12) = ____________________

10. If the probability of a sum is $\frac{5}{36}$ the odds of getting that sum would be ________ to ________. The odds of not getting that sum would be ________ to ________.

STOP! STOP! STOP! *Your answers to #9 and #10 must be* **judged correct by your teacher** *before you proceed to the next question.*

11. Hannah likes to roll dice and would like to create some sets of sums that would result in a fair activity for the three family members to determine responsibility for the weekly chore of garbage. She thinks these three sets of sums would work. Tell whether you agree or not and why.

Mr. Chance takes out the garbage if the sum is 2, 4, 5, or 9.

Mrs. Chance takes out the garbage if the sum is 3, 6, or 8.

Hannah takes out the garbage if the sum is 7, 10, 11 or 12.

This activity is ________ (fair, unfair) because ____________

____

____.

12. Hannah had her turn to try to determine a fair way. You now have yours. Find one or more sets of sums that would result in a fair activity. Remember that to be fair, the three family members' chances of being chosen must be equally likely.

Mr. Chance takes out the garbage if the sum is ____________.

Mrs. Chance takes out the garbage if the sum is ________________ .

Hannah takes out the garbage if the sum is ____________________ .

13. The table below includes names of the 42 men who have served as president of the United States. The chart also includes the number of letters in each name.

Presidents of the United States

President's Name	Number of Letters	President's Name	Number of Letters
George Washington	16	Grover Cleveland	15
John Adams	9	Benjamin Harrison	16
Thomas Jefferson	15	Grover Cleveland	15
James Madison	12	William McKinley	15
James Monroe	11	Theodore Roosevelt	17
John Quincy Adams	15	William Howard Taft	17
Andrew Jackson	13	Woodrow Wilson	13
Martin Van Buren	14	Warren G. Harding	14
William Henry Harrison	20	Calvin Coolidge	14
John Tyler	9	Herbert Hoover	13
James Knox Polk	13	Franklin Delano Roosevelt	23
Zachary Taylor	13	Harry S Truman	12
Millard Fillmore	15	Dwight D. Eisenhower	17
Franklin Pierce	14	John Fitzgerald Kennedy	21
James Buchanan	13	Lyndon Baines Johnson	19
Abraham Lincoln	13	Richard Milhous Nixon	19
Andrew Johnson	13	Gerald R. Ford	11
Ulysses S. Grant	13	James Earl Carter	15
Rutherford B. Hayes	16	Ronald Reagan	12
James A. Garfield	14	George Bush	10
Chester A. Arthur	14	William Jefferson Clinton	23

For the names of these 42 presidents:

The mean is 14.6.

The mode is 13.

The median is 14.

The range is 9 to 23.

Harry S Truman served from 1945 until 1953, and if we wanted to consider names of presidents for the last 50 years of the 20th century, we would need to start with him.

For that reason, the names and number of letters in the names for the last 10 presidents are in bold type.

Find the mean, mode, median, and range for our 10 presidents serving the last half of the 20th century.

The mean is ___________ .

The mode(s) is (are) ___________ .

The median is ___________ .

The range is ___________ to ___________ .

14. How does the length of your name compare with the typical length of our 42 presidents' names? ______________________________

____

How does it compare with the typical length of names of the presidents for the last half of the 20th century? ________________

____

15. Attach to this assessment the task assigned at the beginning of the unit. Be sure your collection of data displays includes your summary which explains how effective each data display is in communicating information.

Keys

Lesson One

Practice (pp. 342-343)

1. Answers will vary.
2. Answers will vary.
3. head or tails
4. head or tails
5. heads, tails; heads, heads; tails, heads; tails, tails
6. $\frac{1}{4}$; $\frac{1}{4}$; $\frac{2}{4}$
7. unfair; family members do not have equally likely chances

Practice (pp. 344-345)

1. Answers will vary.
2. Answers will vary.
3. 6; 1, 2, 3, 4, 5, 6
4. $\frac{1}{3}$
5. $\frac{1}{3}$
6. $\frac{1}{3}$
7. fair; family members have equally likely chances

Practice (pp. 346-348)

1. Answers will vary.
2. Answers will vary.
3. Answers will vary.
4. Answers will vary.
5. 120; 120; 120
6. $\frac{120}{360}$; $\frac{120}{360}$; $\frac{120}{360}$
7. $\frac{1}{3}$
8. 60; 60; 120
9. 40; 40; 40; 120
10. 20; 50; 50; 120
11. $\frac{120}{360}$; $\frac{120}{360}$; $\frac{120}{360}$
12. fair; family members have equally likely chances
13. fair; family members have equally likely chances

Practice (pp. 349-351)

1. Answers will vary.
2. Answers will vary.
3. See table below.

Sums Possible When Two Number Cubes or Dice Are Rolled

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11, roll again
6	7	8	9	10	11, roll again	12, roll again

4. 6; $\frac{6}{33}$
5. 15; $\frac{15}{33}$
6. 12; $\frac{12}{33}$
7. unfair; family members do not have equally likely chances

Practice (pp. 352-354)

1. Answers will vary.
2. Answers will vary.
3. See chart below.

Spinner	Second Spinner	Outcome
red	red or blue	red and red = red red and blue = purple
blue	red or blue	blue and red = purple blue and blue = blue
yellow	red or blue	yellow and red = orange yellow and blue = green

Keys

4. 6
5. 2; 2/6
6. 2; 2/6
7. 2; 2/6
8. fair; family members have equally likely chances

Practice (pp. 355-356)

Answers will vary.

Practice (p. 357)

1. 1 to 3
2. 2 to 2
3. 3 to 1
4. 2 to 2

Practice (p. 358)

1. B
2. G
3. C
4. I
5. F
6. E
7. A
8. D
9. H

Practice (pp. 359-360)

1. cube
2. face
3. congruent
4. protractor
5. angle
6. center of circle
7. degree
8. circle
9. sum
10. data
11. odds
12. ratio

Lesson Two

Practice (p. 362)

Column total: 180; 82; 180; 82

Practice (p. 365)

See chart below.

	Year-Round Calendar		Traditional Calendar	
Monthly	Number of Days Students In School	Number of Days Students Out of School*	Number of Days Students In School	Number of Days Students Out of School*
Mean	15	6.5	15	6.8
Mode	19	1	20	1
Median	18.5	3.5	18.5	3.5
Range	0-21	1-22	0-21	1-22

Practice (pp. 366-369)

1. False; 366 - 180 - 82 = 104
2. True
3. True
4. False; Year-round calendar has 36 school days in July and August.
5. True
6. True
7. False; Means are the same. Both calendars have 82 days out of school.

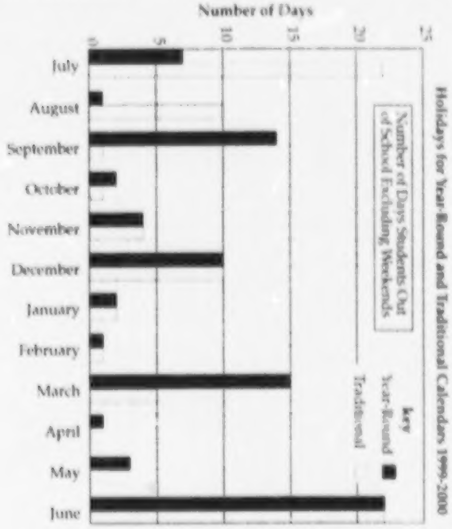
Keys

8. True
9. True
10. True

Practice (p. 372)

See the double bar graph below.

Holidays for Year Round and Traditional Calendar 1999-2000



Practice (pp. 373-375)

1. year round; there are more holidays in September to allow travel to games
2. traditional; there are more consecutive days out of school
3. traditional middle school child would be available to help with 1st grade sibling
4. year-round; there are more holidays in March to allow students to be at home to help with planting
5. year round; there are fewer consecutive days out of school
6. Answers will vary.
7. Answers will vary.

Practice (pp. 376-377)

1. display data
2. mean
3. mode
4. median
5. range (of a set of numbers)
6. difference
7. grid
8. axes
9. labels (for a graph)
10. scales
11. graph
12. percent

Practice (p. 378)

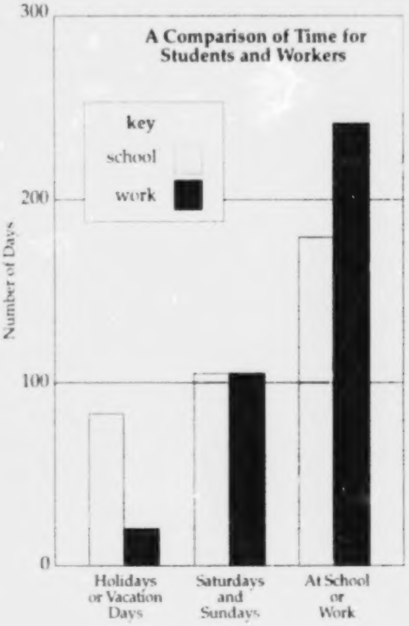
1. A
2. E
3. B
4. D
5. C

Keys

Lesson Three

Practice (p. 381)

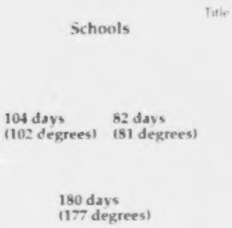
See the double bar graph below.

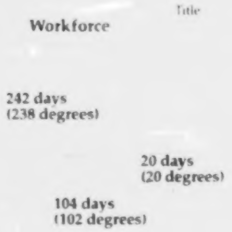


Practice (p. 384)

1. 102
2. 81

Practice (p. 385)





Practice (p. 386)

Answers will vary.

Practice (pp. 390-391)

1. 4; Nebraska; New Mexico; Utah; West Virginia
2. 2; Georgia; Virginia
3. 1 to 52; 51
4. 6; found the two center values in the list of 50 values; both were 6
5. 1; finding the most frequently occurring value
6. 8.7; dividing the total of 435 representatives by the 50 states
7. 4th; Texas; New York; California
8. was
9. does not
10. True

Lesson Four

Practice (pp. 393-399)

1. 10; 15; 5
2. 32,268,301; 163,707; 197; 197
3. 609,311; 656,424; 1; 1
4. 1,483; 88; 16.85; 1,450; 110; 13.18; 3.7
5. find sum of values and divide sum by the number of values; 30.8; order the values from smallest to largest and find center value or mean of center two values; 29; find value with greatest frequency in data set; 29; Answers will vary.

Keys

6. 3rd; 4th; 5th; 3rd; 4th; 5th; 4th; 5th; 5th; 10; 2; 1/10
7. subtracted 1847 from 1931; multiplied age in years by 12 to get age in months; 1008; 1
8. divided total distance by total time; 458
9. 18; 18; 36; 18/36; 18/36; true
10. See table below.

x	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

36; 27; 9; 27/36; 9/36; false

11. even; even; odd; odd; even; even

Practice (p. 400)

1. E
2. L
3. J
4. H
5. F
6. B
7. A
8. C
9. D
10. G
11. I
12. K

Practice (p. 401)

1. B
2. G
3. C
4. D
5. E
6. A
7. F

Practice (p. 402)

1. sum
2. theoretical probability
3. probability
4. odds
5. mean
6. median
7. range
8. mode
9. equally likely

Practice (p. 403)

1. E
2. B
3. C
4. D
5. F
6. A

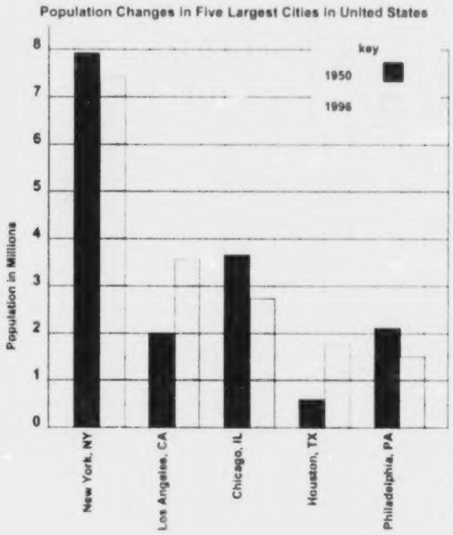
Unit Assessment (pp. 129-137TG)

1. Since the circle graph is based on 100 percent of immigrants, it more clearly shows that more than one-half of the immigrants settled in Florida, Texas, California, and New York.
2. The circle graph shows this more clearly. It is easy to see that the section labeled "all other states" is about one-third, while the six

Keys

sections representing the six individual states combine to cover about two-thirds of the circle.

3. The bar for "other states" is taller than the bars for the six states shown separately. It might make a visual impact and a statement such as "Look, we have immigrants who need assistance getting established, too!"
4. See double bar graph.



5. New York; Chicago; Philadelphia
6. Los Angeles
7. Answers will vary. Some students may prefer the chart, some the graph.

8. See table below.

Sums Possible When Two Number Cubes or Dice Are Rolled

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11, roll again
6	7	8	9	10	11, roll again	12, roll again

9. a. $P(2) = \frac{1}{36}$
   b. $P(3) = \frac{2}{36}$
   c. $P(4) = \frac{3}{36}$
   d. $P(5) = \frac{4}{36}$
   e. $P(6) = \frac{5}{36}$
   f. $P(7) = \frac{6}{36}$
   g. $P(8) = \frac{5}{36}$
   h. $P(9) = \frac{4}{36}$
   i. $P(10) = \frac{3}{36}$
   j. $P(11) = \frac{2}{36}$
   k. $P(12) = \frac{1}{36}$

10. 5 to 31; 31 to 5
11. fair; it gives each person a probability of $\frac{12}{36}$
12. Answers will vary. Any combination of sums that yields a probability of $\frac{12}{36}$ for each person is fair.

Keys

13. Mean = 15.9
    Mode = 12, 19
    Median = 16
    Range = 10 to 23
14. Answers will vary when the student compares the length of his or her name with typical lengths of names of presidents.
15. Display should include the following: one bar graph; one circle graph; one line plot or line graph, pictograph, or stem-and-leaf plot; table or chart; and probability of an event.

    A summary should be provided showing how effective the data displays are in communicating information.

Scoring Recommendations for Unit Assessment

Item Numbers	Assigned Points	Total Points
1, 2, 3, 6, 7	3	15 points
11, 14	4	8 points
5, 10, 12	6	18 points
13	10	10 points
9	11	11 points
4	13 (1 point per bar, 1 point for title, 1 point for each axis label)	13 points
15	25 (4 points per display and 5 points for summary)	25 points
8	0 (because answers to number 9 depend on number 8)	
		Total = 100 points

Benchmark Correlations for Unit Assessment

Benchmark	Addressed in Items
E.1.3.1	1, 2, 3, 4, 5, 6, 15
E.1.3.2	13
E.1.3.3	assessed with E.1.3.1 and E.1.3.2
E.2.3.1	8, 9, 10, 11, 12, 15
E.2.3.2	11
A.1.3.2	1, 5, 6, 7, 10, 14
A.1.3.3	1, 2, 3, 7, 8, 9
A.3.3.3	13

Appendices

Instructional Strategies

Classrooms draw from a diverse pool of talent and potential. The educator's challenge is to structure the learning environment and instructional material so that each student can benefit from his or her unique strengths. Instructional strategies adapted from the Curriculum Frameworks are provided on the following pages as examples that you might use, adapt, and refine to best meet the needs of your students and instructional plans.

Cooperative Learning Strategies—to promote individual responsibility and positive group interdependence for a given task.

Jigsawing: each student becomes an "expert" and shares his or her knowledge so eventually all group members know the content.

Divide students into groups and assign each group member a numbered section or a part of the material being studied. Have each student meet with the students from the other groups who have the same number. Next, have these new groups study the material and plan how to teach the material to members of their original groups. Then have students return to their original groups and teach their area of expertise to the other group members.

Corners: each student learns about a topic and shares that learning with the class (similar to jigsawing).

Assign small groups of students to different corners of the room to examine and discuss particular topics from various points of view. Have corner teams discuss various points of view concerning the topic. Have corner teams discuss conclusions, determine the best way to present their findings to the class, and practice their presentation.

Think, Pair, and Share: students develop their own ideas and build on the ideas of other learners.

Have students reflect on a topic and then pair up to discuss, review, and revise their ideas. Then have the students share their ideas with the class.

Debate: students participate in organized presentations of various viewpoints.

Have students form teams to research and develop their viewpoints on a particular topic or issue. Provide structure in which students will articulate their viewpoints.

Sequence of Activities—to develop understanding by progressing from new ideas through use of concrete manipulatives to an application of the concept using pictures, graphs, diagrams, or numerical representations, and ending with using symbols.

Have students explore concepts with concrete objects followed by pictorial representations of the concept, and then progress to symbolic lessons that include independent problem solving.

Use of Manipulatives—to introduce or reinforce a concept through observation of mathematical concepts in action.

Have students explore the meaning of the concept in a visual style and observe mathematical patterns, procedures, and relationships.

Drill and Practice Activities—to prompt quick recall. Some examples of "drill and practice" activities are skip counting, the number line, triangle flash cards, chalkboard drills, and basic fact games.

Have students practice algorithms and number facts a few at a time at frequent intervals. Timed tests are not recommended because they can put too much pressure on students and can cause them to become fearful and develop negative attitudes toward mathematics learning.

Projects—to prepare and deliver a presentation or produce a product over a period of time.

Have students choose a topic for a project that can be linked with a mathematical concept. The project can be in the form of a term paper, a physical model, a video, a debate, or a mathematically relevant art, music, or athletic performance.

Brainstorming—to elicit ideas from a group.

Have students contribute ideas related to a topic. Accept all contributions without initial comment. After the list of ideas is finalized, have students categorize, prioritize, and defend their contributions.

Free Writing—to express ideas in writing.

Have students reflect on a topic, then have them respond in writing to a prompt, a quotation, or a question. It is important that they keep writing whatever comes to mind. They should not self-edit as they write.

K–W–L (Know–Want to Know–Learned)—to provide structure for students to recall what they know about a topic, deciding what they want to know, and then after an activity, list what they have learned and what they still want or need to learn.

Before engaging in an activity, list on the board under the heading "What We Know" all the information students know or think they know about a topic. Then list all the information the students want to know about a topic under, "What We Want to Know." As students work, ask them to keep in mind the information under the last list. After completing the activity, have students confirm the accuracy of what was listed and identify what they learned, contrasting it with what they wanted to know.

Learning Log—to follow-up K–W–L with structured writing.

During different stages of a learning process, have students respond in written form under three columns:

"What I Think"
"What I Learned"
"How My Thinking Has Changed"

Interviews—to gather information and report.

Have students prepare a set of questions in interview format. After conducting the interview, have students present their findings to the class.

Dialogue Journals—to hold private conversations with the teacher or share ideas and receive feedback through writing (this activity can be conducted by e-mail).

Have students write on topics on a regular basis, responding to their writings with advice, comments, and observations in written conversation.

Continuums—to indicate the relationships among words or phases.

Using a selected topic, have students place words or phases on the continuum to indicate a relationship or degree.

Mini-Museums—to create a focal point.

Have students work in groups to create exhibits that represent areas of mathematics.

Models—to represent a concept in simplified form.

Have students create a product, like a model of Platonic solids, or a representation of an abstract idea, like an algebraic equation or a geometric relationship.

Reflective Thinking—to reflect on what was learned after a lesson.

Have students write in a journal the concept or skill they have learned, comment on the learning process, note questions they still have, and describe their interest in further exploration of the concept or skill. Or have students fill out a questionnaire addressing such questions as: Why did you study this? Can you relate it to real life?

Problem Solving—to apply knowledge to solve problems.

Have students determine a problem, define it, ask a question about it, and then identify possible solutions to research. Have them choose a solution and test it. Finally, have students determine if the problem has been solved.

Predict, Observe, Explain—to predict what will happen in a given situation when a change is made.

Ask students to predict what will happen in a given situation when some change is made. Have students observe what happens when the change is made and discuss the differences between their predictions and the results.

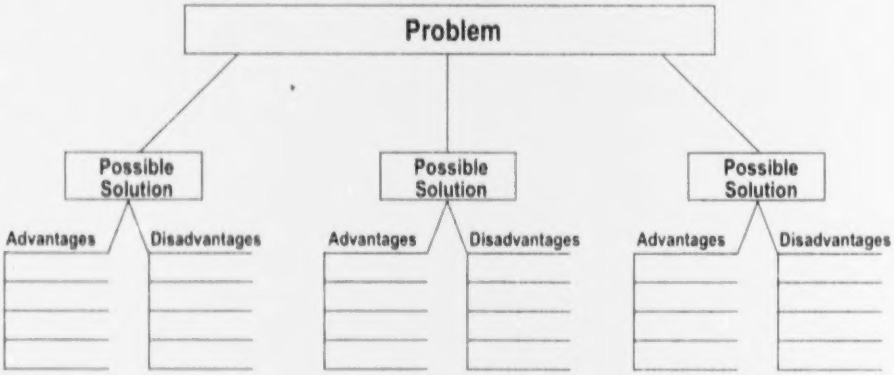
Literature, History, and Storytelling—to bring history to life through the eyes of a historian, storyteller, or author, revealing the social context of a particular period in history.

Have students locate books, brochures, and tapes relevant to a specific period in history. Assign students to prepare reports on the life and times of mathematicians during specific periods of history. Ask students to write their own observations and insights afterwards.

Graphic Organizers—to transfer abstract concepts and processes into visual representations.

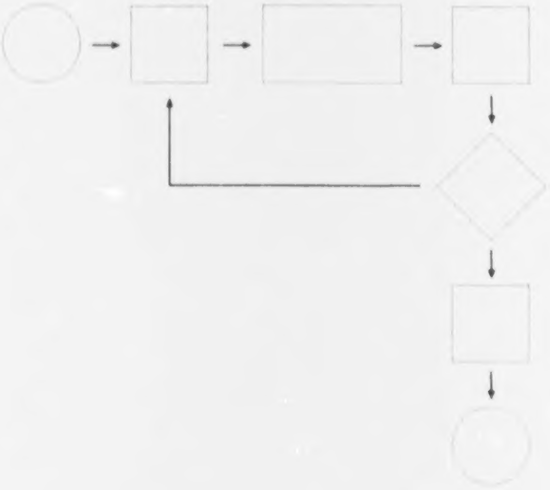
Consequence Diagram/Decision Trees: illustrates real or possible outcomes of different actions.

Have students visually depict outcomes for a given problem by charting various decisions and their possible consequences.


Problem
Possible Solution
Possible Solution
Possible Solution
Advantages
Disadvantages
Advantages
Disadvantages
Advantages
Disadvantages

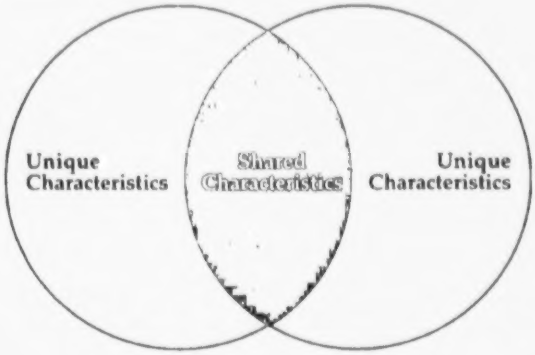
Flowchart: depicts a sequence of events, actions, roles, or decisions.

Have students structure a sequential flow of events, actions, roles, or decisions graphically on paper.



Venn Diagram: analyzes information representing the similarities and differences among, for example, concepts, objects, events, and people.

Have students use two overlapping circles to list unique characteristics of two items or concepts (one in the left part of the circle and one in the right); in the middle have them list shared characteristics.



Portfolio—to capture students' learning within the context of the instruction.

Elements of a portfolio can be stored in a variety of ways; for example, they can be photographed, scanned into a computer, or videotaped. Possible elements of a portfolio could include the following selected student products:

- solution to an open-ended question that demonstrates originality and unusual procedures
- mathematical autobiography
- teacher-completed checklists
- student-or teacher-written notes from an interview
- papers that show the student's correction of errors or misconceptions

- photo or sketch of student's work with manipulatives or mathematical models of multi-dimensional figures
- letter from student to the reader of the portfolio, explaining each item
- description by the teacher of a student activity that displayed understanding of a mathematical concept or relation
- draft, revision, and final version of student's work on a complex mathematical problem, including writing, diagrams, graphs, charts, or whatever is most appropriate
- excerpts from a student's daily journal
- artwork done by student, such as a string design, coordinate picture, scaled drawing, or map
- problem made up by the student, with or without a solution
- work from another subject area that relates to mathematics, such as an analysis of data collected and presented in a graph
- report of a group project, with comments about the individual's contribution, such as a survey of the use of mathematics in the world of work or a review of the uses of mathematics in the media

Learning Cycle—to engage in exploratory investigations, construct meanings from findings, propose tentative explanations and solutions, and relate concepts to our lives.

Have students explore the concept, behavior, or skill with hands-on experience and then explain their exploration. Through discussion, have students expand the concept or behavior by applying it to other situations.

Field Experience—to use the community as a laboratory for observation, study, and participation.

Before the visit, plan and structure the field experience with the students. Engage in follow-up activities after the trip.

Teaching Suggestions

The standards and benchmarks of the Sunshine State Standards are the heart of the curriculum frameworks and reflect Florida's efforts to reform and enhance education. The following pages provide samples of ways in which students could demonstrate achievements of benchmarks through the study of Mathematics.

Number Sense, Concepts, and Operations

1. Have students draw a picture and describe how an elevator could be used to explain integers to a friend. (MAA.1.3.1)

2. Have students use data displays to demonstrate an understanding of the relative size of fractions and percents and answer questions about a given problem. (MAA.1.3.2.a)

   Example:

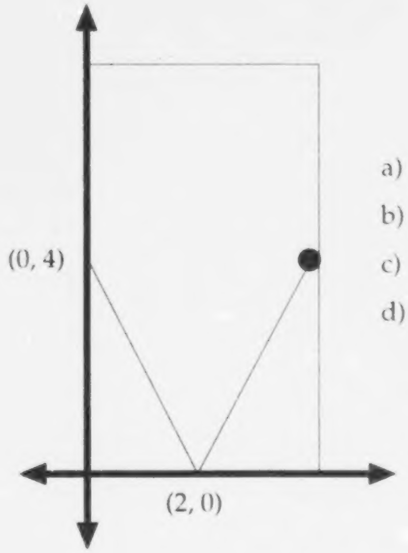
   In the pictograph below, the shaded sector represents the portion of time that an average teenager of Max Lewis Middle School spends on the phone on the weekends. About what fraction of the weekend do these teenagers spend on the phone? About what percent of the weekend do they spend talking on the phone?

3. Have students determine the greatest distance from the sun that each planet in the solar system reaches. Ask students to express the ratio of each distance to that of Earth using scientific notation. (MA.A.1.3.4.b)

4. Have students explain the need for absolute value in the contextual situation. (MA.A.1.3.3.a)

   Example:

   Below is the graph of the path of a pool ball when hit against the side of the table. Which equation accurately represents the line? Explain your choice.



a) $y = -2x + 4$
b) $y = 2x + 4$
c) $y = 2x - 4$
d) $y = |2x + 4|$

5. Have students represent numbers in a variety of ways and answer questions about a given problem. (MA.A.1.3.4.a)

   Example:

   Batting averages for baseball and softball players are reported as a three-digit decimal that is found by dividing the number of hits by the number of times at bat. If Chip has a batting average of .280 and has been at bat 25 times, how many hits does he have? What will his average be if he gets a hit on his next time at bat? What would a batting average of 1.000 mean? How many consecutive hits would he need to have a batting average of .500?

6. Have students use and explain exponential and scientific notation. (MA.A.2.3.1.a)

   Example:

   Write two examples of large numbers containing more than 2 non-zero digits correctly represented in scientific notation (such as distance to planets).

   Write two examples of very small numbers containing more than 2 non-zero digits correctly represented in scientific notation (such as atomic units).

   Explain why these numbers are best represented in scientific notation. Explain what the exponent represents in each.

   Sample Solution:

   4.56×10^{19}

   This number, not represented in scientific notation, would require 20 digits which would make it cumbersome to work with and to write. The exponent 19 means 4.56 multiplied by 10,000,000,000,000,000,000, which gives the equivalent whole number representation.

7. Have students determine and draw the five-bar code for the missing digit. (MA.A.2.3.2.a)

   Example:

   In 1963 the United States Postal Service began using five-digit zip codes in order to expedite its handling of the mail. In order for the codes to be read by scanners, each digit of our decimal system is represented by five bars. When a five-digit zip code is written, it begins and ends with a single long bar, called a framing bar. Use the following zip codes to match the bar codes with the digits 0-9:

66045

02138

35487

Which digit is missing above? Draw the code for the missing digit and explain the strategy you used to find it.

8. Have students use the formula

$$P = 1.2\,\frac{W}{H^2} \quad \text{where} \quad \begin{array}{l} P = \text{pounds per square inch} \\ W = \text{your weight in pounds} \\ H = \text{width of heel in inches,} \end{array}$$

to describe the effect on the pounds of pressure exerted when H increases or decreases. (MA.A.3.3.1.a)

9. Have students perform mathematical operations on the numbers 2, 4, 6, 8, and 10 to form today's date. Ask students to find either the day, the month and day, or the month, day, and year. (MA.A.3.3.2.a)

Example:

To find the date each number (2, 4, 6, 8, and 10) must be used, but can only be used once. For example, March 23, 1995, might be solved by finding 23, 323, or 32395, using the correct order of operations.

Sample Solution:

$$\begin{aligned} 23 &= 10 \times 8 \div 4 + 6 \div 2 \\ &\quad 80 \div 4 + 3 \\ &\quad\quad 20 + 3 \\ &\quad\quad\quad 23 \end{aligned}$$

10. Have student defend the correct application of the algebraic order of operations in a contextual situation. (MA.A.3.3.2.b)

    Example:

    Joel and Rachel want to put a fence around their yard. They know the formula for the perimeter of a rectangle is "two times the length plus the width." The yard is 180 feet long and 200 feet wide. Joel says they need 560 feet of fencing, saying "2 times 180 is 360 and 360 plus 200 is 560." Rachel disagrees, saying they need 760 feet of fencing. Pretend to be in Rachel's place and defend her answer.

11. Have students use the advertisement sections of a newspaper as a resource to write a descriptive plan to complete a shopping trip in a contextual situation. (MA.A.3.3.3.a)

    Example:

    James has a holiday budget of $150 and 6 family members for whom to buy gifts. James' mother will leave him at the mall at 12:00 noon and pick him up at 5:30 p.m. Given this situation, develop a schedule showing the maximum time that should be allotted for finding each gift and a purchase plan with costs that include a 6% sales tax.

12. Have students use rounding and concepts of common percents to estimate real quantities. Estimate and explain the answer given the following information: The average cost of housing in the Florida panhandle is 54.5% of the cost of housing in central Florida, and the average cost of housing in central Florida is $127,500. What is the average dollar cost of a house in the panhandle? Explain your process of estimation and answer. (MA.A.4.3.1.a)

    Solution:

    Because 54.5% is close to 50%, housing in the panhandle area would be close to half the cost in central Florida (rounded to $128,000), or approximately $64,000.

13. Have students use a model to justify common multiples. (MA.A.5.3.1.a)

    Example:

    A double strand of blinking holiday lights has a strand of red lights blinking every 9 seconds and a strand of green lights blinking every 15 seconds. Determine after how many seconds both strands will be on at the same time and justify the answer.

    Sample Solution:

    red-on at 9, 18, 27, 36, 45, 54 seconds
    green-on at 15, 30, 45, 60, 75, 90 seconds

    They will both be on at 45 seconds, the least common multiple of 9 and 15.

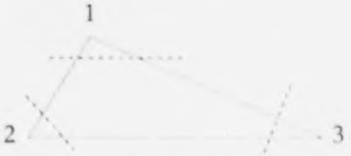
Measurement

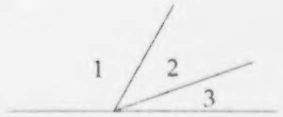
1. Have students use a graphic model to derive a formula for finding the volume of a three-dimensional model. Fold and cut graph paper to build two-dimensional shapes. Compare the area and perimeter of triangles, squares, and trapezoids that have the same base length and height and document these findings on a table with a written interpretation of the finding. With tape build three-dimensional solids using the two-dimensional models as faces. Predict the surface area, and then test predictions. Using graph paper as a guide, estimate the volume of each model. Work with a group to test estimations and reach a group consensus on a working formula for finding the volume of the three-dimensional models. (MA.B.1.3.1.a)

2. Have students use a cut out triangle to write the formula for the sum of the interior angles of a triangle. (MA.B.1.3.2.a)

    Example:

    Cut a triangle from a sheet of paper. Cut each of the three angles from the triangle and lay them so that they form adjacent angles. (See diagram on following page.)



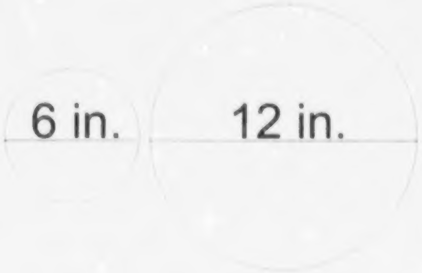


Compare results with results from other class members and determine a rule that applies to all triangles.

3. Have students determine and justify comparable pricing for different sizes of pizza. (MA.B.1.3.3.a)

   Example:

   The eighth grade is having a pizza sale. They have 2 sizes: 6-inch diameter and 12-inch diameter. A 6-inch pizza sells for $2.75. Determine the fair price for a 12-inch pizza and justify the answer.



4. Have students construct and use scale drawings. Scale a picture from a coloring book or greeting card by drawing a 2-cm by 2-cm grid on the picture. Create a 1-cm by 1-cm grid on plain paper and a 3-cm by 3-cm grid on a legal size manila folder. Duplicate the original picture one square at a time with a "key" showing the scale used. (MA.B.1.3.4.a)

5. Have students select appropriateness of direct or indirect measurement for given situation. (MA.B.2.3.1.a)

Example:

Determine whether placing an order for a living room carpet would require a direct or indirect measurement and explain why. Calculate the number of square yards of carpet needed if the room is 12 feet by 15 feet.

6. Have students compute reaction time in seconds, given the speed of a ball in miles per hour. (MA.B.2.3.2.a)

   Example:

   The pitcher of the high school baseball team has been clocked throwing the ball at 70 miles per hour. The distance from the pitcher's mound to home plate is 60 feet 6 inches. Determine, for that speed and that distance, how many seconds the batter has to react?

7. Have students create a line graph to represent a real-world problem. Construct a line graph depicting the energy output of a typical middle school student over a 24-hour period, which includes a school day and an afternoon soccer practice. Label the y-axis energy output in estimated calories and the x-axis time in hours. With the graph provide a written description of the activities. (MA.B.3.3.1.a)

8. Given a list of measurements, have students identify the appropriate measurement for each defined example. (MA.B.4.3.1.a)

Example:

Determine which of the following measures would be most appropriate for each of the described situations.

500 yd   461.6 cm   462 ft   460 m

a) The length of a photo to be framed.

b) The feet of fencing needed for the back yard.

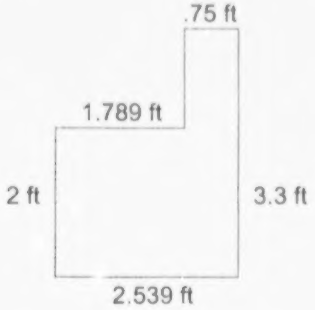
c) The distance to grandma's house.

d) The cloth needed to make costumes for the play.

9. Have students determine perimeter. (MA.B.4.3.1.b)

   Example:

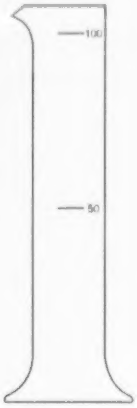
   Find the perimeter of the following figure. Discuss how precise the perimeter is and why.


.75 ft
1.789 ft
2 ft
3.3 ft
2.539 ft

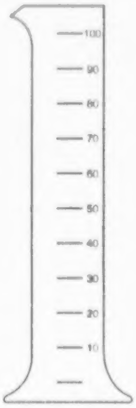
10. Have students choose appropriate graduated cylinder for precision of measurement required. (MA.B.4.3.1.c)

    Example:

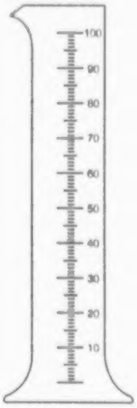
    Determine which of the following graduated cylinders would be best to accurately measure 4.23 ml and explain why.



a.


100
90
80
70
60
50
40
30
20
10

b.


100
90
80
70
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10

c.

11. Have students in small groups estimate the cost of a construction job, given the job's blueprints, specifications, material costs, and labor. (Teachers can request samples of this information from construction contractors. Students could be required to research the material and labor costs. The groups could participate in a bid process and discuss why group estimates vary.) (MA.B.4.3.2.a)

Geometry and Spatial Sense

1. Given a variety of regular polygons (triangle, square, pentagon, hexagon, etc.), have students investigate the relationship between the number of sides, and the number of diagonals of any regular polygon. Ask students to justify the relationship and support the conclusions. (MA.C.1.3.1.a)

2. Have students visually explore the geometric concepts of symmetry, reflections, congruency, similarity, perpendicularity, parallelism and transformations, including flips, slides, and turns. (MA.C.2.3.1.a)

   Example:

   Make squares from self-stick notes and draw the same three-color design. Explore the many designs that can be created by flipping, sliding, and turning one or more of the self-stick notes. Display a favorite design and use geometric terms to describe how it was created and what is being shown by combining the 4 pieces.

3. Have students solve a real-world problem given a context.

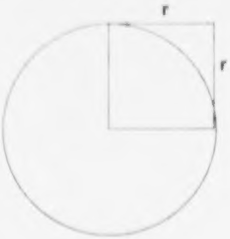
   Example:

   The local convenience store sells an average of 275 soft drinks each day. Over the course of one year, the manager noticed an imbalance in the income from soft drinks. Based on the information on the following page, determine which drink is priced incorrectly (A, B, C, or D) and justify the answer. (MA.C.3.3.1.a)

Volume = $\pi r^2 h$; use 3.14 for π

	radius (cm)	height (cm)	Price
A)	5	10	$ .24
B)	7	12	$ .72
C)	8	15	$ .92
D)	9	18	$1.39

4. Have students apply formulas for the area of one figure to approximate the area of another figure. (MA.C.3.3.1.b)



What is the best approximation for the area of a circle?

r^2 $2r^2$ $3r^2$ $4r^2$

Justify your answer.

5. As students enter the room, give each one an index card with an ordered pair on it. Ask students to choose seats as though the desks were a rectangular coordinate system. Have students then generate a list of the names of all students who represent different lines. (MA.C.3.3.2.a)

   For example: a vertical line,

   $y = 5$; $x + y = 4$; $x - y < 6$; and so on.

6. Have students explain to another student how to get from the classroom she or he is in to another location on the school grounds using symbols and Cartesian maps. (MA.C.3.3.2.b)

Algebraic Thinking

1. Have students determine and use patterns. (MA.D.1.3.1.a)

   Example:

   Answer the following questions:

X	Y
1	5
3	9
7	17
8	a
16	b
c	63
d	23
e	35

   1) What is being done to the numbers in the *X* column to get the numbers in the Y column?

   2) Describe the pattern you would use to find the numbers for "*a*" and "*b*."

   3) Use the pattern described in number 2 to find the numbers for "*c*," "*d*," and "*e*."

   4) Describe how you found *X* when you used your pattern in question number 3.

2. Have students graph and explain the growth of a population over time of a colony of organisms that doubles once a day. (MA.D.1.3.2.a)

3. Have students represent and solve a real-world problem with algebraic expressions. (MA.D.2.3.1.a)

   Example:

   Write an expression that would describe a book that is overdue by 10 days; by 15 days; by "*d*" days.

The school library overdue book fine is as follows:

First week	$1.00
After first week	$1.00 + $.25 a day

4. Have students use algebraic problem-solving strategies to solve real-world problems. (MA.D.2.3.2.a)

   Example:

   Describe the strategy or strategies used to answer the following question. Suppose there are 45 animals on the beach, some turtles and some pelicans. There are 104 legs on the beach. How many turtles and how many pelicans are on the beach?

Data Analysis and Probability

1. Have students participate in a class census. Ask students to pick from a list of topics such as favorite type of music, favorite fast food eatery, favorite professional sports team, etc. (Ideally there are as many topics as students in class.) Have students ask fellow students in class about the topic of his or her census. Record responses in an organized table. Ask students to represent the data in graph form and present it to the class with a written interpretation of what the graph shows about the class. (MA.E.1.3.1.a)

2. Have students collect temperatures every 30 minutes throughout a 12-hour period beginning at 8:00 a.m. and ending at 8:00 p.m. Ask students to find the measures of central tendency and write an argument that defends which measure best describes what the day was like or that states that none of the measures of central tendency alone fairly describe the temperature for the day. (MA.E.1.3.2.a)

3. Have students analyze and make predictions from collected data using calculators to apply formulas for measures of central tendency, and organize data in the form of charts, tables, or graphs. Have student use computer graphing software to organize collected data in a quality display. (MA.E.1.3.3.a)

Example:

Make a quality display of data on salaries from a company (from president to custodian). First find the mean, median, and mode. Then prepare a discussion of why these numbers accurately or inaccurately present the measure of central tendency. Show how eliminating the "extremes" affects the measures of central tendency.

4. Have students compare experimental results with mathematical expectations of probabilities in a context. (MA.E.2.3.1.a)

   Example:

   Have students compute the mathematical probability of two coins tossed in the air at the same time both landing with heads up. Next toss the two coins 20 times and record the results each time. Determine whether the experimental result matches the mathematical expectation and explain why it does or does not. If it does not, explain whether the experiment could be changed to get a better match.

   Example:

   Have students in small groups estimate the percentage of Earth's surface covered by each of the continents and by each of the major oceans, with other land and other water as additional categories. Have students toss and catch an inflatable globe 100 times, with the position of the index finger on the dominant hand recorded. Using statistical data for surface area of Earth and area of each land and water body listed, compute the percentage of Earth covered by each land or water body. Compare experimental probability to both estimation and mathematical expectations.

5. Have students determine all possible outcomes when tossing two differently colored number cubes numbered 1-6, and then determine the odds for and against tossing the cubes and having the sum of the two numbers shown equal 6. (MA.E.2.3.2.a)

Accommodations/Modifications for Students

The following accommodations/modifications may be necessary for students with disabilities and other students with diverse learning needs to be successful in school and any other setting. Specific strategies may be incorporated into each student's individual educational plan (IEP) or 504 plan, or academic improvement plan (AIP) as deemed appropriate.

Environmental Strategies

Provide preferential seating. Seat student near someone who will be helpful and understanding.
Assign a peer tutor to review information or explain again.
Build rapport with student; schedule regular times to talk.
Reduce classroom distractions.
Increase distance between desks.
Allow student to take frequent breaks for relaxation and small talk, if needed.
Accept and treat the student as a regular member of the class. Do not point out that the student is an ESE student.
Remember that student may need to leave class to attend the ESE support lab.
Additional accommodations may be needed.

Organizational Strategies

Help student use an assignment sheet, notebook, or monthly calendar.
Allow student additional time to complete tasks and take tests.
Help student organize notebook or folder.
Help student set timelines for completion of long assignments.
Help student set time limits for assignment completion.
Ask questions that will help student focus on important information.
Highlight the main concepts in the book.
Ask student to repeat directions given.
Ask parents to structure study time. Give parents information about long-term assignments.
Provide information to ESE teachers and parents concerning assignments, due dates, and test dates.
Allow student to have an extra set of books at home and in the ESE classroom.
Additional accommodations may be needed.

Motivational Strategies

Encourage student to ask for assistance when needed.
Be aware of possibly frustrating situations.
Reinforce appropriate participation in your class.
Use nonverbal communication to reinforce appropriate behavior. Ignore nondisruptive inappropriate behavior as much as possible.
Allow physical movement (distributing materials, running errands, etc.).
Develop and maintain a regular school-to-home communication system.
Encourage development and sharing of special interests.
Capitalize on student's strengths.
Provide opportunities for success in a supportive atmosphere.
Assign student to leadership roles in class or assignments.
Assign student a peer tutor or support person.
Assign student an adult volunteer or mentor.
Additional accommodations may be needed.

Presentation Strategies

Tell student the purpose of the lesson and what will be expected during the lesson (e.g., provide advance organizers).
Communicate orally and visually, and repeat as needed.
Provide copies of teacher's notes or student's notes (preferably before class starts).
Accept concrete answers; provide abstractions that student can handle.
Stress auditory, visual, and kinesthetic modes of presentation.
Recap or summarize the main points of the lecture.
Use verbal cues for important ideas that will help student focus on main ideas. ("The next important idea is....")
Stand near the student when presenting information.
Cue student regularly by asking questions, giving time to think, then calling student's name.
Minimize requiring the student to read aloud in class.
Use memory devices (mnemonic aids) to help student remember facts and concepts.
Allow student to tape the class.
Additional accommodations may be needed.

Curriculum Strategies

Help provide supplementary materials that student can read.
Provide *Parallel Alternative Strategies for Students* (*PASS*) materials.
Provide partial outlines of chapters, study guides, and testing outlines.
Provide opportunities for extra drill before tests.
Reduce quantity of material (reduce spelling and vocabulary lists, reduce number of math problems, etc.).
Provide alternative assignments that do not always require writing.
Supply student with samples of work expected.
Emphasize high-quality work (which involves proofreading and rewriting), not speed.
Use visually clear and adequately spaced work sheets. Student may not be able to copy accurately or fast enough from the board or book; make arrangements for student to get information.
Encourage the use of graph paper to align numbers.
Specifically acknowledge correct responses on written and verbal class work.
Allow student to have sample or practice test.
Provide all possible test items to study and then student or teacher selects specific test items.
Provide extra assignment and test time.
Accept some homework papers dictated by the student and recorded by someone else.
Modify length of outside reading.
Provide study skills training and learning strategies.
Offer extra study time with student on specific days and times.
Allow study buddies to check spelling.
Allow use of technology to correct spelling.
Allow access to computers for in-class writing assignments.
Allow student to have someone edit papers.
Allow student to use fact sheets, tables, or charts.
Tell student in advance what questions will be asked.
Color code steps in a problem.
Provide list of steps that will help organize information and facilitate recall.
Assist in accessing taped texts.
Reduce the reading level of assignments.
Provide opportunity for student to repeat assignment directions and due dates.
Additional accommodations may be needed.

Testing Strategies

Allow extended time for tests in the classroom and/or in the ESE support lab.
Provide adaptive tests in the classroom and/or in the ESE support lab (reduce amount to read, cut and paste a modified test, shorten, revise format, etc.).
Allow open book and open note tests in the classroom and/or ESE support lab.
Allow student to take tests in the ESE support lab for help with reading and directions.
Allow student to take tests in the ESE support lab with time provided to study.
Allow student to take tests in the ESE support lab using a word bank of answers or other aid as mutually agreed upon.
Allow student to take tests orally in the ESE support lab.
Allow the use of calculators, dictionaries, or spell checkers on tests in the ESE support lab.
Provide alternative to testing (oral report, making bulletin board, poster, audiotape, demonstration, etc.).
Provide enlarged copies of the answer sheets.
Allow copy of tests to be written upon and later have someone transcribe the answers.
Allow and encourage the use of a blank piece of paper to keep pace and eliminate visual distractions on the page.
Allow use of technology to check spelling.
Provide alternate test formats for spelling and vocabulary tests.
Highlight operation signs, directions, etc.
Allow students to tape-record answers to essay questions.
Use more objective items (fewer essay responses).
Give frequent short quizzes, not long exams.
Additional accommodations may be needed.

Evaluation Criteria Strategies

Student is on an individualized grading system.
Student is on a pass or fail system.
Student should be graded more on daily work and notebook than on tests (e.g., 60 percent daily, 25 percent notebook, 15 percent tests).
Student will have flexible time limits to extend completion of assignments or testing into next period.
Additional accommodations may be needed.

Correlation to Sunshine State Standards

Course Requirements for Mathematics 2
Course Number 1205040

These requirements include the benchmarks from the Sunshine State Standards that are most relevant to this course. The benchmarks printed in regular type are required for this course. The portions printed in *italic type* are not required for this course.

1. Demonstrate understanding and application of concepts about number systems.

Benchmarks		Addressed in Unit(s)	Addressed in Class on Date(s)
MA.A.2.3.1	Understand and use exponential and scientific notation.	1, 4	
MA.A.2.3.2	Understand the structure of number systems other than the decimal system.	1	
MA.A.5.3.1	Use concepts about numbers, including primes, factors, and multiples, to build number sequences.	1, 4	

2. Demonstrate understanding and application of a variety of strategies to solve problems.

Benchmarks		Addressed in Unit(s)	Addressed in Class on Date(s)
MA.A.3.3.1	Understand and explain the effect of addition, subtraction, multiplication, and division on whole numbers, and fractions, including mixed numbers and decimals, including the inverse relationship of positive and negative numbers.	1, 5	
MA.A.3.3.2	Select the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of rational numbers, ratios, proportions, and percents, including the appropriate application of the algebraic order of operations.	1, 3	
MA.A.3.3.3	Add, subtract, multiply, and divide whole numbers, decimals, and fractions, including mixed numbers, to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator.	1, 2, 3, 4, 5	
MA.A.4.3.1	Use estimation strategies to predict results and to check the reasonableness of results.	1, 3, 4, 5	
MA.B.2.3.1	Use direct (measured) and indirect (not measured) measures to compare a given characteristic in either metric or customary units.	2, 3	
MA.B.2.3.2	Solve problems involving units of measure and convert answers to a larger or smaller unit within either metric or customary units.	2, 5	
MA.D.2.3.1	Represent and solve real-world problems graphically, with algebraic expressions, equations, *and inequalities.*	3, 4	
MA.D.2.3.2	Use algebraic problem-solving strategies to solve real-world problems involving linear equations *and inequalities.*	4	

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Correlation to Sunshine State Standards

Course Requirements for Mathematics 2
Course Number 1205040

3. Estimate and measure quantities and use measures to solve problems.

Benchmarks		Addressed in Unit(s)	Addressed in Class on Date(s)
MA.A.4.3.1	Use estimation strategies to predict results and to check the reasonableness of results.	1, 3, 4, 5	
MA.B.1.3.1	Use concrete and graphic models to derive formulas for finding perimeter, area, surface area, circumference, and volume of two- and three-dimensional shapes, including rectangular solids and cylinders.	2, 3, 4	
MA.B.1.3.2	Use concrete and graphic models to derive formulas for finding rates, distance, time, and angle measures.	3, 4, 5	
MA.B.1.3.3	Understand and describe how the change of a figure in such dimensions as length, width, height, or radius affects its other measurements such as perimeter, area, surface area, and volume.	3	
MA.B.1.3.4	Construct, interpret, and use scale drawings such as those based on number lines and maps to solve real-world problems.	2	
MA.B.3.3.1	Solve real-world and mathematical problems involving estimates of measurements including length, time, weight/mass, temperature, money, perimeter, area, and volume in either customary or metric units.	2	
MA.B.4.3.1	Select appropriate units of measurement and determine and apply significant digits in a real-world context. (Significant digits should relate to *both instrument precision and to* the least precise unit of measurement.)	2	
MA.B.4.3.2	Select and use appropriate instruments, technology, and techniques to measure quantities in order to achieve specified degrees of accuracy in a problem situation	2	

Correlation to Sunshine State Standards

Course Requirements for Mathematics 2
Course Number 1205040

4. Describe situations either verbally or by using graphical, numerical, physical, or algebraic mathematical models.

Benchmarks		Addressed in Unit(s)	Addressed in Class on Date(s)
MA.A.1.3.1	Associate verbal names, written word names, and standard numerals with integers, fractions, and decimals; numbers expressed as percents; numbers with exponents; numbers in scientific notation; *radicals*; absolute values; and ratios.	1	
MA.A.1.3.2	Understand the relative size of integers, fractions, and decimals; numbers expressed as percents; numbers with exponents; numbers in scientific notation; *radicals*; absolute value; and ratios.	1, 5	
MA.A.1.3.3	Understand concrete and symbolic representations of rational numbers *and irrational numbers* in real-world situations.	1, 5	
MA.D.1.3.1	Describe a wide variety of patterns, relationships, and functions through models, such as manipulatives, tables, graphs, expressions, equations, *and inequalities*.	1, 2, 3, 4	
MA.D.1.3.2	Create and interpret tables, graphs, equations, and verbal descriptions to explain cause-and-effect relationships.	3, 4, 5	
MA.E.1.3.1	Collect, organize, and display data in a variety of forms, including tables, line graphs, charts, and bar graphs, to determine how different ways of presenting data can lead to different interpretations	5	

5. Demonstrate understanding, representation, and use of numbers in a variety of equivalent forms.

Benchmarks		Addressed in Unit(s)	Addressed in Class on Date(s)
MA.A.1.3.4	Understand that numbers can be represented in a variety of equivalent forms, including integers, fractions, decimals, percents, scientific notation, exponents, *radicals*, and absolute value.	1	

Correlation to Sunshine State Standards

Course Requirements for Mathematics 2
Course Number 1205040

6. Apply statistical methods and probability concepts in real-world situations.

Benchmarks		Addressed in Unit(s)	Addressed in Class on Date(s)
MA.A.4.3.1	Use estimation strategies to predict results and to check the reasonableness of results.	1, 3, 4, 5	
MA.E.1.3.2	Understand and apply the concepts of range and central tendency (mean, median, and mode).	1, 5	
MA.E.1.3.3	Analyze real-world data by applying appropriate formulas for measure of central tendency and organizing data in a quality display, using appropriate technology, including calculators and computers.	5	
MA.E.2.3.1	Compare experimental results with mathematical expectations of probabilities. (**Note:** Problems may include the use of manipulatives to obtain experimental results for simple and compound probabilities, comparison to mathematical expectations, and a discussion of the validity of the experiments.)	5	
MA.E.2.3.2	Determine the odds for and against a given situation.	5	

7. Use geometric properties and relationships.

Benchmarks		Addressed in Unit(s)	Addressed in Class on Date(s)
MA.C.1.3.1	Understand the basic properties of, and relationships pertaining to, regular and irregular geometric shapes in two and three dimensions.	3, 4	
MA.C.2.3.1	Understand the geometric concepts of symmetry, reflections, congruency, similarity, perpendicularity, parallelism, and transformations, including flips, slides, turns, and enlargements.	3	
MA.C.3.3.1	Represent and apply geometric properties and relationships to solve real-world and mathematical problems.	3	
MA.C.3.3.2	Identify and plot ordered pairs in all four quadrants of a rectangular coordinate system (graph) *and apply simple properties of lines.*	3, 4	

Glossary

The glossary adapted from the *Florida Curriculum Framework: Mathematics* is provided on the following pages for your use with the Sunshine State Standards and instructional practices.

absolute value	the number of units a number is from 0 on a number line. *Example*: The absolute value of both 4 and -4, written $|4|$ and $|4|$, is 4.
additive identity	the number zero
additive inverse	the opposite of a number *Example*: 19 and -19 are additive inverses of each other.
algebraic expression	a combination of variables, numbers, and at least one operation *Example*: $5x + 7$, $3t$, or $\frac{1}{2}(x - yz)$
algebraic order of operations	the order in which operations are done when performing computations on expressions • do all operations within parentheses or the computations above or below a division bar • find the value of numbers in exponent form; multiply and divide from left to right • add and subtract from left to right *Example*: 5 + 10 ÷ 2 - 3 x 2 is 5 + 5 - 6, or 10 - 6 which is 4.
analog time	time displayed on a timepiece having hour and minute hands.

associative property of addition	for all real numbers *a*, *b*, and *c*, their sum is always the same, regardless of how they are grouped *Example*: In algebraic terms: $(a + b) + c = a + (b + c)$; in numeric terms: $(5 + 6) + 9 = 5 + (6 + 9)$.
associative property of multiplication	for all real numbers *a*, *b*, and *c*, their product is always the same, regardless of how they are grouped *Example*: In algebraic terms: $(a \bullet b) \bullet c = a \bullet (b \bullet c)$; in numeric terms: $(5 \bullet 6) \bullet 9 = 5 \bullet (6 \bullet 9)$.
central tendency	a measure used to describe data *Example*: mean, mode, median
chance	the possibility of a particular outcome in an uncertain situation
commutative property of addition	two or more factors can be added in any order without changing the sum *Example*: In algebraic terms: $a + b + c = c + a + b = b + a + c$; in numeric terms: $9 + 6 + 3 = 6 + 3 + 9 = 3 + 9 + 6$.
commutative property of multiplication	two or more factors can be multiplied in any order without changing the product *Example*: In algebraic terms: $a \bullet b \bullet c = b \bullet c \bullet a = c \bullet b \bullet a$.

complex numbers	numbers that can be written in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$
composite number	a whole number that has more than two whole-number factors *Example*: 10 is a composite number whose factors are 1, 10, 2, 5.
concrete representation	a physical representation *Example*: graph, model
congruent	two things are said to be congruent if they have the same size and shape
customary system	a system of weights and measures frequently used in the United States *Example*: The basic unit of weight is the pound, and the basic unit of capacity is the quart.
digit	a symbol used to name a number *Example*: There are ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. In the number 49, 4 and 9 are digits.
digital time	a time displayed in digits on a timepiece
dilatation	the process of reducing and/or enlarging a figure

distributive property of multiplication over addition	multiplying a sum by a number gives the same results as multiplying each number in the sum by the number and then adding the products *Example*: In algebraic terms: $ax + bx = (a + b)\,x$ and $x\,(a + b) = ax + bx$; in numeric terms: $3 \bullet (4 + 5) = 3 \bullet 4 + 3 \bullet 5$.
equation	a mathematical sentence that uses an equals sign to show that two quantities are equal *Example*: In algebraic terms: $a + b = c$; in numeric terms: $3 + 6 = 9$.
equivalent forms	different forms of numbers, for instance, a fraction, decimal, and percent, that name the same number *Example*: ½ = .5 = 50%
estimate	an answer that is close to the exact answer *Example*: An estimate in computation may be found by rounding, by using front-end digits, by clustering, or by using compatible numbers to compute.
exponents (exponential form)	the number that indicates how many times the base occurs as a factor. *Example*: 2^3 is the exponential form of $2 \times 2 \times 2$, with 2 being the base and 3 being the exponent.

expression	a mathematical phrase that can include operations, numerals, and variables *Example*: In algebraic terms: $2l + 3x$; in numeric terms: 13.4 - 4.7.
factor	a number that is multiplied by another number to get a product; number that divides another number exactly *Example*: The factors of 12 are 1, 2, 3, 4, 6, 12.
fractal	a geometric shape that is self-similar and has fractional dimensions *Example*: Natural phenomena such as the formation of snowflakes, clouds, mountain ranges, and landscapes involve patterns. The pictorial representations of these patterns are fractals and are usually generated by computers.
function	a relationship in which the output value depends upon the input according to a specified rule *Example*: With the function $f(x) = 3x$, if the input is 7, the output is 21.
histogram	a bar graph that shows the frequency of data within intervals
identity property of addition	adding zero to a number does not change the number's value *Example*: $x + 0 = x$; $7 + 0 = 7$; $\frac{1}{2} + 0 = \frac{1}{2}$

identity property of multiplication	multiplying a number by 1 does not change the number's value *Example*: $y \bullet 1 = y$; $2 \bullet 1 = 2$
inequality	a mathematical sentence that shows quantities that are not equal, using $<$, $>$, $\leq$, $\geq$, or $\neq$
infinite	has no end or goes on forever
integers	the numbers in the set {... -4, -3, -2, -1, 0, 1, 2, 3, 4 ...}
inverse operations	operations that undo each other *Example*: Addition and subtraction are inverse operations. Multiplication and division are inverse operations. For instance, $20 - 5 = 15$ and $15 + 5 = 20$; $20 \div 5 = 4$ and $4 \times 5 = 20$.
inverse property of addition	the sum of a number and its additive inverse is 0 *Example*: $3 + -3 = 0$
inverse property of multiplication	the product of a number and its multiplicative inverse is 1 *Example*: In algebraic terms: For all fractions, a/b where $a, b \neq 0$, $a/b \times b/a = 1$; in numeric terms: $3 \bullet {}^{1}/_{3} = 1$; the multiplicative inverse is also called reciprocal.

irrational numbers	a real number that cannot be expressed as a repeating or terminating decimal *Example*: The square roots of numbers that are not perfect squares, for instance, $\sqrt{13}$; 0.121121112....
limit	a number to which the terms of a sequence get closer so that beyond a certain term all terms are as close as desired to that number
linear equation	an equation that can be graphed as a line on the coordinate plane
matrices	a rectangular array of mathematical elements (as the coefficients of simultaneous linear equations) that can be combined to form sums and products with similar arrays having an appropriate number of rows and columns
mean	the sum of the numbers in a set of data divided by the number of pieces of data; the arithmetic average
median	the number in the middle (or the averages of the two middle numbers) when the data are arranged in order
midpoint	the point that divides a line segment into two congruent line segments

mode	the number or item that appears most frequently in a set of data .
multiples	the numbers that result from multiplying a number by positive whole numbers *Example*: The multiples of 15 are 30, 45, 60, ...
natural (counting) numbers	the numbers in the set {1, 2, 3, 4,...}
number theory	the study of the properties of integers *Example*: primes, divisibility, factors, multiples
numeration	the act or process of counting and numbering
ordered pair	a pair of numbers that can be used to locate a point on the coordinate plane. *Example*: An ordered pair that is graphed on a coordinate plane is written in the form: (x-coordinate, y-coordinate), for instance, (8, 2).
operations	any process, such as addition, subtraction, multiplication, division, or exponentiation, involving a change or transformation in a quantity
patterns	a recognizable list of numbers or items

parallel lines	lines that are in the same plane but do not intersect
permutation	an arrangement, or listing, of objects or events in which order is important
perpendicular lines	two lines or line segments that intersect to form right angles
planar cross-section	the area that is intersected when a two-dimensional plane intersects a three-dimensional object
plot	to locate a point by means of coordinates, or a curve by plotted points, and to represent an equation by means of a curve so constructed
power	a number expressed using an exponent *Example*: The power 5^3 is read *five to the third power*, or *five cubed*.
prime	a number that can only be divided evenly by two different numbers, itself and 1 *Example*: The first five primes are 2, 3, 5, 7, 11.
probability	the number used to describe the chance of an event happening; how likely it is that an event will occur

proof	the logical argument that establishes the truth of a statement; the process of showing by logical argument that what is to be proved follows from certain previously proved or accepted propositions
proportion	an equation that shows that two fractions (ratios) are equal *Example*: In algebraic terms: $a/b = c/d,\ b \neq 0,\ d \neq 0$; in numeric terms: $\frac{3}{6} = \frac{1}{2}$, 3:6 = 1:2.
radical	an expression of the form $\sqrt[b]{a}$ *Example*: $\sqrt{68}$, $\sqrt[3]{27}$
range	the difference between the greatest number and the least number in a set of data; the set of output values for a function
ratio	a comparison of two numbers by division *Example*: The ratio comparing 3 to 7 can be stated as 3 out of 7, 3 to 7, 3 : 7, or $\frac{3}{7}$.
rational number	a number that can be expressed as a ratio in the form a/b where a and b are integers and $b \neq 0$ *Example*: $\frac{1}{2}$, $\frac{3}{5}$, -7, 4.2, $\sqrt{49}$
real numbers	the set of numbers that includes all rational and irrational numbers

rectangular coordinate system	a system formed by the perpendicular intersection of two number lines at their zero points, called the origin, and the horizontal number line is called the x-axis, the vertical number line is called the y-axis, and the axes separate the coordinate plane into four quadrants
recursive definition	a definition of sequence that includes the values of one or more initial terms and a formula that tells how to find each term of a sequence from previous terms
reflection	the figure formed by flipping a geometric figure about a line to obtain a mirror image
reflexive property	a number or expression is equal to itself *Example:* $a = a$, $cd = cd$
right triangle trigonometry	finding the measures of missing sides or angles of a triangle given the measures of the other sides or angles
rotation	a transformation that results when a figure is turned about a fixed point a given number of degrees
scale	the ratio of the size of an object or the distance in a drawing to the actual size of the object or the actual distance

scientific notation	a short-hand way of writing very large or very small numbers *Example*: The number is expressed as a decimal number between 1 and 10 multiplied by a power of 10, for instance, $7.59 \times 10^5 = 759,000$.
sequences	an ordered list of numbers with either a constant ratio (geometric) or a constant difference (arithmetic)
series	an indicated sum of successive terms of an arithmetic or geometric sequence
similar	objects or figures are similar if their corresponding angles are congruent and their corresponding sides are in proportion, and they are the same shape, but not necessarily the same size
surface area	the sum of the areas of all the faces of a three-dimensional figure
symmetry	the correspondence in size, form, and arrangement of parts on opposite sides of a plane, line, or point
tessellation	a repetitive pattern of polygons that covers an area with no holes and no overlaps *Example*: floor tiles

transformation	an operation on a geometric figure by which each point gives rise to a unique image *Example*: Common geometric transformations include translations, rotations, and reflections.
translation (also called a *slide*)	a transformation that results when a geometric figure is moved by sliding it without turning or flipping it, and each of the points of the figure move the same distance in the same direction
variable	a symbol, usually a letter, used to represent one or more numbers in an expression, equation, or inequality *Example*: In $5a$; $2x = 8$; $3y + 4 \neq 10$, a, x, and y are variables.
whole numbers	the numbers in the set $\{0, 1, 2, 3, 4, \ldots\}$

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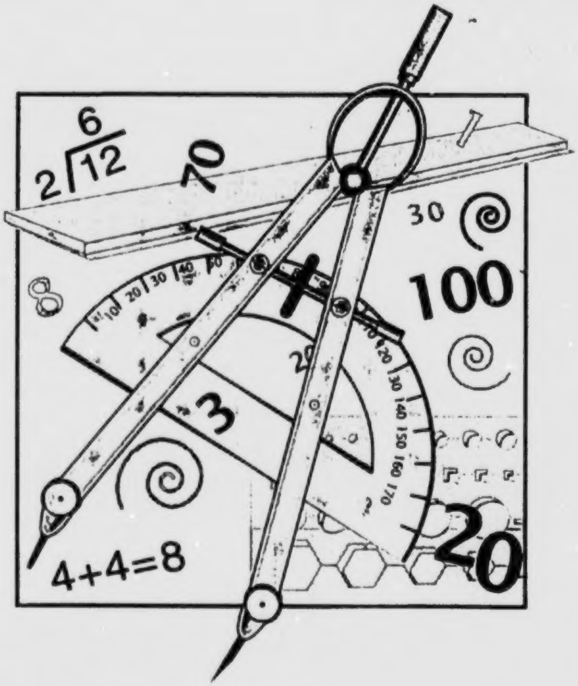

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Mathematics 2

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Mathematics 2

Course No. 1205040

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LEON COUNTY SCHOOLS

Exceptional Student Education

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Unit 1: Number Sense, Concepts, and Operations

This unit emphasizes how numbers and number operations are used in various ways to solve problems.

Unit Focus

Number Sense, Concepts, and Operations

- Associate verbal names, written words, and standard numerals with whole numbers, integers, and decimals; numbers with exponents; and numbers in scientific notation. (A.1.3.1)
- Understand relative size of integers, fractions, and decimals. (A.1.3.2)
- Understand concrete and symbolic representations of rational numbers in real-world situations. (A.1.3.3)
- Understand that numbers can be represented in a variety of equivalent forms, including fractions, decimals, and percents. (A.1.3.4)
- Understand and use exponential and scientific notation. (A.2.3.1)
- Understand the structure of number systems other than the decimal system. (A.2.3.2)
- Understand and explain the effects of addition, subtraction, multiplication, and division on whole numbers, fractions, mixed numbers, and decimals, including the inverse relationship of positive and negative numbers. (A.3.3.1)
- Select the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of rational numbers and percent, including the appropriate application of the algebraic order of operations. (A.3.3.2)

- Add, subtract, multiply, and divide whole numbers, decimals, and fractions to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to predict results and to check the reasonableness of results. (A.4.3.1)
- Use concepts about numbers, including primes, factors, and multiples. (A.5.3.1)

Algebraic Thinking

- Describe a wide variety of patterns and relationships. (D.1.3.1)

Data Analysis and Probability

- Understand and apply the concepts of range and central tendency (mean, median, and mode). (E.1.3.2)

Caterer or Chef

- determines how much of each ingredient is needed to purchase for recipes
- may need to change from customary units to metric units

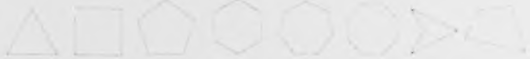
Vocabulary

Study the vocabulary words and definitions below.

absolute value	a number's distance from zero (0) on the number line *Example*: The absolute value of both 4, written $\lvert 4 \rvert$, and negative 4, written $\lvert -4 \rvert$, equals 4.
addends	numbers used in addition *Example*: In 14 + 6 = 20, 14 and 6 are addends.
chart	see *table*
composite number	any whole number that has more than two factors *Example*: 16 has five factors—1, 2, 4, 8, and 16.
decimal number	any number written with a decimal point in the number *Example*: A decimal number falls between two whole numbers, such as 1.5 falls between 1 and 2. Decimal numbers smaller than 1 are sometimes called decimal fractions, such as five-tenths is written 0.5.
denominator	the bottom number of a fraction, indicating the number of equal parts a whole was divided into *Example*: In the fraction $\frac{2}{3}$ the denominator is 3, meaning the whole was divided into 3 equal parts.
difference	the result of a subtraction *Example*: In 16 - 9 = 7, 7 is the difference.

dividend	a number that is to be divided by the divisor *Example*: In $7\overline{)42}$, $42 \div 7$, $\frac{42}{7}$, 42 is the dividend.
divisor	a number by which another number, the dividend, is divided *Example*: In $7\overline{)42}$, $42 \div 7$, $\frac{42}{7}$, 7 is the divisor.
estimation	the use of rounding and/or other strategies to determine a reasonably accurate approximation without calculating an exact answer
expanded form	a method of writing numbers using place value and addition *Example*: $324 = 300 + 20 + 4$ or $(3 \times 100) + (2 \times 10) + (4 \times 1)$
exponent (exponential form)	the number of times the base occurs as a factor *Example*: 2^3 is the exponential form of $2 \times 2 \times 2$. The numeral two (2) is called the *base*, and the numeral three (3) is called the *exponent*.
factor	a number or expression that divides exactly another number *Example*: 1, 2, 4, 5, 10, and 20 are factors of 20.
fraction	any numeral representing some part of a whole; of the form $\frac{a}{b}$ *Example*: One-half written in fractional form is $\frac{1}{2}$.

greatest common factor (GCF)	the largest of the common factors of two or more numbers *Example*: For 6 and 8, 2 is the greatest common factor.
improper fraction	a fraction that has a numerator greater than or equal to the denominator *Example*: $\frac{5}{4}$ or $\frac{3}{3}$ are improper fractions.
integers	the numbers in the set {..., -4, -3, -2, -1, 0, 1, 2, 3, 4,...}
irrational numbers	a real number that cannot be expressed as a ratio of two numbers *Example*: $\sqrt{2}$
least common multiple (LCM)	the smallest of the common multiples of two or more numbers *Examples*: For 4 and 6, 12 is the least common multiple.
mixed number	a number that consists of both a whole number and a fraction *Example*: $1\frac{1}{2}$ is a mixed number.
multiples	the numbers that result from multiplying a given number by the set of whole numbers *Example*: The multiples of 15 are 0, 15, 30, 45, 60, 75, etc.
numerator	the top number of a fraction, indicating the number of equal parts being considered *Example*: In the fraction $\frac{2}{3}$, the numerator is 2.

octagon	a polygon with eight sides
pattern (relationship)	a predictable or prescribed sequence of numbers, objects, etc.; also called a *relation* or *relationship*; may be described or presented using manipulatives, tables, graphs (pictures or drawings), or algebraic rules (functions) *Example*: 2, 5, 8, 11...is a pattern. The next number in this sequence is three more than the preceding number. Any number in this sequence can be described by the algebraic rule, $3n - 1$, by using the set of counting numbers for n.
percent	a special-case ratio in which the second term is always 100 *Example*: The ratio is written as a whole number followed by a percent sign, such as 25% which means the ratio of 25 to 100.
pi (π)	the symbol designating the ratio of the circumference of a circle to its diameter, with an approximate value of either 3.14 or $\frac{22}{7}$
polygon	a closed plane figure whose sides are straight and do not cross *Example*: triangle (3 sides), quadrilateral (4 sides), pentagon (5 sides), hexagon (6 sides), heptagon (7 sides), octagon (8 sides); concave, convex 
positive numbers	numbers greater than zero

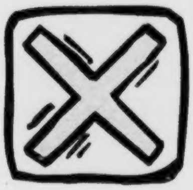
prime factorization	writing a number as the product of prime numbers *Example*: $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$
prime number	any whole number with only two factors, 1 and itself *Example*: 2, 3, 5, 7, 11, etc.
product	the result of a multiplication *Example*: In $6 \times 8 = 48$, 48 is the product.
proper fraction	a fraction that has a numerator less than the denominator; fractions that have a value greater than zero but less than one *Example*: $\frac{3}{5}$ is a proper fraction.
proportion	a mathematical sentence stating that two ratios are equal *Example*: The ration of 1 to 4 equals 25 to 100, that is $\frac{1}{4} = \frac{25}{100}$.
quotient	the result of a division *Example*: In $42 \div 7 = 6$, 6 is the quotient.
ratio	the quotient of two numbers used to compare two quantities *Example*: The ratio of 3 to 4 is $\frac{3}{4}$.
rational numbers	a real number that can be expressed as a ratio of two integers
real numbers	all rational and irrational numbers
relationship (relation)	see *pattern*

scientific notation	a shorthand method of writing very large or very small numbers using exponents in which a number is expressed as the product of a power of 10 and a number that is greater than or equal to one (1) and less than 10 *Example*: The number is written as a decimal number between 1 and 10 multiplied by a power of 10, such as $7.59 \times 10^5 = 759{,}000$. It is based on the idea that it is easier to read exponents than it is to count zeros. If a number is already a power of 10, it is simply written 10^{27} instead of 1×10^{27}.
standard form	a method of writing the common symbol for a numeral *Example*: The standard numeral for five is 5.
sum	the result of an addition *Example*: In $6 + 8 = 14$, 14 is the sum.
table (or chart)	an orderly display of numerical information in rows and columns
whole number	any number in the set {0, 1, 2, 3, 4...}

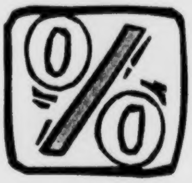
Unit 1: Number Sense, Concepts, and Operations

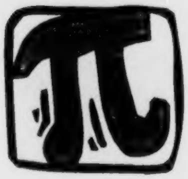
Introduction

A confident user of mathematics needs an understanding of numbers and operations. The user needs to understand what numbers mean, the relative size of numbers, various ways of representing numbers, and the effects of operating with numbers. Quick recall of single-digit addition combinations and the counterparts for subtraction, multiplication, and division is essential. Knowing and being able to use accurate methods to add, subtract, multiply, and divide are the foundations upon which problem solving is built.



In middle school there is a great deal of focus on fractions, decimals, and percents. Scientific notation is found to be an effective way to represent very small and very large numbers. Both negative numbers and positive numbers are found in problems to be solved. The use of a common **irrational number**, π (**pi**), is necessary when finding the circumference or area of a circle. Work with these aspects will be applied as measurement, geometry, algebraic thinking, probability, and statistics are studied in other units.

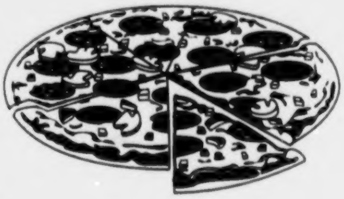




Lesson One Purpose

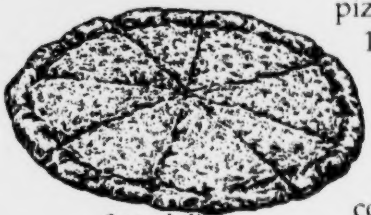
- Understand the relative size of fractions and decimals. (A.1.3.2)
- Understand concrete and symbolic representations of rational numbers in real-world situations. (A.1.3.3)
- Understand that numbers can be represented in a variety of equivalent forms, including fractions and decimals. (A.1.3.4)
- Understand and explain the effects of addition, subtraction, multiplication, and division on whole numbers, fractions, mixed numbers, and decimals, including the inverse relationship of positive and negative numbers. (A.3.3.1)
- Add, subtract, multiply, and divide decimals and fractions to solve problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)

Number Sense



6 equal slices

A friend of the writer of this unit was once asked how much pizza she usually eats. Her response was the following: "I always eat two slices." The writer and her math students began an investigation of the size of a slice of pizza in pizza restaurants in Tallahassee, Florida. A 12-inch pizza was cut in 6 slices at some places and in 8 slices at others. Variation also existed in larger and smaller pizzas.



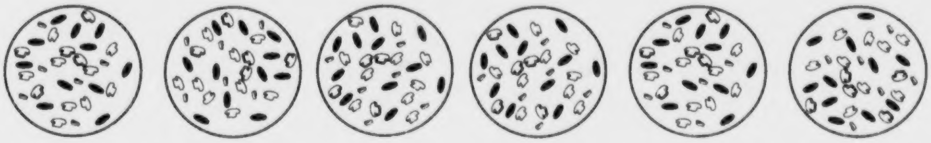
8 equal slices

A group of 12-inch pizzas cut into congruent parts when the number of parts vary from one pizza to another is a visual way to compare fractions.

Practice

Show how the following pizzas could be **sliced** *to* **represent the fractions** *below. Then* **arrange** *the list of* **fractions** *below from* **smallest to largest** *on the lines provided.*

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{3}$



1. ________

2. ________

3. ________

4. ________

5. ________

6. ________

Practice

The **least common multiple (LCM)** *of the* **denominators** *of the* **fractions** *on* **previous page** *is* **60** *because it is the* **smallest number** *that can be* **evenly divided by 2, 3, 4, and 5**. *Use* **60** *as the* **denominator** *for the* **fractions below** *to complete the following. The first problems are done for you.*

1. $\frac{1}{2} = \frac{?}{60}$ $\underline{\frac{30}{60}}$ *Example:* $\frac{1}{2} \times \frac{30}{30} = \frac{30}{60}$

2. $\frac{1}{3} = \frac{?}{60}$ ________

3. $\frac{1}{4} = \frac{?}{60}$ ________

4. $\frac{3}{4} = \frac{?}{60}$ ________

5. $\frac{1}{5} = \frac{?}{60}$ ________

6. $\frac{2}{3} = \frac{?}{60}$ ________

7. Arrange your answers for numbers 1 - 6 above from smallest to largest.

a. $\underline{\frac{1}{5}}$ = $\underline{\frac{12}{60}}$

b. ________ = ________

c. ________ = ________

d. ________ = ________

e. ________ = ________

f. ________ = ________

8. If your answers in number 7 verify your answers to the practice on previous page, continue. If not, reconsider your work in each question.

Practice

To find the **decimal number equivalent for a fraction**, *the* **numerator is divided by the denominator.** *Use paper and pencil, a calculator, or mental mathematics to* **find** *the* **decimal number equivalent for each fraction**. **Round** *to the nearest* **hundredths**. *The first problems are done for you.*

(paper and pencil) (calculator)

1. $\frac{1}{2}$ = .50 *Example:* $\begin{array}{r} .50 \\ 2\overline{)1.00} \end{array}$ $1 \div 2 = 0.5 = .50$

2. $\frac{1}{3}$ = ______

3. $\frac{1}{4}$ = ______

4. $\frac{3}{4}$ = ______

5. $\frac{1}{5}$ = ______

6. $\frac{2}{3}$ = ______

7. Arrange your answers for numbers 1- 6 above from smallest to largest.

a. $\frac{1}{5}$ = 0.20

b. ______ = ______

c. ______ = ______

d. ______ = ______

e. ______ = ______

f. ______ = ______

8. If this work verifies your answers to the practice on pages 11 and 12, continue. If not, reconsider your work in each question.

9. If you were asked to explain to another person how to *order* the set of fractions given in the practice on page 11, which of the three methods would you use from the practices on pages 11-13 for ordering fractions?

____

Explain why you prefer that method. ____________________

____

____

____

Practice

Use the list below to complete the following statements.

decimal number
denominator
fraction
least common multiple (LCM)
numerator

____________________ 1. any number written with a decimal point in the number

____________________ 2. any numeral representing some part of a whole

____________________ 3. the smallest of the common multiples of two or more numbers

____________________ 4. the bottom number of a fraction, indicating the number of equal parts a whole was divided into

____________________ 5. the top number of a fraction, indicating the number of equal parts being considered

Practice

Use the set of **fractions** *below to answer the following. Remember to change* **improper fractions** *into* **mixed numbers** *and write each answer in lowest terms.*

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{2}{3}, \frac{3}{4}$

Think about addition.

1. In adding any two of the fractions, what is the greatest **sum** possible?

   Explain how you got your answer. ________________________________

2. In adding any two of the fractions, what is the smallest *sum*

   possible? ________________________________

   Explain how you got your answer. ________________________________

Think about multiplication.

3. The product of two **positive numbers** greater than (>) 1 exceeds the value of either of the two **factors**.
   Example: 6 x 8 = 48; 48 > 6 and 48 > 8

   The product of two positive numbers greater than 0 but less than 1

   ________________________ (exceeds, does not exceed) the value of

   either of the two factors.

Think about subtraction.

4. In subtracting any two of the fractions, what is the greatest **difference** possible? ______________________________

Explain how you got your answer. ______________________

____

____

____

____

Circle the letter of the correct answer.

Think about division.

5. When dividing two **proper fractions**, the **quotient** ____________ .

   a. is always greater than the **dividend** and greater than the **divisor**

   b. is sometimes greater than the dividend and greater than the divisor

   c. is never greater than the dividend and greater than the divisor

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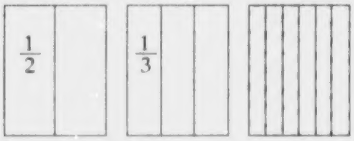
Practice

Complete the following showing your work. Write each answer in lowest terms. The first and last problems are done for you.

If the denominators of two fractions to be added are the same ($\frac{1}{5} + \frac{3}{5}$), we add the numerators and write the sum over the common denominator.

If the denominators are not the same ($\frac{1}{2} + \frac{1}{3}$), we must rename the fractions so they have a common denominator.



$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

or

$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

We can make lists of **multiples** of each denominator and identify the smallest common multiple in the list.

Multiples of 2: 2 4 6 8 10

Multiples of 3: 3 6 9 12

1. $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

2. $\frac{1}{2} + \frac{1}{4} =$ ________________

3. $\frac{1}{2} + \frac{1}{5} =$ ________________

4. $\frac{1}{2} + \frac{2}{3} =$ ________________

5. $\frac{1}{2} + \frac{3}{4} =$ ________________

6. $\frac{1}{3} + \frac{1}{4} =$ ________________

7. $\frac{1}{3} + \frac{1}{5} =$ ________________

8. $\frac{1}{3} + \frac{2}{3} =$ ________________

9. $\frac{1}{3} + \frac{3}{4} =$ ____________________

10. $\frac{1}{4} + \frac{1}{5} =$ ____________________

11. $\frac{1}{4} + \frac{2}{3} =$ ____________________

12. $\frac{1}{4} + \frac{3}{4} =$ ____________________

13. $\frac{1}{5} + \frac{2}{3} =$ ____________________

14. $\frac{1}{5} + \frac{3}{4} =$ ____________________

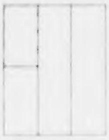
15. $\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12}$

16. In each of these problems, is the sum less than or greater than either of the **addends**? ____________________________________

17. If your answers in practice problems 1-15 verify your answers to practice problems 1-2 on page 16, continue. If not, reconsider your work.

Practice

Complete the following showing your work. Write each answer in lowest terms. The first and last problems are done for you.



$\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$

1. $\frac{1}{2} \times \frac{1}{3} =$ $\frac{1 \times 1}{2 \times 3} = \frac{1}{6}$

2. $\frac{1}{2} \times \frac{1}{4} =$ ____________________

3. $\frac{1}{2} \times \frac{1}{5} =$ ____________________

4. $\frac{1}{2} \times \frac{2}{3} =$ ____________________

5. $\frac{1}{2} \times \frac{3}{4} =$ ____________________

6. $\frac{1}{3} \times \frac{1}{4} =$ ____________________

7. $\frac{1}{3} \times \frac{1}{5} =$ ____________________

8. $\frac{1}{3} \times \frac{2}{3} =$ ____________________

9. $\frac{1}{3} \times \frac{3}{4} =$ ____________________

10. $\frac{1}{4} \times \frac{1}{5} =$ ____________________

11. $\frac{1}{4} \times \frac{2}{3} =$ ____________________

12. $\frac{1}{4} \times \frac{3}{4} =$ ____________________

13. $\frac{1}{5} \times \frac{2}{3} =$ ____________________

14. $\frac{1}{5} \times \frac{3}{4} =$ ____________________

15. $\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12} = \frac{1}{2}$

16. In each of the multiplication problems, is the product less than or

    greater than either of the factors? ____________________

    Would this be true if we were finding the product of a proper fraction and

    an improper fraction, such as $\frac{2}{3} \times \frac{9}{2} = 3$? ____________________

17. If your answers in practice problems 1-15 verify your answers to practice problem 3 on page 16, continue. If not, reconsider your work.

Practice

Match each definition with the correct term. Write the letter on the line provided.

_____ 1. a number or expression that divides exactly another number; 1, 2, 4, 5, 10, and 20 are _________ of 20.

_____ 2. numbers used in addition; in 14 + 6 = 20, 14 and 6 are _________ .

_____ 3. a number by which another number is divided; in $7\overline{)42}$, $42 \div 7$, $\frac{42}{7}$, 7 is the _________ .

_____ 4. a number that is to be divided; in $7\overline{)42}$, $42 \div 7$, $\frac{42}{7}$, 42 is the _________ .

_____ 5. the result of a division; in $42 \div 7 = 6$, 6 is the _________ .

_____ 6. the result of a subtraction; in 16 - 9 = 7, 7 is the _________ .

_____ 7. the result of a multiplication; in 6 x 8 = 48, 48 is the _________ .

_____ 8. the result of an addition; in 6 + 8 = 14, 14 is the _________ .

A. addends

B. difference

C. dividend

D. divisor

E. factor

F. product

G. quotient

H. sum

Practice

Complete the following showing your work. Write each answer in lowest terms. The first and last problems are done for you.

1. $\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$

2. $\frac{1}{2} - \frac{1}{4} =$ ____________________

3. $\frac{1}{2} - \frac{1}{5} =$ ____________________

4. $\frac{1}{3} - \frac{1}{4} =$ ____________________

5. $\frac{1}{3} - \frac{1}{5} =$ ____________________

6. $\frac{1}{4} - \frac{1}{5} =$ ____________________

7. $\frac{2}{3} - \frac{1}{2} =$ ____________________

8. $\frac{2}{3} - \frac{1}{3} =$ ____________________

9. $\frac{2}{3} - \frac{1}{4} =$ ____________________

10. $\frac{2}{3} - \frac{1}{5} =$ ____________________

11. $\frac{3}{4} - \frac{1}{2} =$ ____________________

12. $\frac{3}{4} - \frac{1}{3} =$ ____________________

13. $\frac{3}{4} - \frac{1}{4} =$ ____________________

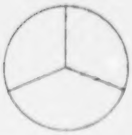
14. $\frac{3}{4} - \frac{1}{5} =$ ____________________

15. $\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$

16. If your answers in practice problems 1-15 verify your answers to practice problem 4 on page 17, continue. If not, reconsider your work.

Practice

Complete the following showing your work. Write each answer in lowest terms. The first and last problems are done for you.



$\frac{1}{2} \div \frac{1}{3} = 1\frac{1}{2}$

How many times is $\frac{1}{3}$ contained in $\frac{1}{2}$?

We can see that all of one $\frac{1}{3}$ and half of another $\frac{1}{3}$ are contained in $\frac{1}{2}$.

1. $\frac{1}{2} \div \frac{1}{3} = \underline{\frac{1}{2} \boxed{\times \frac{3}{1}} = \frac{3}{2} = 1\frac{1}{2}}$

invert and multiply

2. $\frac{1}{2} \div \frac{1}{4} =$ ____________________

3. $\frac{1}{2} \div \frac{1}{5} =$ ____________________

4. $\frac{1}{2} \div \frac{2}{3} =$ ____________________

5. $\frac{1}{2} \div \frac{3}{4} =$ ____________________

6. $\frac{1}{3} \div \frac{1}{4} =$ ____________________

7. $\frac{1}{3} \div \frac{1}{5} =$ ____________________

8. $\frac{1}{3} \div \frac{2}{3} =$ ____________________

9. $\frac{1}{3} \div \frac{3}{4} =$ ____________________

10. $\frac{1}{4} \div \frac{1}{5} =$ ____________________

11. $\frac{1}{4} \div \frac{2}{3} =$ ____________________

12. $\frac{1}{4} \div \frac{3}{4} =$ ____________________

13. $\frac{1}{5} \div \frac{2}{3} =$ ____________________

14. $\frac{1}{5} \div \frac{3}{4} =$ ____________________

15. $\frac{2}{3} \div \frac{3}{4} =$ ____________________

16. $\frac{1}{3} \div \frac{1}{2} =$ ____________________

17. $\frac{1}{4} \div \frac{1}{2} =$ ____________________

18. $\frac{1}{5} \div \frac{1}{2} =$ ____________________

19. $\frac{2}{3} \div \frac{1}{2} =$ ____________________

20. $\frac{3}{4} \div \frac{1}{2} =$ ____________________

21. $\frac{1}{4} \div \frac{1}{3} =$ ____________________

22. $\frac{1}{5} \div \frac{1}{3} =$ ____________________

23. $\frac{2}{3} \div \frac{1}{3} =$ ____________________

24. $\frac{3}{4} \div \frac{1}{3} =$ ____________________

25. $\frac{1}{5} \div \frac{1}{4} =$ ____________________

26. $\frac{2}{3} \div \frac{1}{4} =$ ____________________

27. $\frac{3}{4} \div \frac{1}{4} =$ ____________________

28. $\frac{3}{4} \div \frac{1}{5} =$ ____________________

29. $\frac{2}{3} \div \frac{1}{5} =$ ____________________

30. $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 1\frac{1}{8}$

Circle the letter of the correct answer.

31. When dividing two proper fractions, the quotient __________ .

    a. is always greater than the dividend and greater than the divisor

    b. is sometimes greater than the dividend and greater than the divisor

    c. is never greater than the dividend and greater than the divisor

32. In the practice pages 18-19, you were asked to find the sum of

    $\frac{1}{2} + \frac{1}{3}$ but you were not asked to find the sum of $\frac{1}{3} + \frac{1}{2}$.

    Explain why. ______________________________

33. In the practice pages 20-21, you were asked to find the product of

    $\frac{1}{2} \times \frac{1}{3}$ but you were not asked to find the product of $\frac{1}{3} \times \frac{1}{2}$.

    Explain why. ______________________________

Practice

Use the set of **decimal numbers** *below to answer the following questions.*

0.20 0.25 0.33 0.50 0.67 0.75

1. If any two of the decimal numbers are added, what is the greatest sum possible? ____________

2. If any two of the decimal numbers are added, what is the least sum possible? ____________

3. If any two of the decimal numbers are multiplied, is the product greater or smaller than either of the factors? ____________

4. If any two of the decimal numbers are subtracted, what is the greatest difference possible? ____________

Circle the letter of the correct answer.

5. When dividing two decimal numbers whose values are greater than 0 but less than 1, the quotient ____________ .
   a. is always greater than the dividend and greater than the divisor
   b. is sometimes greater than the dividend and greater than the divisor
   c. is never greater than the dividend and greater than the divisor

Practice

Draw **three circles, three squares,** *and* **three regular octagons**. *Then* **draw lines** *to* **divide each of the nine figures** *into* **four equal** *parts.*

If these drawings represented pizzas and you wanted to cut them in such a way that four people could share each one equally, how might you do it?

- You might cut one as you would expect the typical restaurant to cut it.
- You might cut another one as a creative chef trying to add interest to the pizza being served.

(**Remember:** A regular octagon is an 8-sided **polygon** with all sides equal and all angles equal.)

Practice

Use the list below to write the correct term for each definition on the line provided.

addends
decimal number
denominator
fraction
improper fraction
least common multiple (LCM)
mixed number
multiples
numerator
positive numbers
proper fraction
sum

__________________ 1. numbers used in addition

__________________ 2. any numeral representing some part of a whole

__________________ 3. the result of an addition

__________________ 4. any number written with a decimal point in the number

__________________ 5. the bottom number of a fraction, indicating the number of equal parts a whole was divided into

__________________ 6. the smallest of the common multiples of two or more numbers

__________________ 7. the top number of a fraction, indicating the number of equal parts being considered

__________________ 8. numbers greater than zero

__________________ 9. a fraction that has a numerator less than the denominator

__________________ 10. a fraction that has a numerator greater than or equal to the denominator

__________________ 11. a number that consists of both a whole number and a fraction

__________________ 12. the numbers that result from multiplying a given number by the set of whole numbers

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. a polygon with eight sides

______ 2. a number that is to be divided

______ 3. a closed plane figure whose sides are straight and do not cross

______ 4. the result of a division

______ 5. the result of a multiplication

______ 6. the result of a subtraction

______ 7. a number by which another number is divided

______ 8. a number or expression that divides exactly another number

A. difference

B. dividend

C. divisor

D. factor

E. octagon

F. polygon

G. product

H. quotient

Lesson Two Purpose

- Understand and use exponential and scientific notation. (A.2.3.1)
- Use concepts about numbers, including primes, factors and multiples. (A.5.3.1)
- Understand and explain the effects of multiplication on whole numbers and decimals, including inverse relationships. (A.3.3.1)
- Associate verbal names, written word names, and standard numerals with integers, fractions, and decimals; numbers expressed as percents; numbers with exponents; numbers in scientific notation; and absolute value. (A.1.3.1)
- Understand that numbers can be represented in a variety of equivalent forms, including integers, fractions, decimals, percents, scientific notation, exponents, and absolute value. (A.1.3.4)

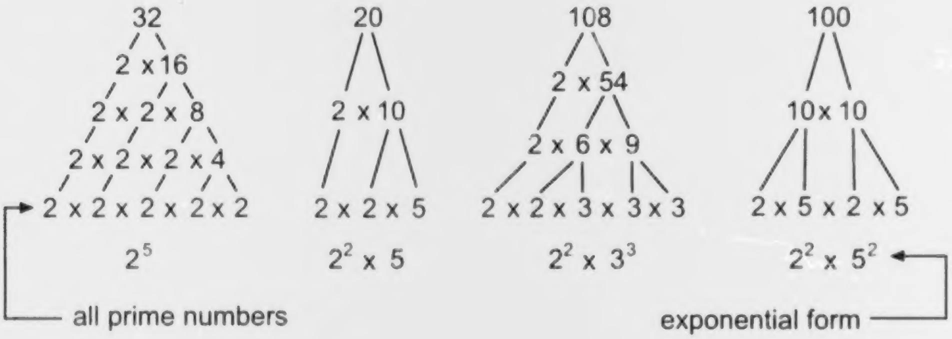
Writing Numbers

Numbers are often written in different ways for different purposes. The number 28,407 is written in **standard form**. The **expanded form** for a number can be written in several different ways. Being able to read, write, understand, and use numbers regardless of the form is important. This will be the focus of this lesson.

Factor Trees

Samuel was using *factor trees* to find the **prime factorization** of some **composite numbers** in his math class. He wrote each number as the product of **prime numbers**. Samuel knew that a prime number has exactly two factors, 1 and itself.

Samuel's factor trees looked like this:


32
2 x 16
2 x 2 x 8
2 x 2 x 2 x 4
2 x 2 x 2 x 2 x 2
2^5
20
2 x 10
2 x 2 x 5
2^2 x 5
108
2 x 54
2 x 6 x 9
2 x 2 x 3 x 3 x 3
2^2 x 3^3
100
10 x 10
2 x 5 x 2 x 5
2^2 x 5^2
all prime numbers
exponential form

(**Remember:** The numbers 0 and 1 are neither prime nor composite.)

Samuel liked using **exponents** when he had more than one *factor* of 2 or more than one factor of some other number. He understood that

$2 \times 2 \times 2 \times 2 \times 2$ has a value of 32
and that it can also be written as 2^5 or 2 to the 5th power,
meaning 2 is used as a factor 5 times.

Samuel knew that an *exponent* represents the number of times the *base* occurs as a factor. He understood that

2^5 is the **exponential form** of
$2 \times 2 \times 2 \times 2 \times 2$
and in 2^5, the numeral two (2) is called the *base*
and the numeral five (5) is called the *exponent*.

Expressions written with exponents are called *powers*.

Samuel decided to list the *prime factors* of 16 and 20. The **greatest common factor** (**GCF**) of the two numbers is the product of the *lesser* power of each *common* prime factor. He circled all pairs of the prime factors common to both. Then he multiplied to find the product of the common factors.

$16 = 2 \times 2 \times 2 \times 2 = 2^4$
$20 = 2 \times 2 \times 5 = 2^2 \times 5$

GCF of 16 and 20 = $2 \times 2 = 4$

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Samuel also knew he could find their least common multiple (LCM). The *least common multiple* is the product of the *greater* power of each *prime* factor that occurs in *either*. Zero is not prime and is not considered when finding the LCM.)

He circled all pairs of the prime factors common to both. Then he circled each remaining factor. Samuel then found the product of the prime factors using each *common* prime factor only *once*.

He listed the prime factorization of 16 and 20.

$$\begin{aligned} 16 &= \boxed{2 \times 2} \times \boxed{2} \times \boxed{2} = 2^4 \\ 20 &= \boxed{2 \times 2} \times \boxed{5} = 2^2 \times 5 \\ \text{LCM} &= 2 \times 2 \times 2 \times 2 \times 5 \\ &= 2^4 \times 5 \end{aligned}$$

LCM of 16 and 20 = $2^4 \times 5 = 80$

Other methods include the following:

$$\begin{aligned} 16 &= 2^4 \\ 20 &= 2^2 \times 5 \end{aligned}$$

The LCM is the product of the highest power of any factor in the prime factorizations.

$$2^4 \times 5 = 80$$

or

Multiples of 16: 16 32 48 64 80

Multiples of 20: 20 40 60 80

When listing multiples of two numbers, the first multiple appearing in both list is the LCM.

Practice

Find the **prime factorization** *of each of the following numbers. When writing your final answer, place the factors in order from* **smallest to largest** *and use* **exponents** *when one number appears as a* **factor two or more times.**

1. 24 = ____________________

2. 81 = ____________________

3. 125 = ____________________

4. 40 = ____________________

5. 42 = ____________________

Using your prime factorization answer from above, find the **greatest common factor** *of each of the following numbers.*

6. 24 and 40 = ____________________

7. 24 and 42 = ____________________

Find the **least common multiple** *of each of the following numbers.*

8. 24 and 40 = ____________________

9. 40 and 42 = ____________________

Practice

Find the **value** *of each of the following.*

1. 3^4 = __________

2. 10^3 = __________

3. 8^3 = __________

4. 4^3 = __________

5. 2^6 = __________

6. If you have access to different kinds of calculators, you might find it helpful to use one to find the values asked for in questions 1-5. Exploring how to use calculators and reading their manuals may help you find more than one function on the calculator useful. If so, explain which you prefer and why. ________________________

____

____

____

Standard and Expanded Forms of Numbers

The **chart** below shows how powers of 10 are related to *place value*. Each place value to the left is 10 times the place value to the right.

billions	hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones (or units)	tenths	hundredths	thousandths
1,000,000,000	100,000,000	10,000,000	1,000,000	100,000	10,000	1,000	100	10	1	0.1	0.01	0.001
10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	$\frac{1}{10^1}$	$\frac{1}{10^2}$	$\frac{1}{10^3}$

whole-number part ↑ | ↑ decimal point | ↑ decimal part

Samuel was writing numbers in expanded form in his math class and found that using exponents again saved him some work. Consider the following.

standard form	**expanded form**
3,269	= (3 x 1000) + (2 x 100) + (6 x 10) + (9 x 1)
	or
3,269	= (3 x 10^3) + (3 x 10^2) + (6 x 10^1) + (9 x 10^0)

5,123, 876, 455 = (5 x 1,000,000,000) + (1 x 100,000,000) + (2 x 10,000,000) + (3 x 1,000,000) + (8 x 100,000) + (7 x 10,000) + (6 x 1,000) + (4 x 100) + (5 x 10) + (5 x 1)

or

5,123,876,455 = (5 x 10^9) + (1 x 10^8) + (2 x 10^7) + (3 x 10^6) + (8 x 10^5) + (7 x 10^4) + (6 x 10^3) + (4 x 10^2) + (5 x 10^1) + (5 x 10^0)

(**Remember:** Any nonzero number to the zero power is 1.)

Practice

Use the directions above each section to answer the following.

The numbers below are written in **standard form**. *Write the* **expanded form** *for each of the following numbers in* **standard form** *and use* **exponents** *to show the* **powers of ten**.

1. 4,598,644 ______________________________

2. 2,000 ______________________________

3. 5,800,640 ______________________________

4. 24,000,000,000 ______________________________

The numbers below are shown in **expanded form**. *Write the* **standard form** *for each of the following.*

5. $(8 \times 10^5) + (3 \times 10^4) + (2 \times 10^3) + (6 \times 10^2) + (8 \times 10^1) + (6 \times 10^0)$

6. $(4 \times 10^6) + (1 \times 10^4) + (2 \times 10^2) + (5 \times 10^0)$

Numbers in the News

Samuel's teacher brought several newspaper articles to share with the class that illustrated another way numbers are sometimes written. Consider the following.

Instead of using $4,800,000 (standard form) to show four million, eight-hundred thousand dollars (word form), the article used $4.8 million.

The newspaper article used $4.8 million, instead of $4,800,000.

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Practice

Write the number in **standard form** *and its* **word name** *for the following.*

1. $16.2 million

2. $4.5 billion

Write the following numbers as a **newspaper** *might show them in a headline or article.*

3. $5,900,000

4. $8,300,000,000

Scientific Notation

ERROR
C
=
/
*
7
8
9
-
4
5
6
+
1
2
3
=
0
M
.

Samuel was exploring large numbers on a calculator he had used in elementary school. He entered 99,999,999 x 99,999,999. The calculator display said "Error." He tried it again and again and continued to get the same response. He got his scientific calculator and entered 99,999,999 x 99,999,999, and the calculator display showed 9.9999998E15. He thought, "This is wacky! One calculator gives me a message that says 'Error,' and the other makes an error doing the problem!" He took both calculators to school the next day, and his teacher helped him understand. The first calculator had the capacity to add, subtract, multiply, and divide, but it could not handle numbers of more than 8 digits. When it exceeded its capacity, it displayed an "error" message.

The second calculator was a scientific calculator equipped to handle large and small numbers through the use of **scientific notation**. It had not made an error but had used its shortened form of scientific notation. The calculator displayed 9.9999998E15, which is the calculator's shorthand for $9.\overline{9} \times 10^{15}$.

Scientists use scientific notation for numbers that are very small and numbers that are very large. The number is written as a product of a number between 1 and 10 and a power of 10.

1,234,000,000 would be written as 1.234×10^{9}. To help Samuel understand this, his teacher gave him a calculator and told him to find the following products. He was then to write some statements summarizing his observations.

$45 \times 1 = 45$

$45 \times 10 = 450$

$45 \times 100 = 4{,}500$

$45 \times 1{,}000 = 45{,}000$

$45 \times 10{,}000 = 450{,}000$

$45 \times 100{,}000 = 4{,}500{,}000$

$45 \times 1{,}000{,}000 = 45{,}000{,}000$

Practice

Use the information on the previous page to test each of Samuel's observations. Write **True** *if the observation is correct. Write* **False** *if the observation is not correct.*

1. The product of any number and 1 is the number itself.

2. When a **whole number** is multiplied by 10, the units digit in the product is zero.

3. When a whole number is multiplied by a *multiple* of 10 such as 100 or 1,000, the product begins with the digits of the original factor followed by the number of zeroes in the multiple of 10.
   (45 x 1,000 = 45,000; the product begins with 45 and is followed by 3 zeroes since 1,000 has 3 zeroes.)

4. If I multiplied 5,600 by 10,000, I would first write 5,600 and then follow it with 4 zeroes: 56,000,000.

Using a Scientific Calculator

Samuel's teacher then told him to do the following problems using his scientific calculator that had a key for *exponents*. He was to write some statements summarizing his observations.

$45 \times 10^{0} = 45$

$45 \times 10^{1} = 450$

$45 \times 10^{2} = 4{,}500$

$45 \times 10^{3} = 45{,}000$

$45 \times 10^{4} = 450{,}000$

$45 \times 10^{5} = 4{,}500{,}000$

$45 \times 10^{6} = 45{,}000{,}000$

Samuel's observations are on the following page.

Practice

Use the information on the previous page to test each of Samuel's observations. Write **True** *if the observation is correct. Write* **False** *if the observation is* not *correct.*

1. These are the same answers I got when I did the last set of problems.
2. If 45 x 1 is 45 and 45 x 10^0 is 45, then 10^0 must be equal to 1.
3. The value of 10^1 is 10 and 10 has one zero.
4. The value of 10^2 is 100 and 100 has two zeroes.
5. The value of 10^6 is 1,000,000 and 1,000,000 has six zeroes.
6. The value of 10^{10} would have 10 zeroes and would be 10,000,000,000 which is 10 million.

Decimals and Scientific Notation

Before Samuel's teacher could get back to check on him, he was ready to find some answers for himself. He wondered, "What happens when I multiply decimal numbers by 10, 100, etc.?"

2.34 x 1 = 2.34
2.34 x 10 = 23.4
2.34 x 100 = 234

When he multiplied by 1, the product was 2.34 as he expected. When he multiplied by 10, the decimal point moved one place to the right. When he multiplied by 100, the decimal point moved two places to the right. He then tried the following.

.00004 x 1 = .00004

.00004 x 10 = 0.0004

.00004 x 100 = 0.004

.00004 x 1,000 = 0.04

.00004 x 10,000 = 0.4

.00004 x 100,000 = 4

.00004 x 1,000,000 = 40

The decimal point moved 1 place to the right when the decimal number was multiplied by 10, the decimal point moved 2 places to the right when multiplied by 100 and so on.

.00004 x 10 = 0.0004	The decimal point moved 1 place to the right when the decimal number was multiplied by 10.
.00004 x 100 = 0.004	The decimal point moved 2 places to the right when the decimal number was multiplied by 100.
.00004 x 1000 = 0.04	The decimal point moved 3 places to the right when the decimal number was multiplied by 1000.

Since 10 is 10^1, the decimal point moves one place to the right. Since 100 is 10^2, the decimal moves two places to the right. Samuel saw a **pattern**. He also realized that moving the decimal one place to the right makes the number 10 times greater. Moving two places to the right makes the number 100 times greater.

The last piece of the puzzle fell into place when Samuel did the following calculation.

$2.59 \times 0.1 = 0.259$

$2.59 \times 0.01 = 0.0259$

$2.59 \times 0.001 = 0.00259$

$2.59 \times 0.0001 = 0.000259$

If

$10^3 = 1{,}000$ and

$10^2 = 100$ and

$10^1 = 10$ and

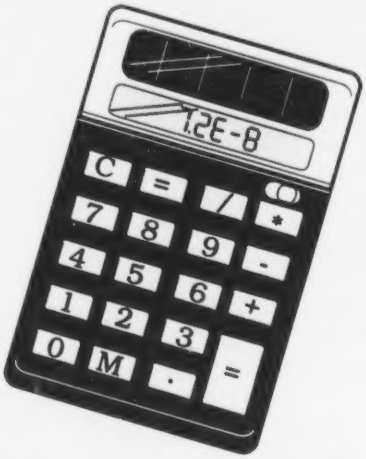
$10^0 = 1$

Does

$10^{-1} = 0.1$ or $\frac{1}{10}$?

Does

$10^{-2} = 0.01$ or $\frac{1}{100}$?



He tried these with his calculator, and it worked. Scientific notation was beginning to make sense.

He entered 0.00009 x 0.0008 =, and his calculator display read:

7.2E-8

In decimal form, this number is: .000000072

The next page shows how Samuel calculated this number with pencil and paper.

This is how Samuel did the problem with pencil and paper.

$$\begin{array}{r} .00009 \\ \text{x}\ \ .0008 \\ \hline 00072 \\ 00000 \\ 00000 \\ 00000 \\ \hline .000000072 \end{array}$$

If the decimal point in 7.2 is moved 8 places to the left (note the 8 is negative in the scientific notation format), the answer is the same as Samuel got when he used paper and pencil.

His teacher said the E-8 is his *calculator's shorthand* for scientific notation but that he should use the correct form when writing a number using scientific notation. So Samuel used the correct form for writing the number in scientific notation.

$$7.2 \times 10^{-8}$$

Samuel saw that numbers in scientific notation with negative exponents equal very small numbers and that numbers in scientific notation with positive exponents may equal very large numbers.

Practice

The following numbers are written in **scientific notation**. *Write them in* **standard form**.

1. 8.6×10^{8} ______________________________

2. 4.3×10^{6} ______________________________

3. 3.7×10^{9} ______________________________

4. 4.875×10^{5} ______________________________

5. 6.1544×10^{7} ______________________________

6. 4.1×10^{-5} ______________________________

7. 3.6×10^{-7} ______________________________

8. 5.567×10^{-6} ______________________________

Practice

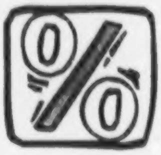
The following numbers are written in **standard form**. *Write them in* **scientific notation**.

(**Remember:** You need a number greater than 1 but less than 10 multiplied by a power of 10.)

1. 2,400,000 = 2.4 x ______________________
2. 75,000,000,000 = 7.5 x ______________________
3. 699,000 = 6.99 x ______________________
4. 4,348,000,000 = ______________________
5. 6,000,000 = ______________________
6. 0.875 = 8.75 x ______________________
7. 0.0095 = ______________________
8. 0.00000000863 = ______________________

Percent

Samuel's teacher had two more topics for him related to writing numbers. The first was **percent**. Samuel had some experience with percent. He knew that his community collects a 7 percent sales tax (which means 7 cents on the dollar). He also knew that when his family eats at a restauant, they usually leave a 15 percent tip for the server (15 cents on the dollar). This calculation might be done with mental mathematics, with paper and pencil, or with a calculator. He changed 7% to 0.07 and 15% to 0.15. He also knew how to write these as fractions.



$7\% = 0.07 = \frac{7}{100}$

$15\% = 0.15 = \frac{15}{100}$

(**Remember**: A percent is a special fraction with a denominator.)

Practice

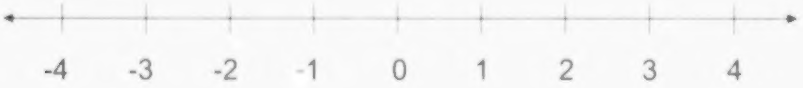
Complete the following **table** *writing the* **equivalent forms** *of the number.*

Percent	Decimal	Fraction
25%	0.25	$\frac{25}{100}$ or $\frac{1}{4}$
40%		
	0.10	
		$\frac{}{100}$ or $\frac{1}{2}$
75%		
	0.03	
		$\frac{}{100}$ or $\frac{1}{5}$
		$\frac{}{100}$ or $\frac{1}{3}$
90%		
	0.15	

Absolute Value

Samuel was almost on the home stretch. He had one more topic to master.

Absolute value indicates the *distance* a number is from zero. We know that 2 is 2 units from zero, and we also know -2 is two units from zero. If we think of a number line, we often think of 2 being 2 units to the right of zero and -2 being 2 units to the left of zero. If we think of a thermometer hanging outside a window, we think of 2 being two units above zero and of -2 being two units below zero. In mathematics, the placement of a vertical segment to the left and to the right of a number ( ) stands for the absolute value of the number or the distance the number is from zero.



For example:

$|4| = 4$

$|-4| = 4$

(**Remember:** 4 and -4 are four units from zero, so the absolute value of 4 is 4 and the absolute value of -4 is 4.)

Practice

Find the **absolute value** *of each of the following.*

1. |-18| = ______

2. | 2 | = ______

3. |400| = ______

4. |-900| = ______

5. |-621| = ______

Practice

Use the list below to write the correct term for each definition on the line provided.

absolute value
composite number
expanded form
exponent (exponential form)
greatest common factor (GCF)
pattern
percent
prime factorization
prime number
scientific notation
standard form
table
whole number

________________ 1. a special-case ratio in which the second term is always 100
Example: 25% means the ratio of 25 to 100

________________ 2. a method of writing the common symbol for a numeral
Example: five = 5

________________ 3. a number's distance from zero (0) on the number line
Example: both $|4|$ and $|-4| = 4$

________________ 4. writing a number as the product of prime numbers
Example: $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

________________ 5. a shorthand method of writing very large or very small numbers
Example: $7.59 \times 10^5 = 759{,}000$

________________ 6. a method of writing numbers using place value and addition
Example: $324 = 300 + 20 + 4$ or
$(3 \times 100) + (2 \times 10) + (4 \times 1)$

__________ 7. the number of times the base occurs as a factor
Example: $2^3 = 2 \times 2 \times 2$

__________ 8. any whole number with only two factors, 1 and itself
Example: 2, 3, 5, 7, 11,...

__________ 9. any whole number that has more than two factors
Example: 16 has five factors—1, 2, 4, 8, and 16.

__________ 10. the largest of the common factors of a pair of numbers
Example: $6 = \boxed{2} \times 3$ and
$8 = \boxed{2} \times 2 \times 2$, so
2 is the answer

__________ 11. a predictable or prescribed sequence of numbers, objects, etc.; also called a *relation* or *relationship*

__________ 12. any number in the set {0, 1, 2, 3, 4...}

__________ 13. an orderly display of numerical information in rows and columns

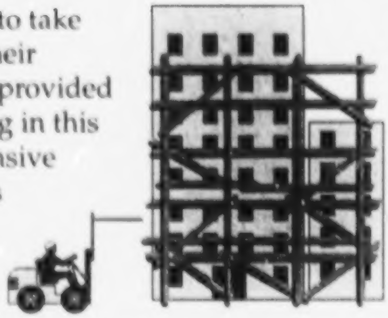
Lesson Three Purpose

- Understand the structure of number systems other than the decimal system. (A.2.3.2)
- Use concepts about prime numbers. (A.5.3.1)
- Select the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of rational numbers and percents. (A.3.3.2)
- Add, subtract, multiply, and divide whole numbers and decimals to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to predict results and to check the reasonableness of results. (A.4.3.1)
- Associate verbal names, written word names, and standard numerals with whole numbers and decimals. (A.1.3.1)
- Understand the relative size of integers and decimals. (A.1.3.2)
- Describe a wide variety of patterns and relationships. (D.1.3.1)
- Understand that numbers can be represented in a variety of equivalent forms, including fractions, decimals and percents. (A.1.3.4)
- Understand and apply the concepts of range and central tendency (mean, median, and mode). (E.1.3.2)

Everyday Problem Solving

In our everyday lives, mathematics is more likely to occur in a problem solving setting than to come to us as a page of 50 problems to be added, subtracted, multiplied or divided. Gaining experience in problem solving is beneficial whether the problem involves mathematics or not.

Construction workers often use scaffolding to take them from lower levels to higher levels in their building process. Some scaffolding is being provided to support you as you begin problem solving in this book. The taller the building, the more extensive the scaffolding. The same will be true in this unit, the tougher the problem, the more extensive the scaffolding. You will eventually provide your own scaffolding, but for now, it is being provided.



Practice

A newspaper article in June, 1999, reported that the average number of pairs of shoes owned and worn on a regular basis by women was 12.1 and by men 7.7. It also reported that the average price per pair of women's shoes was $36.68 and men's was $61.06. Determine whether **men or women spend more** *for shoes and* **how much more** *rounded to the* **nearest cent**.

Understanding the Problem

- We know that we cannot have $\frac{1}{10}$ of a pair of shoes. We know that we cannot have $\frac{7}{10}$ of a pair of shoes. To find the average number of pairs of shoes, the total number of pairs of women's shoes was divided by the total number of women, and the same was done for the men. We, therefore, understand how the average number of pairs of shoes could be 12.1.
- We also know that one pair of shoes is worn at a time, but shoes may be changed as often as desired.
- We also know that men's shoes typically cost more per pair than women's, according to the article.
- To answer the question, the amount spent by women must be compared with the amount spent by men. If the average price per pair is multiplied by the average number of pairs, spending amounts can be found and compared.

Estimating a Solution

1. $36.68 x 12.1 (women's shoes)

   a. If $36.68 is rounded up to 40 and 12.1 is rounded down to 10, the product is ___________, and one estimate or **estimation** of women's spending might be $ ___________ .

b. If $36.68 is rounded to 37 and 12.1 is rounded to 12, 10 times 37 is ____________, and the cost of two more pairs of shoes ($37 + $37) would make the estimate ____________ (more or less) than $400 but ____________ (more or less) than $450.

2. $61.06 x 7.7 (men's shoes)

a. If $61.06 is rounded down to 60 and 7.7 is rounded to 8, an estimate is ____________.

b. The estimates indicate ____________ (women or men) spend more on shoes.

Solving the Problem

Women's Shoes

$$\begin{array}{r} 36.68 \\ \times\ 12.1 \\ \hline 3668 \\ 7336 \\ 3668 \\ \hline 443.828 \end{array}$$

- To determine decimal placement, an estimate indicates an amount close to 400. The product is 443.828.
- To determine decimal placement, it can also be noted that thirty-six and sixty-eight hundredths is being multiplied by twelve and one tenth. (Recall: $\frac{1}{10} \times \frac{1}{100} = \frac{1}{1000}$)
- Since hundredths times tenths is thousandths, three digits after the decimal are needed.
- The product is 443.828 or four hundred forty-three and eight hundred twenty-eight thousandths.

- Since this represents dollars and cents, the answer should be rounded to the nearest hundredth or cent and is $443.83.
- The average amount spent by women for shoes is $443.83.

3. Find the average amount spent by men for shoes. Show your work and explain how you determined the place for the decimal.

____

____

____

____

____

____

____

4. ___________ (men or women) spend $ ___________ more than ___________ (men or women) on shoes according to the data provided.

275

Practice

Of the 9.6 million total tickets for the 2000 Olympics held in Sydney, Australia, 75 percent went to Australians. Find the **number of tickets** *that did* **not** *go to* **Australians**.

Understanding the Problem

1. Newspapers and magazines often choose to write large numbers as shown here. 9.6 million means 9.6 times one million or 9.6 x 1,000,000. Write the number in standard form that you will use to solve the problem.

2. If 75 percent of the tickets went to Australians, what percent did *not* go to Australians?

3. For computation, 75 percent would need to be changed to a decimal or a fraction. The decimal equivalent is 0.75 and the fraction is $\frac{75}{100}$ or $\frac{3}{4}$. Give the decimal equivalent and the fraction for the answer you found in number 2 above.

Estimating a Solution

4. Round 9.6 million to the nearest million.

5. If 75% or $\frac{3}{4}$ of that amount went to Australians, about how many tickets were left for others?

Solving the Problem

6. Use paper and pencil or a calculator to solve the problem and compare your solution with your estimate.

    ____

7. My estimate was ______________________ (amount)

    (more, less) than the answer.

Practice

A theater is being planned, and each row of seats is to have two more than the previous row. The first row will have 14 seats. Find the **sum** *of the* **number of seats on rows 1 through 10**. *(You may draw an* **X** *to represent theater seats.)*

Drawing a Picture to Understand the Problem

Row 1 14 seats

Row 2 16 seats

Row 3 18 seats

Row 4 ______ seats

Row 5 ______ seats

Row 6 ______ seats

Row 7 ______ seats

Row 8 ______ seats

Row 9 ______ seats

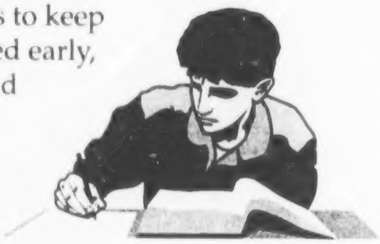
Row 10 ______ seats

Solving the Problem

1. 14 + 16 + 18 + 20 + ____ + ____ + ____ + ____ + ____ + ____ = ____

Looking for a Pattern

A story is told of a student who seemed to always finish first, and his teacher looked for things to keep him busy. One day when the student finished early, the teacher told him to go to his seat and find the sum of the first 100 counting numbers. The student went to his seat, and in no time at all, he was back at the teacher's desk with an answer. The teacher asked how he had finished so quickly, and the student showed him his work which looked like this:



$1 + 2 + 3 + 4 + 5 + \ldots\ldots + 96 + 97 + 98 + 99 + 100 =$

- The sum of 1 and 100 is 101.
- The sum of 2 and 99 is 101.
- The sum of 3 and 98 is 101.
- The sum of 4 and 97 is 101.
- The sum of 5 and 96 is 101.

This *pattern* or **relationship** continues until we reach the center of the list and find the sum of 50 and 51.

There are 100 numbers in the list and there will be 50 pairs with a sum of 101.

$$50 \times 101 \text{ is } 5{,}050$$

The sum of the first 100 counting numbers is 5,050. This can be applied to the problem on the previous page.

Practice

Use the **previous page** *and the practice on pages 64-65 to help you complete the following.*

Solving the Problem

1. The sum of the number of seats on Row 1 and Row 10 is ________.
2. The sum of the number of seats on Row 2 and Row 9 is ________.
3. The sum of the number of seats on Row 3 and Row 8 is ________.
4. The sum of the number of seats on Row 4 and Row 7 is ________.
5. The sum of the number of seats on Row 5 and Row 6 is ________.
6. There are 10 rows and each of the 5 pairs have a

   5 x ________ = ________.

7. The sum of the seats on Rows 1 through 10 is ________.

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Practice

In the 2000 Olympics, 10,200 athletes were expected to compete in 28 sports over 16 days. Find the **average number (mean) of athletes per sport** *rounded to the* **nearest whole number**.

Understanding the Problem

1. There are two numbers in the problem that provide general information, and there are two numbers that will be used in solving the problem. The numbers providing general information are ________ (the year during which the Olympics take place) and ________ (the length in days of the Olympics). The numbers to be used in solving the problem are ________ (the number of athletes) and ________ (the number of sports).

- The mean of the total number of athletes per sport is found by dividing the total number of athletes by the number of sports.
- The answer is to be rounded to the nearest whole number. *For example*: 4.1, 4.2, 4.3, and 4.4 would all round to 4 while 4.5, 4.6, 4.7, 4.8, 4.9 would all round to 5.

Estimating a Solution

2. The number of athletes might be rounded to the nearest thousand which is ________, and the number of sports might be rounded to the nearest ten which is ________ .

3. Using the answers from number 2 above, an estimate would be ________ .

Solving the Problem

4. Solve problem number 2 above using paper and pencil or a calculator. ________

Practice

Alicia and her classmates use the Roman numerals MMIV *to represent the year in which they will graduate from high school. Determine what* **year** *the* **Roman numerals represent**.

Understanding the Problem

This problem requires a knowledge of Roman numerals which you have probably seen in a variety of places but which you may not use very often. The Romans used them through the 9th century A.D. and limited use remains today. Many movies use Roman numerals to show the year in which the movie was made. You may see Roman numerals above the doors of some buildings indicating the year in which they were built. A teacher might return a perfect paper to a student with a C at the top since the value of a C in Roman numerals is 100.

The basic Roman letters and their numeric equivalents are as follows:

I	1
V	5
X	10
L	50
C	100
D	500
M	1000

The position of a letter tells the user whether it is to be added or subtracted.

For example:

- IV means 4 since the smaller value of I precedes the larger value of V; I, or 1, is subtracted from V, or 5, to get 4. The same principles will be true of the following examples.
- VI means 6 since the smaller value follows the larger value.
- XL means 40 since the smaller value precedes the larger value.
- LX means 60 since the smaller value follows the larger value.
- XX means 20 since the values are the same.

Solving the Problem

1. Use the information on the previous page to write what the year MMIV represents. ____

   ____

2. Write the year you expect to graduate from high school or college using Roman numerals. ____________________________________

   ____

3. Write the year of your birth using Roman numerals. ____________

   ____

Practice

The strength of an earthquake is described by a Richter scale that goes from 0 to 9 or greater. Each increase of 1.0 in the scale indicates an earthquake strength 10 times stronger. Determine **how many times stronger** *the 1960 earthquake in Chile with a magnitude of 9.5 would have been than the one in northern Iran in 1997 with a 7.5 magnitude.*

Understanding the Problem

1. The Richter scale describes ______________________________

   ______________________________.

2. An earthquake of magnitude 2.0 would be __________ times stronger than an earthquake of magnitude 1.0.

3. An earthquake of magnitude 3.0 would be __________ times stronger than an earthquake of magnitude 2.0.

4. An earthquake of magnitude 3.0 would be __________ times stronger than an earthquake of magnitude 1.0.

Solving the Problem

5. An increase from 7.5 to 8.5 is an increase of __________ and from 8.5 to 9.5 is an increase of __________. An increase from 7.5 to 9.5 is an increase of __________.

6. The 1960 earthquake in Chile with a magnitude of 9.5 would have been __________ times stronger than the one in northern Iran in 1997 with a 7.5 magnitude.

Practice

In 1998 a new $20 bill was issued preceded by a new $100 bill in 1996 and a new $50 bill in 1997. As of March 31, 1997, there were 4,093,739,605 $20 bills in circulation. What is the dollar value of the $20 bills in circulation? Complete the following.

1. Write 4,093,739,605 in words.

   ____

   ____

   ____

2. To estimate an answer, 4,093,739,605 might be rounded to the nearest ____________ . Multiplying by 20 might be done by first doubling the rounded number of bills and then multiplying by 10. Each of these calculations is often done with mental mathematics. An estimate is ____________ .

3. Solve the problem by multiplication followed by rounding your answer. ____________

4. The number of $20 bills in circulation was about 4.1 billion. What would your answer have been if there had been 4.9 billion $20 bills in circulation? ____________

Practice

The state of Delaware has an area of 2,489 square miles, and the state of Kentucky has an area of 40,411 square miles. Write the **areas** *rounded to the* **nearest thousand**. *Make a* **comparison** *and determine about* **how many times greater** *the* **area of Kentucky is than the area Delaware**.

Understanding the Problem

- The goal is to be able to compare the sizes of the two states.
- A statement might be written to say: The area of the state of Kentucky is about ? times greater than the area of the state of Delaware.
- An estimate is being used.
- The estimate is to be based on rounding numbers as directed.

Solving the Problem

1. Delaware's area of 2,489 square miles = __________ rounded to the nearest thousand.
2. Kentucky's area of 40,411 square miles = __________ rounded to the nearest thousand.
3. When you compare the areas of Delaware and Kentucky both rounded to the nearest thousand, about how many times greater is the area of Kentucky than the area of Delaware? __________

Thinking Critically

4. If 40,411 is divided by 2,489, the quotient is 16.2 rounded to the nearest tenth. The area of Delaware to the nearest hundred is __________. The area of Kentucky to the nearest thousand is __________.

5. About how many times greater is the area of Kentucky than the area of Delaware? __________

6. Was the estimate for number 5 more accurate than the estimate for number 3 when you rounded both numbers to the nearest thousand as number 1 and 2 previously directed? __________

7. Which method of estimation do you prefer and why?

Practice

Three students ran for student council representative from their homeroom. Twenty-five votes were cast. The number of votes cast for each of the 3 candidates was a **prime number**. *Determine* **how many ways** *this could have happened.*

Understanding the Problem

- Three candidates were in the race, and each received some votes.
- The number of votes each candidate received was a prime number.
- A prime number has two and only two factors, 1 and itself.
- The number of ways this could have happened is to be determined.
- The problem does not say that no two candidates got the same number of votes, so that will need to be considered.

Solving the Problem

1. Since 25 votes were cast, a list of prime numbers up to 25 would be helpful.

   _____ , _____ , _____ , _____ , _____ , _____ , _____ , _____ , _____ , _____

2. Try combinations having a sum of 25 until you find one that works. Continue until you are confident that you have found them all. List all of the combinations that work, and you have solved the problem.

   _____ + _____ + _____ = 25

   _____ + _____ + _____ = 25

   _____ + _____ + _____ = 25

   _____ + _____ + _____ = 25

Practice

Robert's aunt plans to give him $1.00 for his first birthday and to double the amount of her gift each birthday through his tenth. His uncle plans to give him $100.00 for his first birthday and $100.00 for each birthday through his tenth. Find the **difference** *in the* **total amounts** *of the two gift plans.*

Understanding the Problem

- Robert's aunt will give him $1.00, $2.00, $4.00, and so on, doubling the amount each year for the 10 years of her gifts.
- Robert's uncle will give him $100.00, $100.00, $100.00 each year for the 10 years of his gifts.
- The total for the uncle's gifts can be done with mental arithmetic.
- The total for the aunt's gifts will require a list to be made followed by mental arithmetic or work with paper and pencil or a calculator.
- Once the total for each of the plans is found, the difference in the totals is needed.

Just for fun, guess which plan you believe will have the greater total.

Solving the Problem

1. Aunt's gifts:

   ___ + ___ + ___ + ___ + ___ + ___ + ___ + ___ + ___ + ___ = ___

2. Uncle's gifts:

   10 x ___ = ___

3. Difference

   The gifts from Robert's ______(aunt or uncle) will exceed the gifts

   from Robert's __________ (aunt or uncle) by $ __________ .

Lesson Four Purpose

- Understand and use exponential and scientific notation. (A.2.3.1)
- Understand the structure of number systems other than the decimal system. (A.2.3.2)
- Use concepts about numbers, including primes, factors, and multiples. (A.5.3.1)
- Associate verbal names, written word names and standard numerals with integers; numbers with exponents; and numbers in scientific notation. (A.1.3.1)
- Understand that numbers can be represented in a variety of equivalent forms. (A.1.3.4)
- Select the appropriate operation to solve problems, including the appropriate application of the algebraic order of operations. (A.3.3.2)

Mystery Numbers

This lesson will focus on a set of problems dealing with "Mystery Numbers." You will apply your knowledge about numbers and use the following clues to find each mystery number. You will then focus on the order in which operations are done within a problem. There is an agreement about this among mathematicians that will be reviewed in this lesson.

Practice

Use the clues below to find **Mystery Number One**. (**Roman numerals** *are used in the clues. You may want to refer to Lesson Three for a review of Roman numerals.)*

- *Clue I*: The number has II *digits* or number symbols when written in standard form but requires VII letters when written in Roman numerals.
- *Clue II*: III of the VIII factors of the number are II, III, and XIII, and III of the other factors are greater than XIII.
- *Clue III*: The sum of the factors is CLXVIII.
- *Clue IV*: This number is found in the year that Hannah Gray became president of the University of Chicago, the first woman to head a major United States university.

1. The Mystery Number One is ____________ in standard form and

____________ in Roman numerals.

Use the clues below to find **Mystery Number Two**.

- *Clue 1*: The number has three digits.
- *Clue 2*: Its prime factorization contains four different factors with none appearing two or more times.
- *Clue 3*: The number is a multiple of 21.
- *Clue 4*: The number is less than 4.0×10^2.
- *Clue 5*: The sum of 7 times this number and 1 represents the number of miles of interstate highways in Florida.

2. The Mystery Number Two is ____________ .

Practice

Use the clues below to find **Mystery Number Three**.

- *Clue 1*: The number has four digits.
- *Clue 2*: It is divisible by 5 but not 10.
- *Clue 3*: The tens digit is greater than 4 and is the second power of 3.
- *Clue 4*: The hundreds digit is greater than 4 and a power of 2.
- *Clue 5*: The thousands digit is less than 4 and the only positive number that is not prime and not composite.
- *Clue 6*: The number represents the year that the first pizza restaurant opened in New York City.

1. The Mystery Number Three is ____________ .

Use the clues below to find **Mystery Number Four**.

- *Clue 1*: The number is a prime number greater than 500 and less than 600.
- *Clue 2*: When divided by 2, 5, or 10 there is a remainder of 1.
- *Clue 3*: The hundreds digit is one more than a power of 2.
- Clue 4: The tens digit is 1 less than a power of 2.
- *Clue 5*: The sum of the digits is 13. This number tells us how many athletes from 32 nations competed for medals at the 1998 Paralympic Games in Japan.

2. The Mystery Number Four is ____________ .

Practice

Use the clues below to find **Mystery Number Five**.

- *Clue 1*: The number has 10 digits and is a multiple of one million.
- *Clue 2*: The digit in the billions place is the smallest positive multiple of 2 and 3.
- *Clue 3*: The digit in the hundred-millions place is the second power of two.
- *Clue 4*: The digit in the ten-millions place is the fourth smallest prime number.
- *Clue 5*: The digit in the millions place is the value of 10^0.
- *Clue 6*: The number represents the amount of money in dollars taken in by Nike, an American clothing business, in 1996.

1. The Mystery Number Five is ______________________________ .

In scientific notation the mystery number would be ____________

____ .

If printed in words, the mystery number would be ____________ .

____

____

If shortened to use in a news headline, it might be written as

____________________________________ .

Practice

Create two **Mystery Number problems**. *It is often easier to* **think of an interesting number first and then write your clues**. *Be careful with your vocabulary. Get at least* **two people** *to try to guess your mystery numbers before turning in your problems. Their feedback may be informative if changes need to be made.*

My Mystery Number One ______________________________

My Mystery Number Two ______________________________

Practice

Use the following information about **algebraic order of operations** *to answer the following statements.*

Study the following.

3 + 5 x 2 = 13 True
3 + 5 x 2 = 16 False

5 + 2 x 2 = 9 True
5 + 2 x 2 = 14 False

1. For the examples marked true, the operation of ________________________ (addition or multiplication) preceded the operation of ________________________ (addition or multiplication).

Study the following.

9 - 4 x 2 = 1 True
9 - 4 x 2 = 10 False

8 - 2 x 3 = 2 True
8 - 2 x 3 = 18 False

2. For the examples marked true, the operation of ________________________ (subtraction or multiplication) preceded the operation of ________________________ (subtraction or multiplication).

Study the following.

$12 \div 2 \times 3 = 18$ True
$12 \div 2 \times 3 = 2$ False

$20 \div 4 \times 5 = 25$ True
$20 \div 4 \times 5 = 1$ False

3. For the examples marked true, the operation of

_______________________ (division or multiplication) preceded the

operation of _______________________ (division or multiplication).

Study the following.

$8 - 3 + 2 = 7$ True
$8 - 3 + 2 = 3$ False

4. For the example marked true, the operation of

_______________________ (subtraction or addition) preceded the

operation of _______________________ (subtraction or addition).

Rules and More Rules

Without these rules, communication would be difficult, and we would not have agreement on answers to simple problems of arithmetic. Learn the rules and practice their application.

1. Work first within parentheses.
2. Then calculate all powers, from left to right.
3. Then do multiplications **and** divisions in order as they occur, from left to right.
4. Then do additions **and** subtractions in order as they occur, from left to right.

Please note the **and** in *multiplications* **and** *divisions* and in *additions* **and** *subtractions*. This tells you that if multiplication occurs *before* division, do it *first*. If division occurs *before* multiplication, do it *first*. The same is true for addition and subtraction. The words "from **left** to **right**" are very important words.

Study the following.

20 - 4 x 3 =

There are no parentheses. There are no powers. We look for multiplication or division and find multiplication. We multiply. We look for addition or subtraction and find subtraction. We subtract.

20 - 4 x 3 =
20 - 12 =
8

Study the following.

$$8 \div 4 + 8 \div 2 =$$

There are no parentheses. There are no powers. We look for multiplication or division and find division. We divide. We look for addition or subtraction and find addition. We add.

$$\begin{aligned} 8 \div 4 + 8 \div 2 &= \\ 2 + 4 &= \\ 6 \end{aligned}$$

If the rules were ignored, one might divide 8 by 4 and get 2, then add 2 and 8 to get 10, then divide 10 by 2 to get 5. Agreement is needed.

Study the following.

$$12 - 2^3 =$$

There are no parentheses. We look for powers and find 2^3. We calculate this. We look for multiplication or division and find none. We look for addition or subtraction and find subtraction. We subtract.

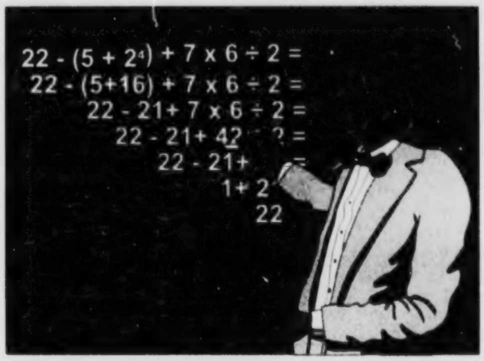
$$\begin{aligned} 12 - 2^3 &= \\ 12 - 8 &= \\ 4 \end{aligned}$$

Study the following.

$$22 - (5 + 2^4) + 7 \times 6 \div 2 =$$

We look for parentheses and find them. We must do what is inside the parentheses first. We find addition and a power. We do the power first and then the addition. We look for multiplication or division and find both. We do them in the order they occur, left to right, and the multiplication occurs first. We look for addition or subtraction and find both. We do them in the order they occur, left to right and the subtraction occurs first.

$$\begin{aligned} 22 - (5 + 2^4) + 7 \times 6 \div 2 &= \\ 22 - (5+16) + 7 \times 6 \div 2 &= \\ 22 - 21 + 7 \times 6 \div 2 &= \\ 22 - 21 + 42 \div 2 &= \\ 22 - 21 + 21 &= \\ 1 + 21 &= \\ 22 & \end{aligned}$$



Practice

Complete the following.

1. $4 + (3 \times 2^3) \div 4 =$ ____________

2. $5 - 2 + 6 =$ ____________

3. $14 \div 7 \times 2 =$ ____________

4. $16 - 3 \times 5 =$ ____________

5. $48 - (6 - 2 + 8) + 2 \times 5 =$ ____________

6. $2^5 + (5 - 2) \times 6 \div 2 =$ ____________

7. $18 - 7 + 3 - 4 =$ ____________

8. $6 \div 3 \times 8 \times 2 =$ ____________

9. $(6 - 2) + 3^2 =$ ____________

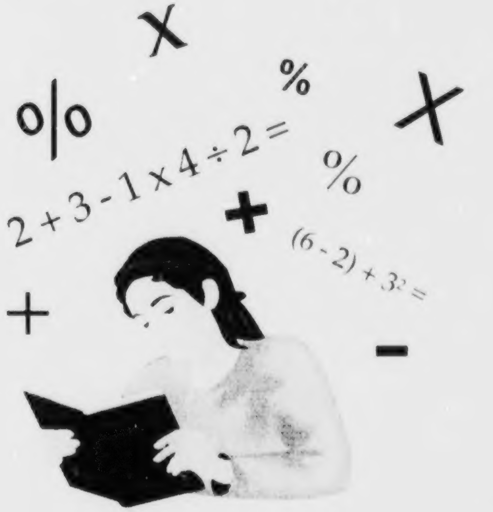
10. $2 + 3 - 1 \times 4 \div 2 =$ ____________

Lesson Five Purpose

- Select the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of numbers, ratios, proportions, and percent. (A.3.3.2)
- Add, subtract, multiply, and divide numbers to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to check the reasonableness of results. (A.4.3.1)

Percent, Ratio, and Proportion

This lesson will be similar to Lesson Three but the majority of problems to be solved will deal with percent or **ratio** and **proportion**.



Practice

A bookstore sells membership cards for $18.00, and members get 15 percent off purchases during their one-year membership. Purchases must **exceed** *what amount for the savings to be* **greater** *than the cost of membership?*

Using Reasoning to Solve Problems with Percents

1. Restate the words "15 percent" using a number and the percent symbol. __________

2. Percent is a *special-case ratio* in which the second term is always 100. Fifteen percent means 15 to 100. If $100.00 is spent at the bookstore __________ is saved. If $200 is spent at the bookstore, __________ is saved.

3. Using this information, one could quickly estimate that spending must be greater than __________ but less than __________ for the membership to be beneficial.

4. The actual amount will be closer to __________ ($100 or $200).

5. If $15 is saved on $100, then $3 more must be saved to pay for the membership. Since 3 is one-fifth of 15, one-fifth of $100 must also be spent and one-fifth of $100 is __________ . If purchases equal __________, the savings is the same as the cost of membership. Purchases must exceed this amount for savings to exceed the cost of membership.

Using Guess and Check to Solve Problems with Percent

6. *Guess*: If I spend \$50, how much is saved?

   Check: 15 percent means 15 to 100 or $\frac{15}{100}$ or 0.15. If \$50 is multiplied by 0.15, the result is \$ ________ . This is ________ (more or less) than \$18.

7. *Guess*: If I spend \$100, how much is saved?

   Check: 15 percent means 15 to 100 or $\frac{15}{100}$ or 0.15. If \$100 is multiplied by 0.15, the result is \$ ________ . This is ________ (more or less) than \$18.

8. *Guess*: If I spend \$150, how much is saved?

   Check: If \$150 is multiplied by 0.15, the result is \$ ________ . This is ________ (more or less) than \$18.

9. *Guess*: If I spend \$125, how much is saved?

   Check: If \$125 is multiplied by 0.15 the result is \$ ________ . This is close to \$18 but slightly more.

10. *Guess*: If I spend \$120, how much is saved?

    Check: If \$120 is multiplied by 0.15 the result is ________ . This means that I must spend \$120 to save enough to pay for my membership and more than \$120 for savings to exceed the cost of membership.

Using Concept of Equivalence to Solve Problems with Percents

Fifteen percent savings means $15 is saved for each $100 spent, so it would help to find an equivalent ratio having 18 for the first term.

$$\frac{15}{100} = \frac{18}{?}$$

11. What must 15 be multiplied by to get 18? ____________ This is the same as asking 18 divided by 15 is what? ____________

12. If 15 is multiplied by ____________ to get 18, then 100 must be multiplied by the same factor which gives ____________ times 100 is ____________ .

Choosing a Strategy

13. Of the three strategies used for finding percent, tell which you prefer and why. ____

____

____

____

____

____

____

____

____

____

306

14. If you know another way to solve the problem, use this space to show and explain it.

Practice

In an orchestra, 60 percent of the musicians typically play string instruments, and the rest play woodwinds, brasses, or percussion instruments. If an orchestra has 130 members, **how many would not** *play strings?*

Understanding the Problem

1. If 60 percent play strings, then ____________ percent do not.

2. If 60 out of 100 play strings, then ____________ out of 100 do not.

   This is the same as saying 6 out of 10 play strings and ____________ out of 10 do not. It is also the same as saying 3 out of 5 play strings and ____________ out of 5 do not.

√ The decimal equivalent for 60% is 0.60 and the fraction is $\frac{60}{100}$ or $\frac{3}{5}$.

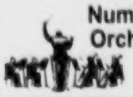
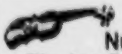
√ There are 130 members so 60 of the first 100 play strings and 60% of the remaining 30 play strings.

√ The question to be answered is how many *do not* play strings.

Making a Table

The table begins with 100 in the orchestra because we know that 60 percent means 60 out of 100 and that the orchestra has more than 100 people.

3. Complete the table.

 Number in Orchestra	 Number Playing Strings	Number Not Playing Strings
100	60	40
105	63	42
110	66	44
115		
120		
125		
130		

Illustrating the Problem

4. Draw a picture to illustrate the problem and tell how you would use your picture to solve it. You might draw sets of 10 people and show which play strings and which do not. You might want to illustrate it in another way.

Practice

Use the directions below to answer the following.

Using the Concept of Equivalence

$\frac{60}{100} = \frac{?}{130}$

1. What must 100 be multiplied by to get 130? ______________

2. When 60 is multiplied by that number, the product is ______________ .

3. If ______________ play strings, then 130 to ______________ do not.

or

$\frac{40}{100} = \frac{?}{130}$

4. What must 100 be multiplied by to get 130?

5. When 40 is multiplied by that number, the product is ______________ .

6. ______________ do not play strings.

or

Since $\frac{40}{100}$ can be *simplified* to $\frac{2}{5}$,

$\frac{2}{5} = \frac{?}{130}$

7. What must 5 be multiplied by to get 130? ______________

8. When 2 is multiplied by that number, the product is ______________ .

9. ______________ do not play strings.

Finding 40 Percent of 130 Using Multiplication

10. Find the product of 0.40 x 130, or the product of $\frac{40}{100}$ x 130, or the product of $\frac{4}{10}$ x 130, or the product of $\frac{2}{5}$ x 130. ______________

Choosing Strategies

11. Which of the strategies helps you understand the problem and its solution best? ______________

12. Which of the strategies is most efficient for you in terms of your time and your skills in mathematics? ______________

Practice

A pair of jeans has a regular price of $36 but is on sale at 25 percent off. Find the **sale price**. *Use one of the* **strategies** *applied in the problems in the practices on pages 90-97 to solve this problem. Show your work.*

1. The sale price of the jeans would be ____________.

Practice

A state is considering increasing its sales tax from 5 percent to 7 percent. Proponents encourage people to support an increase since it represents just pennies per dollar spent. Opponents argue against the increase and remind voters of tax on costly products such as automobiles. How much more **sales tax** *would a consumer pay on a $25,000 automobile if the* **increase** *is approved by voters?*

Understanding the Problem

- Proponents are people in favor of an issue.
- Opponents are people opposed to an issue.
- The difference in the tax on $25,000 at 7 percent and 5 percent must be found.
- A 5 percent tax means the consumer pays 5 cents in tax for each $1.00 spent on taxable items.
- A 7 percent tax means the consumer pays 7 cents in tax for each $1.00 spent on taxable items.

Solving the Problem—Using Multi-Steps to Get the Full Picture

1. Find 7 percent of $25,000.

   $25,000 x 0.07 = ____________

or

2. What is 100 multiplied by to give 25,000? ____________ When 7 is multiplied by that same number, what is the result? ____________

   $\frac{7}{100} \times \frac{?}{?} = \frac{?}{25{,}000}$

3. $25000 \times 0.05 =$ ____________

or

4. What is 100 multiplied by to give 25,000? ____________ When 5 is multiplied by that same number, what is the result? ____________

$\frac{5}{100} = \frac{?}{?} = \frac{?}{25{,}000}$

5. Subtract the amount of tax at 5 percent from the amount of tax at 7 percent to find the difference. ____________

This helps the problem solver fully grasp the amount of tax actually paid under each plan. Some consumers are surprised at the amount of tax on costly items.

Using a Single Step Strategy

If the proposed increase is to be 2 percent, then the difference will be 2 percent of the $25,000.

6. $0.02 \times 25{,}000 =$ ____________

or

7. $\frac{2}{100} = \frac{?}{25{,}000}$

8. The increase in tax on a $25,000 purchase would be ____________ .

Practice

In the 20th century, 18 men held the office of President of the United States, and 11 of them were Republican. Rounded to the **nearest whole number**, *what* **percent** *were Republican?*

Understanding the Problem

1. There were ____________ Republicans out of a total of ____________ .

   - This information allows us to write a fraction, and we can find its decimal equivalent.
   - If 9 of the presidents had been Republicans, it would have been 9 out of 18, and 9 divided by 18 is 0.50.
   - We would then write this decimal as a percent and get 50%. To write the percent needed to answer the problem, a fraction and decimal need to be written.

Using a Fraction/Decimal/Percent Strategy

2. The fraction representing the number of Republicans out of the number of men serving as president would be ____________ .

3. The decimal equivalent for that fraction would be ____________ .

4. The percent equivalent would be ____________ . (Remember to round to the nearest whole number.)

315

Using a Concept of Equivalence

5. Since percent means per 100, we need to know the fraction equivalent having a denominator of 100 for the fraction $\frac{11}{18}$.

$\frac{11}{18} = \frac{?}{100}$

6. What must 18 be multiplied by to get 100? ____________

$\frac{11}{18} \times \frac{?}{?} = \frac{?}{100}$

7. When 11 is multiplied by that same factor, what is the product?

____

8. $\frac{?}{100}$ = ____________ %. (Hint: Use the answer to number 7 above as the numerator and remember to round it to the nearest whole number.)

9. ____________ percent of the 18 men serving as President of the United States during the 20th century were Republicans.

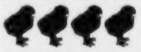
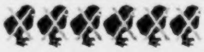
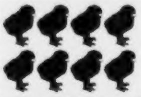
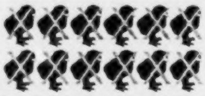
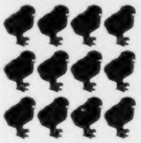
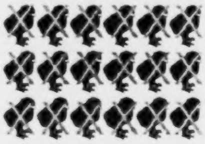
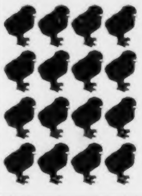
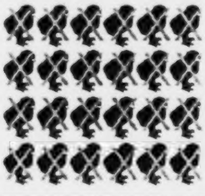
Practice

The Conservation Minister for New Zealand reported in May, 1999 that stoats were killing 6 in 10 kiwi chicks each year, leaving about 15,000 chicks. Based on this information, **how many** *kiwi chicks were there* **originally***?*

Understanding the Problem

- A kiwi is a flightless bird found in New Zealand.
- Stoats are in the weasel family.
- For each group of 10 kiwi chicks, 6 were killed by stoats.
- About 15,000 kiwi chicks were left.
- The original number of kiwi chicks is what must be found.

Using a Picture or Table

Kiwi Chicks		Number Living*	Number Killed by Stoats	Original Number
Living Chicks	Killed Chicks			
		4	6	10
		4 + 4 = 8	6 + 6 = 12	20
		4 + 4 + 4 = 12	6 + 6 + 6 = 18	30
		4 + 4 + 4 + 4 = 16	6 + 6 + 6 + 6 = 24	40
		15,000	????	????

**The term living is used for the kiwi chicks not killed by stoats. Some of those might not live for other reasons, but that is not a consideration in this problem.*

You might want to draw a picture here to illustrate the problem.

Solving the Problem

1. In each group of 10 kiwi chicks, ____________ lived and ____________ were killed by stoats.

2. If the table continued until we had a total of 15,000 living kiwi chicks, we would find the original number. Enough groups of 10 are needed for 4 in each group to make 15,000.

   ________________________ groups of 10 are needed.

3. ________________________ groups of 10 makes a total of ________________________ kiwi chicks.

4. Show what you might do to determine if your answer is reasonable.

   ____

   ____

   ____

   ____

5. The report stated 6 in 10 were killed by stoats. Does the number killed compare to the total number as 6 compares to 10? __________ If so, your work is reasonable. If not, take another look at your work.

Using a Concept of Equivalence

If 6 out of 10 kiwi chicks were killed by stoats, then 4 out of 10 lived.

$$\frac{4}{10} = \frac{15{,}000}{?}$$

$$\frac{4}{10} \times \underline{\qquad} = 15{,}000$$

6. Since 4 must be multiplied by ______________________ to get 15,000, ten must be multiplied by the same factor to get the denominator that represents the original number of kiwi chicks. That result is ______________________ .

7. 4 chicks out of 10 is equivalent to 15,000 chicks out of ______________________ .

8. The number of kiwi chicks prior to 6 out of 10 being killed by stoats was ______________________ .

9. The number of kiwi chicks killed by stoats was ______________________ .

Practice

The ratio of boys to girls at Bos Middle School is 5 to 4. The school wants the same boy-girl ratio for students participating in team sports in the school. The number of boys involved in team sports is 135. How many girls are needed to **maintain** *the* **ratio**?

Understanding the Problem

1. Out of 9 students at Bos Middle School, 5 are boys and 4 are girls. Out of 18 students, ____________ are boys and 8 are girls. Out of 27 students, 15 are boys and ____________ are girls.

- There are 135 boys in team sports.
- For every 5 boys in team sports, 4 girls are needed in team sports.
- How many girls are needed?

Solving the Problem

2. Show how you might draw a picture or make a table to begin working on the problem.

3. If using the *concept of equivalence*, $\frac{5}{4} = \frac{135}{?}$, show how you would find the value of the question mark which represents the number of girls needed.

The number of girls needed to maintain the ratio is ____________ .

Checking to See If Answer Is Reasonable

4. Explain why you believe your answer is a reasonable and correct solution to the problem. ____

____

____

____

____

Practice

In April, 1999, a hailstorm occurred in Sydney, Australia. A newspaper reported that damage claims for insured automobiles would total $30 million in Australian currency. At that time, the American dollar was worth one and one-half Australian dollars. Find the **value** *of expected claims in American* **dollars**.

Understanding the problem

- Hail is small masses or pellets of ice which fall during showers or storms.
- The pellets vary in size.
- The number of pellets and the size of the pellets are both factors in damage to automobiles as well as other property.
- Claims were expected to amount to $30 million in Australian currency.
- $1.00 in American currency was the same as $1.50 in Australian currency at the time.
- The equivalent in American dollars to $30 million in Australian is to be found.

Solving the problem

1. $30 million can be written in standard form as

   ______________________ .

2. If $1.00 in American money was worth $1.50 in Australian money, then what amount of American money would be worth $30 million in Australian money? Using the *concept of equivalence*, the following proportion could be written and solved: $\frac{1}{1.5} = \frac{??}{??}$ Replace the ?? in the *denominator* with the correct number.

$\frac{1}{1.5} = \overline{\qquad\qquad\qquad}$ ⟵ (denominator)

3. Solve the problem.

$\frac{1}{1.5} =$ ____________________

4. At the time of the hailstorm, $30 million in Australian money would have had a value of ____________________ in American money.

Practice

If the ratio of water coverage to land coverage on our planet is approximately 2.4 square miles to 1 square mile, **how much** *of our planet is covered by water if approximately 57,900,000 square miles are covered by land?*

Use a method of your choice to solve this problem. Show your work.

1. Water covers ______________________ square miles of our planet.

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. the result of an addition

______ 2. the result of a division

______ 3. the result of a multiplication

______ 4. any whole number with only two factors, 1 and itself

______ 5. a number or expression that divides exactly another number

______ 6. a special-case ratio in which the second term is always 100

______ 7. a shorthand method of writing very large or very small numbers

______ 8. any numeral representing some part of a whole

______ 9. the quotient of two numbers used to compare two quantities

______ 10. the use of rounding or other strategies to determine a reasonably accurate approximation

______ 11. a predictable or prescribed sequence of numbers, objects, etc.

______ 12. a mathematical sentence stating that two ratios are equal

A. estimation

B. factor

C. fraction

D. pattern (relationship)

E. percent

F. prime number

G. product

H. proportion

I. quotient

J. ratio

K. scientific notation

L. sum

Unit 2: Measurement

This unit emphasizes how estimation and measuring are used to solve problems.

Unit Focus

Number Sense, Concepts, and Operations

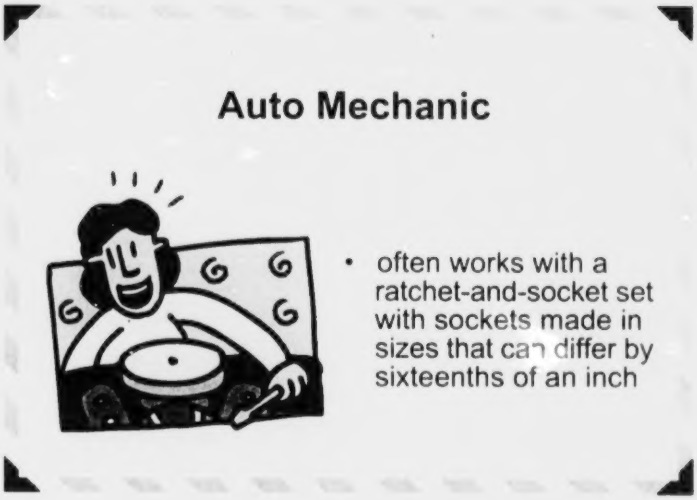
- Add, subtract, multiply, and divide whole numbers, decimals, and fractions to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)

Measurement

- Use concrete and graphic models to derive formulas for finding perimeter, area, surface area, circumference, and volume of two- and three-dimensional shapes, including rectangular solids and cylinders. (B.1.3.1)
- Construct, interpret, and use scale drawings such as those based on the number lines and maps to solve real-word problems. (B.1.3.4)
- Use direct (measured) and indirect (not measured) measures to compare a given characteristic in either metric or customary units. (B.2.3.1)
- Solve problems involving units of measure and convert answers to larger or smaller unit within either the metric or customary system. (B.2.3.2)
- Solve real-world and mathematical problems involving estimates of measurements including length, areas, and volume in either customary or metric system. (B.3.3.1)
- Select appropriate units of measurement. (B.4.3.1)
- Select and use appropriate instruments and techniques to measure quantities in order to achieve specified degrees of accuracy in a problem situation. (B.4.3.2)

Algebraic Thinking

- Describe a wide variety of patterns, relationships, and functions through models, such as manipulatives, tables, expressions, and equations. (D.1.3.1)



Auto Mechanic

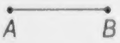
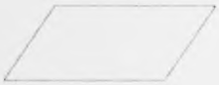
- often works with a ratchet-and-socket set with sockets made in sizes that can differ by sixteenths of an inch

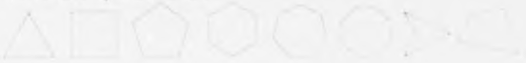
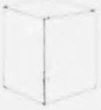
Vocabulary

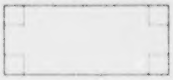
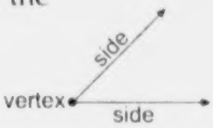
Study the vocabulary words and definitions below.

area (*A*)	the inside region of a two-dimensional figure measured in square units *Example*: A rectangle with sides of four units by six units contains 24 square units or has an area of 24 square units.
apex	the highest point of a triangle, cone, or pyramid; the vertex (corner) opposite a given base
base (*b*)	the line or plane upon which a figure is thought of as resting
capacity	the amount of space that can be filled *Example*: Both capacity and volume are used to measure three-dimensional spaces; however *capacity* usually refers to *fluids*, whereas *volume* usually refers to *solids*.
composite number	any whole number that has more than two factors *Example*: 16 has five factors—1, 2, 4, 8, and 16.
congruent	figures or objects that are the same shape and the same size
cube	a rectangular prism that has six square faces
cubic units	units for measuring volume

customary units	the units of measure developed in England and used in the United States *Example*: length: inches, feet, yards, miles; weight: ounces, pounds, tons; volume: cubic inches, cubic feet, cubic yards; capacity: fluid ounces, cups, pints, quarts, gallons
decimal number	any number written with a decimal point in the number *Example*: A decimal number falls between two whole numbers, such as 1.5 falls between 1 and 2. Decimal numbers smaller than 1 are sometimes called decimal fractions, such as five-tenths is written 0.5.
diameter (*d*)	a line segment from any point on the circle passing through the center to another point on the circle
estimation	the use of rounding and/or other strategies to determine a reasonably accurate approximation without calculating an exact answer
face	one of the plane surfaces bounding a three-dimensional figure; a side
formula	a way of expressing a relationship using variables or symbols that represent numbers
grid	a network of evenly spaced, parallel horizontal and vertical lines

height (*h*)	a line segment extending from the vertex or *apex* (highest point) of a figure to its base and forming a right angle with the base or basal plane *h* *h*
length (*l*)	a one-dimensional measure that is the measurable property of line segments
line segment	a portion of a line that has a defined beginning and end  *Example*: The line segment *AB* is between point *A* and *B* and includes point *A* and point *B*.
metric units	the units of measure developed in France and used in most of the world; uses the base 10, like the decimal system *Example:* length: millimeters, centimeters, meters, kilometers; weight: milligrams, grams, kilograms; volume; cubic millimeters, cubic centimeters, cubic meters; capacity: milliliters, centiliters, liters, kiloliters
mixed number	a number that consists of both a whole number and a fraction *Example*: $1\frac{1}{2}$ is a mixed number.
parallelogram	a polygon with four sides and two pairs of parallel sides 

pattern (relationship)	a predictable or prescribed sequence of numbers, objects, etc.; also called a *relation* or *relationship;* may be described or presented using manipulatives, tables, graphics (pictures or drawings), or algebraic rules (functions) *Example*: 2, 5, 8, 11...is a pattern. The next number in this sequence is three more than the preceding number. Any number in this sequence can be described by the algebraic rule, $3n - 1$, using the set of counting numbers for n.
perimeter (*P*)	the length of the boundary around a figure; the distance around a polygon
plane	an undefined, two-dimensional (no depth) geometric surface that has no boundaries specified; a flat surface
plane figure	a two-dimensional figure, with height and width but no depth
polygon	a closed plane figure whose sides are straight lines and do not cross *Example*: triangle (3 sides), quadrilateral (4 sides), pentagon (5 sides), hexagon (6 sides), heptagon (7 sides), octagon (8 sides); concave, convex 
polyhedron	a three-dimensional figure in which all surfaces are polygons
prism	a three-dimensional figure (polyhedron) with congruent, polygonal bases and lateral faces that are all parallelograms 

rectangle	a polygon with four sides and four right angles	
relationship (relation)	see *pattern*	
right angle	an angle whose measure is exactly 90°	
scale model	a model or drawing based on a ratio of the dimensions for the model and the actual object it represents *Example*: a map	
square units	units for measuring area; the measure of the amount of an area that covers a surface	
sum	the result of an addition *Example*: In 6 + 8 = 14, 14 is the sum.	
table (or chart)	an orderly display of numerical information in rows and columns	
triangle	a polygon with three sides	
vertex	the common endpoint from which two rays begin or the point where two lines intersect; the point on a triangle or pyramid opposite to and farthest from the base; (plural: *vertices*); vertices are named clockwise or counter-clockwise	

volume (*V*)	the amount of space occupied in three dimensions and expressed in cubic units *Example*: Both capacity and volume are used to measure empty spaces; however, *capacity* usually refers to *fluids*, whereas *volume* usually refers to *solids*.
weight	measures that represent the force that attracts an object to the center of Earth; in the customary system, the basic unit of weight is the pound
whole number	any number in the set {0, 1, 2, 3, 4…}
width (*w*)	a one-dimensional measure of something side to side

Unit 2: Measurement

Introduction

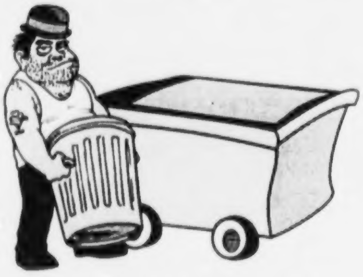
How tall are you? What is your weight? What size jeans do you wear? What is the circumference of your head? How fast can you walk or run? How many times does your heart beat each minute when you are at rest? What is your blood pressure? How many square feet are in the floor of your classroom? What is the volume of the school's dumpster? What is the difference in the weight of a large hamburger patty and a small hamburger patty at your favorite burger restaurant? How much water do you use when taking a shower or brushing your teeth? How many kilowatt hours were used by your family or by our school in a recent month (usually found on utility bills)?

What is your weight?

How much water do you use when brushing your teeth?

How much water do you use when taking a shower?

What size jeans do you wear?



What is the volume of the school's dumpster?

What is your blood pressure?

Lesson One Purpose

- Use direct (measured) and indirect (not measured) measures to compare a given characteristic in either metric or customary units. (B.2.3.1)
- Solve problems involving units of measure and convert answers to a larger or smaller unit within either the metric or customary system. (B.2.3.2)
- Use concrete and graphic models to derive formulas for finding area and volume. (B.1.3.1)
- Solve real-world and mathematical problems involving estimates of measurements including length, area, and volume. (B.3.3.1)

Measuring

Measuring and using measurements, whether in **metric units** or **customary units**, are part of our daily lives. The more we measure and use measurements, the better we are at making reasonable **estimations** of measurements. This lesson should sharpen your skills in these areas and increase your awareness of how frequently measurement is used in our daily lives.

Practice

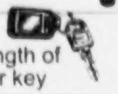
Complete the following on measuring length using **linear units**.

1. Use a ruler to draw a **line segment** 6 inches long. Place tick marks on your line segment to indicate 0, 1, 2, 3, 4, 5, and 6 inches. Add shorter tick marks to show $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, and $5\frac{1}{2}$ inches. Look at the ruler you are using and determine the fractional part of an inch indicated by the shortest tick mark. Most rulers include marks for $\frac{1}{16}$ inch.

2. Use a metric ruler to draw a line segment 6 centimeters long. Place tick marks on your line segment to indicate 0, 1, 2, 3, 4, 5, and 6 centimeters. Look at the ruler you are using and determine the fractional part of a centimeter indicated by the shortest tick mark. Most rulers include marks for $\frac{1}{10}$ centimeter, which is the same as 1 millimeter.

Practice

Estimation is important in the study of measurement. Complete the **table** *below to provide* **estimates** *in inches and feet or centimeters and meters for each item. After completing the column for estimates, use a ruler, tape measure, or meter stick to* **measure** *and complete that column. Compare your estimates with your actual measures and* **summarize** *your strengths and weaknesses.*

Object	Estimate in Inches / Feet	Actual Measurement	Estimate in Centimeters / Meters	Actual Measurement
length of paperclip				
length of pencil				
 length of car key				
height of doorway				
diameter of a penny				
diagonal of a TV screen				
thickness of a book				

1. Summary of my strengths and weaknesses in estimating measures are as follows:

____

____

____

____

____

2. For each measure in the table, list an item having that approximate **length**.

Measure	Item
1 centimeter	
10 centimeters	
100 centimeters	
2 meters	
1 inch	
12 inches	
1 yard	
your choice	

3. It is sometimes helpful to use known lengths of some part of your body as a basis for estimates. Use rulers, meter sticks, or tape measures to complete the following.

Body Part(s)	Actual Measure in Centimeters / Meters	Actual Measure in Inches / Feet
arm span (as far as you can reach fingertip to fingertip)		
height		
length of longest finger		
length of shortest finger		
circumference of head		
distance from chin when centered over body to fingertip when arm is outstretched		
length of foot with shoe removed		

Practice

Complete the following on **measuring area** *using* **square units**.

1. Use a large sheet of paper or a large brown paper bag from a grocery store to draw a square measuring 12 inches on each side. Draw another square measuring 1 inch on each side. You now have a model for 1 square foot and for 1 square inch. Cut out each of your squares to use in the next practice.

2. ____________ square inches would be needed to cover 1 square foot.

3. The **area** of your desktop is approximately ____________ square ____________ .

4. The area of the floor of your classroom is approximately ____________ square ____________ .

5. The area of the palm of your hand is approximately ____________ square ____________ .

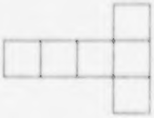
6. Since a yard is 3 feet long, a square yard would need to be a square measuring 3 feet on each side. It would take ____________ square feet to cover 1 square yard.

7. If you were ordering carpet for your classroom, approximately how many square yards would be needed? ____________

Practice

Complete the following on **measuring volume** *using* **cubic units**.

1. Use your square inch from the previous practice to make five additional copies. Arrange them in one of the following patterns or a similar one. Tape the **square units** together and fold your pattern to make a *cubic inch* or a three-dimensional figure with six faces. You may do the same for a cubic centimeter if you choose.



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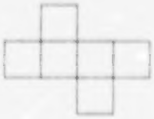

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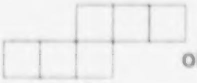

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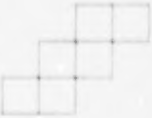
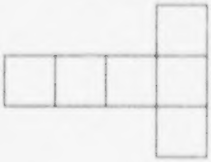
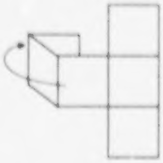


See example below of how to make a cubic inch.

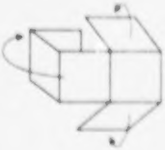
step 1



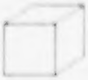
step 2



step 3



step 4



2. Use your square foot from the previous practice to make five additional copies. Cut them out and tape the six together to form a model of a cubic foot. The **formula** for **volume** of a **cube** is commonly listed as $V = S^3$ or volume of a cube equals side times side times side. How many cubic inches would it take to fill your cubic foot?

3. Estimate the number of cubic feet in your classroom. ____________

   How might you check your estimate?

   ____

   ____

   ____

   ____

   ____

4. The number of **cubic units** in a freezer or refrigerator is important information when considering a purchase of one. Estimate the number of cubic feet in the refrigerator or freezer in your home.

   ____

Practice

Complete the following on **measuring capacity** *using* **liquid measurements**. *Use a standard measuring cup, a typical drinking glass, an empty 2-liter soft drink bottle, an empty 1-gallon milk bottle, an empty milk container from the school cafeteria, an empty juice container from the school cafeteria to complete the following.*

1. A typical drinking glass will fill a standard measuring cup approximately __________ time(s).

2. A typical 2-liter soft drink bottle will fill a standard measuring cup approximately __________ time(s).

3. A typical 1-gallon milk bottle will fill a standard measuring cup approximately __________ time(s).

4. A typical carton of milk sold to students in the school cafeteria will fill a standard measuring cup approximately __________ time(s).

5. A typical carton of juice sold to students in the school cafeteria will fill a standard measuring cup approximately __________ time(s).

Practice

Answer the following on **measuring weight**.

1. Find labels on five or more items in your home that list **weight** in ounces or grams. List each item and its weight.

____

____

____

____

____

2. Find labels on five or more items in your home that list weight in pounds or kilograms. List each item and its weight.

____

____

____

____

____

Practice

Use the list below to write the correct term for each definition on the line provided.

- **area**
- **capacity**
- **cube**
- **cubic units**
- **customary units**
- **diameter**
- **estimation**
- **formula**
- **length**
- **line segment**
- **metric units**
- **square units**
- **table**
- **volume**
- **weight**

________________ 1. the units of measure developed in France and used in most of the world; uses the base 10, like the decimal system

________________ 2. the units of measure developed in England and used in the United States

________________ 3. a portion of a line that has a defined beginning and end

________________ 4. a one-dimensional measure that is the measurable property of line segments

________________ 5. the inside region of a two-dimensional figure measured in square units

________________ 6. a line segment from any point on the circle passing through the center to another point on the circle

________________ 7. units for measuring area

________________ 8. the amount of space occupied in three dimensions and expressed in cubic units; usually refers to solids

________________ 9. units for measuring volume

__________ 10. the amount of space that can be filled; usually refers to fluids

__________ 11. measures that represent the force that attracts an object to the center of Earth

__________ 12. the use of rounding and/or other strategies to determine a reasonably accurate approximation without calculating an exact answer

__________ 13. a way of expressing a relationship using variables or symbols that represent numbers

__________ 14. a rectangular prism that has six square faces

__________ 15. an orderly display of numerical information in rows and columns

Lesson Two Purpose

- Use concrete and graphic models to derive formulas for finding perimeter, area, surface area, circumference, and volume of two-and three-dimensional shapes, including rectangular solids and cylinders. (B.1.3.1)

Making Connections

Seeing **relationships** or **patterns** can enable us to make many connections in mathematics and build upon what we know. In this lesson, you will use your knowledge of **rectangles** to learn about **parallelograms** and **triangles**. You will use knowledge of area as you find surface area and volume.

Practice

Complete the following activity on **measuring area** *and* **perimeter**. *Use 24 square tiles or 24 squares of paper and grid paper (on the following page) to arrange and form each* **rectangle** *listed in the table below. The* **width** *is provided each time. Determine the* **length** *that results when 24 tiles are arranged to form a rectangle of the specified width.*

For example: If the width is 1 unit, there will be one row of tiles with 24 tiles in the row. The area would be 24 square units since 24 tiles were used to cover the area of the rectangle. The **perimeter** would be 50 units since perimeter represents the **sum** of the lengths of all sides.

1. Use your tiles or paper squares to form each rectangle.

2. Sketch each rectangle on a sheet of **grid** paper and indicate the dimensions.

3. Complete the following table.

Width of Rectangle	Length of Rectangle	Area of Rectangle	Perimeter of Rectangle
1 unit	24 units	24 square units	50 units
2 units		24 square units	
3 units		24 square units	
4 units		24 square units	

4. Describe the relationship between the dimensions of the rectangle (the length and width) and the area of the rectangle.

____

____

____

347

Practice

Complete the following activity using **area** *and* **perimeter**. *Use square tiles or squares of paper and grid paper to make each* **rectangle** *listed in the table below. Each rectangle you will make should have a* **perimeter** *of 24 units. Find the* **area** *of each.*

1. Use your tiles or paper squares to form each rectangle.
2. Sketch each rectangle on a sheet of grid paper and indicate the dimensions.
3. Complete the table.

Width of Rectangle in Units	Length of Rectangle in Units	Area of Rectangle in Square Units	Perimeter of Rectangle in Units
1 unit	11 units	11 square units	24 units
2 units			24 units
3 units			24 units
4 units			24 units
5 units			24 units
6 units			24 units

4. Describe the relationship between the dimensions of the rectangle (the length and width) and the perimeter of the rectangle.

____

____

____

Practice

Complete the following on **parallelograms**.

figure 1

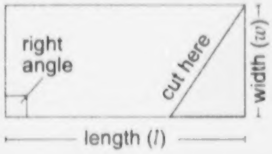


figure 2

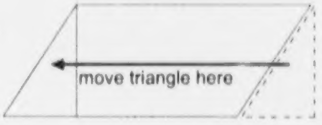
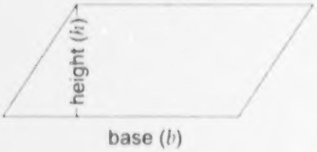


figure 3



1. In figure 1, a triangle has been drawn in one end of the rectangle. On a separate sheet of paper, trace figure 1.
2. Cut out the rectangle from your tracing of figure 1. Now cut out the triangle and place it at the opposite end of your original rectangle as shown in figure 2 above.

   Note that the original *area* of the rectangle is preserved. The area has *not* changed. One part was simply moved to the opposite end which changed the *shape* but not the area.
3. Describe the relationship between the length and width of the original rectangle and the **base** and **height** of the newly formed parallelogram.

Practice

Complete the following activity on **parallelograms**. *Note that in each of the following parallelograms (a rectangle is also a parallelogram), a diagonal line has been drawn, dividing the shape into* **two congruent triangles**. *In figure 2, the* **base** (*b*) *and* **height** (*h*) *of a triangle are indicated through the use of dotted lines. Use a similar system to show the base and height of one triangle in each of the other parallelograms. You may wish to draw the figures first on grid paper and then cut out the shapes to better "see" the relationships.*

figure 1

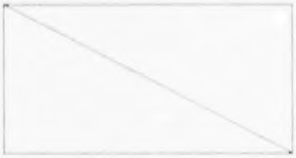


figure 2

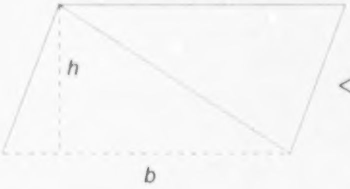
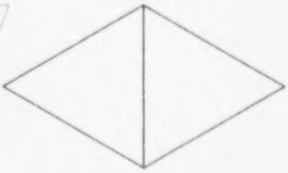


figure 3



1. Describe the relationship between two **congruent** triangles and a parallelogram and the relationship between the area of a triangle and a parallelogram where bases and heights are the same measures.

2. The formula for the area of a rectangle is commonly listed as $A = lw$: area (A) equals length (l) times width (w). Sketch a rectangle on grid paper on the following page and explain why this formula will provide the area.

____

____

____

____

____

____

____

____

____

____

____

____

____

____

____

3. The formula for the area of a parallelogram is commonly listed as $A = bh$: area (A) equals base (b) times height (h). Sketch a non-rectangular parallelogram on grid paper on the following page and explain why this formula will provide the area.

4. The formula for area of a triangle is commonly listed as $A = \frac{1}{2}bh$. Sketch a triangle on grid paper on the previous page and explain why this formula will provide the area.

(**Remember:** b = base; h = height.)

Practice

Complete the following activity on **measuring surface, area,** *and* **volume**.

1. Arrange 24 unit cubes into rectangular **prisms** having the lengths and widths shown in the table below and indicate the height for each prism. The volume should be 24 cubic units each time.

volume
one cubic unit

2. Determine the number of square units required to cover the surface of the prism and include it in the table.

surface area
six square units each ☐

Length of Prism	Width of Prism	Height of Prism	Volume of Prism	Surface Area of a Prism
1 unit	24 units	1 unit	24 cubic units	98 square units
1 unit	12 units		24 cubic units	
1 unit	8 units		24 cubic units	
1 unit	6 units		24 cubic units	
2 units	6 units		24 cubic units	
2 units	4 units		24 cubic units	

3. Describe the relationship between the dimensions of the prism (length, width, and height) and the volume of the prism.

____

____

____

____

4. The formula commonly listed for the volume of a rectangular prism is $V = lwh$. Explain why this formula will provide the volume for each of the rectangular prisms you built.

(**Remember:** l = length; w = width; h = height.)

5. Each **face** of a rectangular prism is a rectangle. It has six faces. If the prism is a cube, all of the faces are congruent squares. If the prism is not a cube, like a shoebox, the rectangular top and bottom are congruent as are the front and back, and the two ends. Explain how this could be helpful in determining the surface area of a rectangular prism.

6. The formula commonly listed for the surface area of a rectangular prism is $S.A. = 2(lw) + 2(lh) + 2(wh)$. Explain why this formula will find the surface area of a rectangular prism.

(**Remember:** l = length; w = width; h = height.)

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. a polygon with four sides and four right angles

______ 2. a polygon with four sides and two pairs of parallel sides

______ 3. a polygon with three sides

______ 4. the length of the boundary around a figure

______ 5. the result of an addition

______ 6. the line or plane upon which a figure is thought of as resting

______ 7. a line segment extending from the vertex or *apex* (highest point) of a figure to its base and forming a right angle with the base or basal plane

______ 8. figures or objects that are the same shape and the same size

______ 9. a three-dimensional figure (polyhedron) with congruent, polygonal bases and lateral faces that are all parallelograms

______ 10. one of the plane surfaces bounding a three-dimensional figure; a side

______ 11. a network of evenly spaced, parallel horizontal and vertical lines

______ 12. a predictable or prescribed sequence of numbers, objects, etc.

______ 13. a one-dimensional measure of something side to side

A. base

B. congruent

C. face

D. grid

E. height

F. parallelogram

G. pattern (relationship)

H. perimeter

I. prism

J. rectangle

K. sum

L. triangle

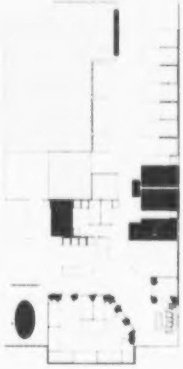
M. width

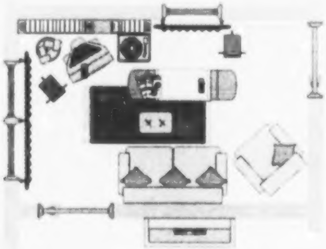
Lesson Three Purpose

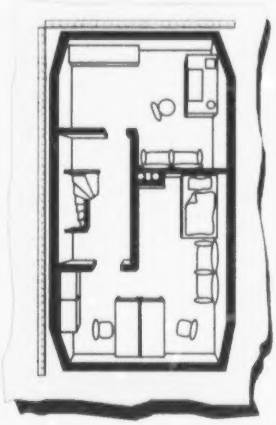
- Construct, interpret, and use scale drawings such as those based on number lines and maps to solve real-world problems. (B.1.3.4)
- Solve real-world and mathematical problems involving estimates of measurements including length, perimeter, and area in either customary or metric units. (B.3.3.1)
- Select and use appropriate instruments and techniques to measure quantities in order to achieve specified degrees of accuracy in a problem situation. (B.4.3.2)
- Select appropriate units of measurement. (B.4.3.1)

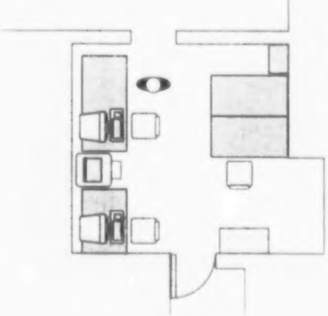
Scale Drawings

Scale drawings are frequently used in plans for homes, buildings, and subdivisions as well as maps. Sometimes it is necessary for us to construct **scale models**, but we more frequently use scale drawings. In this lesson, we will explore scale drawings.



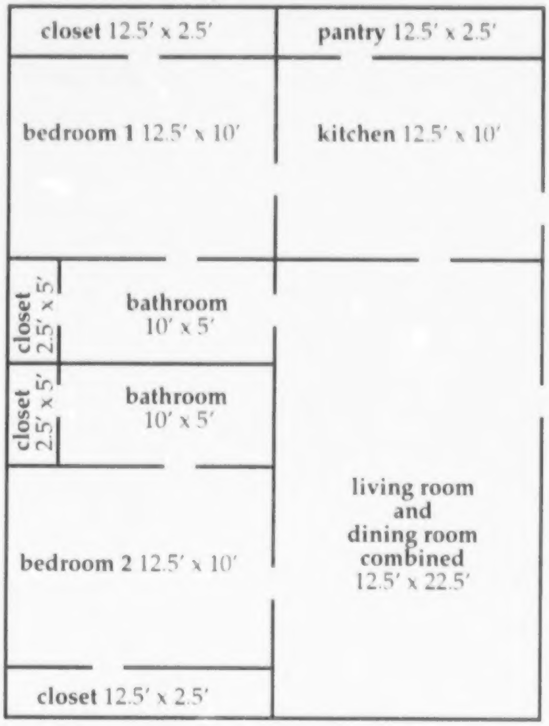






Practice

Use the **scale drawing** *of the* **apartment** *shown below to answer the following questions.*


closet 12.5′ x 2.5′
pantry 12.5′ x 2.5′
bedroom 1 12.5′ x 10′
kitchen 12.5′ x 10′
closet 2.5′ x 5′
bathroom 10′ x 5′
closet 2.5′ x 5′
bathroom 10′ x 5′
living room and dining room combined 12.5′ x 22.5′
bedroom 2 12.5′ x 10′
closet 12.5′ x 2.5′

door

1″ = 5′

1. Determine the number of each of the following in the apartment:

bedrooms ____________________

closets ____________________

bathrooms ____________________

2. Give the dimensions of each of the following in the apartment:

living and dining room ____________________

bedroom 1 ____________________

3. Find the area of each of the following in the apartment:

living and dining room ________________ square feet

kitchen ________________ square feet

bedroom 2 excluding the closet ________________ square feet

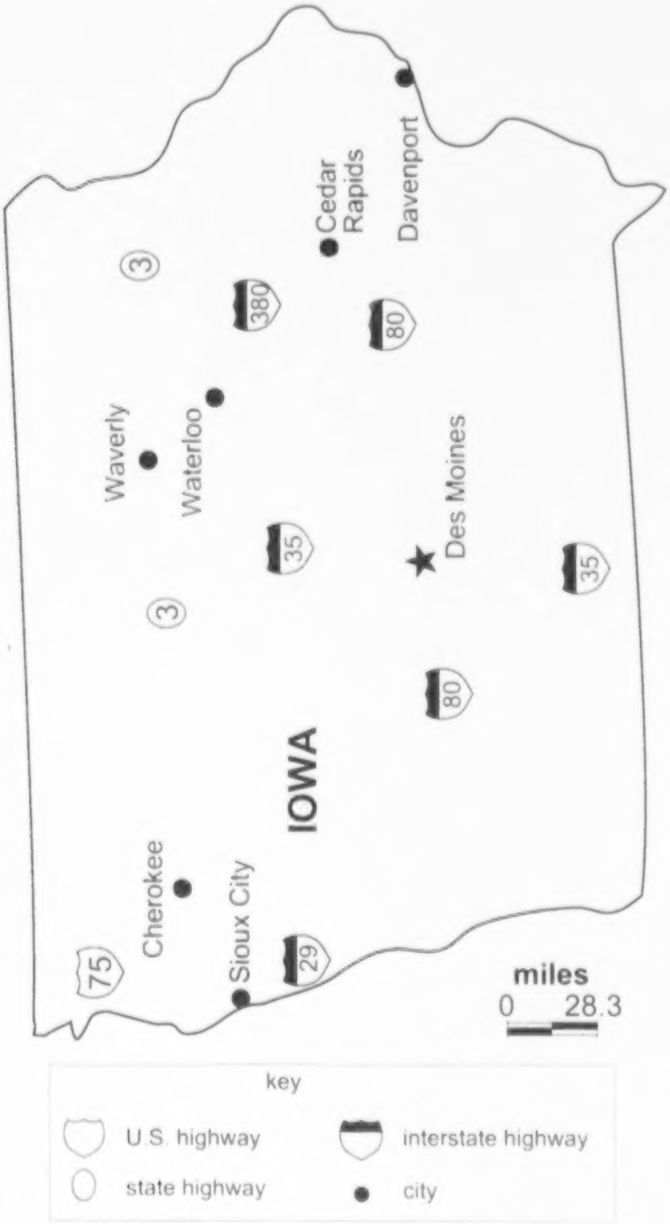
4. Using the scale provided, determine the total area in square feet of the apartment. ____

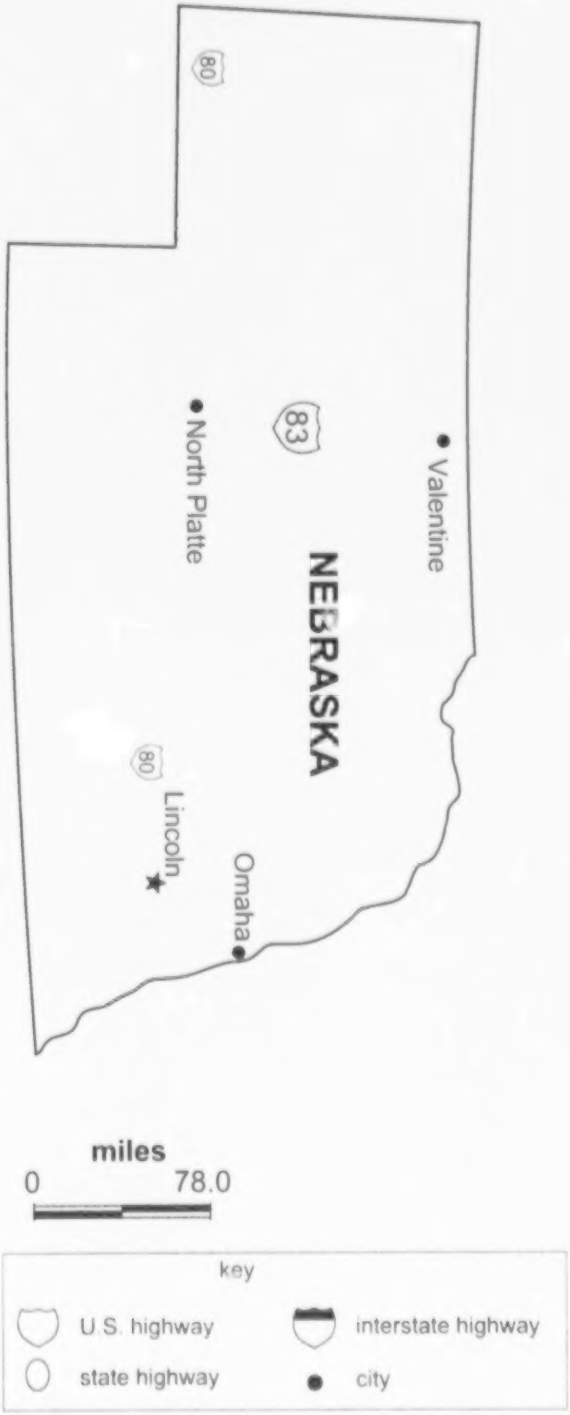
Practice

Construct a **scale drawing** *of your* **classroom**. *Choose a convenient scale using* **customary** *or* **metric units** *and clearly show your scale on the drawing. You will need to carefully measure the room before you begin your drawing.*

Practice

Use the following **maps** *of* **Iowa** *and* **Nebraska** *to answer the statements that follow.*


IOWA
Davenport
Cedar Rapids
Waterloo
Waverly
Des Moines
Cherokee
Sioux City
3
380
80
35
29
75
miles
0 28.3
key
U.S. highway
interstate highway
state highway
city


NEBRASKA
Valentine
North Platte
Lincoln
Omaha
83
80
80
miles
0
78.0
key
U.S. highway
interstate highway
state highway
city

1. A quick glimpse at the two maps without considering the scale might cause the viewer to believe that the area of Iowa is __________ (greater, less) than the area of Nebraska.

2. The scale on the Iowa map shows a fairly short unit representing 28.3 miles, and the length of that fairly short unit is __________ inch.

3. The northern border of Iowa measures approximately __________ inches, which is the same as __________ half-inches. If this is multiplied by 28.3, the approximate length of the northern border of Iowa in miles is found, and it is __________ miles.

4. A quick estimate of the length of the northern border of Iowa could be found by finding the product of 9 and 30, which is __________ miles.

5. The length of the southern border is approximately __________ inches while the length of the widest part midway between the northern and southern borders is approximately 5.5 inches long.

6. If the average or typical distance from the western border to the eastern border of Iowa is approximately 4.7 inches on the map, which is the same as __________ half-inches, then the actual distance is approximately __________ miles.

7. The width of the state might be found by measuring from the northern border to the southern border. This measurement is __________ inches. This appears to be about the same when measuring along the western border as well as the eastern border or midway between them.

8. The width in miles of Iowa would be approximately __________ miles.

9. To find the approximate area of Iowa, the typical length can be multiplied by the typical width, and the result is __________ square miles.

10. A reference source reports the actual area of Iowa is 56,290 square miles. The approximation found using the scale drawing __________ (is, is not) reasonably accurate if used as an estimate.

11. The scale for the map of Nebraska is 1 inch = __________ miles. (You will note that the scale for this map is different from the scale for the map of Iowa.)

12. Determine the approximate area in square miles for the state of Nebraska. ____________

Show and explain all work.

____

____

____

____

13. The actual area of Nebraska is reported as 77,355 square miles. State whether or not your estimate is reasonable. If it is not, how might you improve your strategies to arrive at a more reasonable estimate?

____

____

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____

____

14. Valentine and North Platte are towns on U.S. Highway 83 running north and south near the center of Nebraska. Determine the approximate distance in miles between these two towns.

15. Cherokee and Waverly are towns on State Highway 3 running east and west in the northern part of Iowa. Determine the approximate distance in miles between these two towns.

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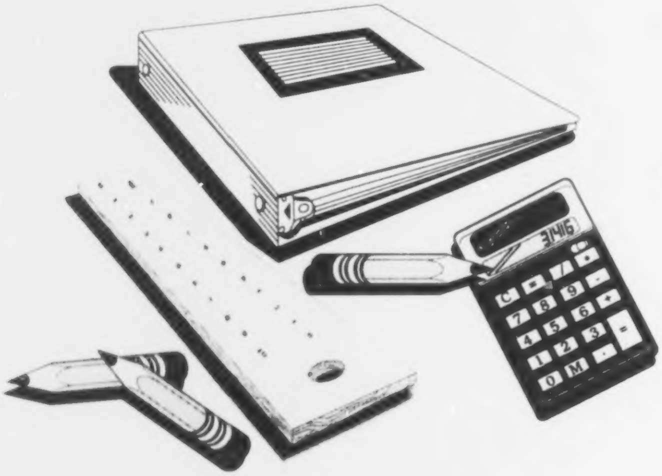
Lesson Four Purpose

- Add, subtract, multiply, and divide whole numbers, decimals, and fractions to solve real-world problems, using appropriate methods of computing such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use concrete and graphic models to derive formulas for finding perimeter, area, surface area, circumference, and volume of two- and three-dimensional shapes, including rectangular solids and cylinders. (B.1.3.1)
- Solve real-world and mathematical problems involving estimates of measurements including length, perimeter, and area in either customary or metric units. (B.3.3.1)
- Describe a wide variety of patterns, relationships, and functions through models, such as manipulatives, tables, expressions, and equations. (D.1.3.1.)

Solving Problems through Measurement

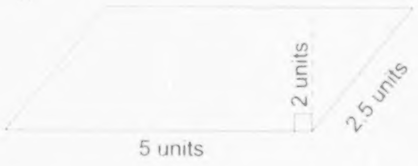
In the real world, we encounter problems that are fairly routine and some that are non-routine. You will be given some problems that can be considered fairly routine to solve on page 160 of this lesson and some that are more non-routine on pages 161-165. You'll be asked which you enjoyed more and why at the end of the lesson.



Practice

Find the **area** *and* **perimeter** *for each of the following* **polygons**.

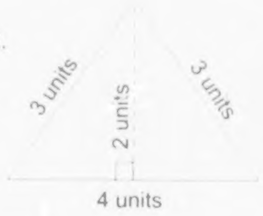
1.


2 units
2.5 units
5 units

area ____________________

perimeter ________________

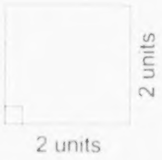
2.


3 units
2 units
3 units
4 units

area ____________________

perimeter ________________

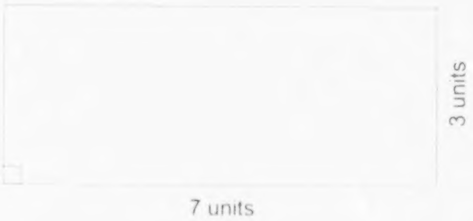
3.


2 units
2 units

area ____________________

perimeter ________________

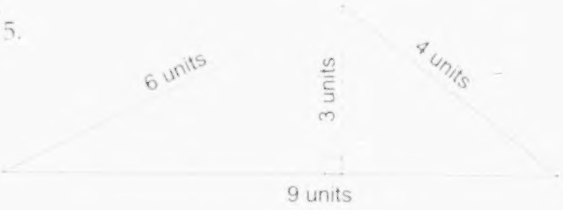
4.


3 units
7 units

area ____________________

perimeter ________________

5.


6 units
3 units
4 units
9 units

area ____________________

perimeter ________________

right angle = □

Area
square: $A = s \times s$ or s^2
rectangle: $A = lw$
parallelogram: $A = bh$
triangle: $A = \frac{1}{2} bh$

Key
A = area
b = base
h = height
l = length
s = side
w = width

Practice

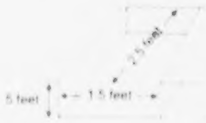
Solve each of the following problems. Show all work.

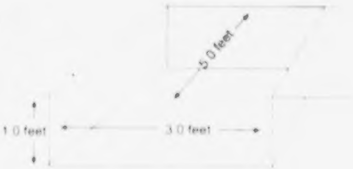
1. A rectangle has an area of 30 square units and its dimensions are **whole numbers**. On the grid paper below, sketch at least three such rectangles and show the dimensions of each.

2. A rectangle has an area of 30 square units and one of its dimensions is a whole number, and the other is a **decimal number** or **mixed number**. On the grid paper above, sketch at least one such rectangle and show the dimensions.

3. A rectangle has a perimeter of 48 units and its length is twice its width. On the grid paper above, sketch this rectangle and show the dimensions.

4. A hole that is 1.5 feet wide, 2.5 feet long, and 0.5 feet deep required one-half hour of digging. At this rate, how long would it take to dig a hole 3.0 feet wide, 5.0 feet long, and 1.0 feet deep. (The answer is NOT one hour!) Remember to show all work in solving this problem.





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5. A square having a side measure of 10 units was cut along its horizontal line of symmetry, and the result was two congruent rectangles. Using grid paper, sketch the original square and the two resulting rectangles. Find the total perimeter of the two rectangles.

____

6. The volume of a cube is 125 cubic units. Find the dimensions of the cube and its surface area. Explain your work.

7. A parallelogram that is not a rectangle has an area of 20 square units and its base and height are whole numbers. On the grid paper below, sketch at least two such parallelograms and show the dimensions of each.

8. A right triangle has an area of 12 square units, and its base and height are whole numbers. On the grid paper below, sketch at least two such right triangles and show the dimensions of each.

9. The dimensions of a rectangle are the smallest two consecutive **composite numbers** (whole numbers that have more than two factors). Find the perimeter and area of the rectangle.

____

____

10. A two-car garage has an area of 400 square feet and its dimensions can be expressed in whole numbers. Consider the possible dimensions listed in the following table that would produce an area of 400 square feet. Choose the one that you think is most reasonable for a two-car garage and explain your reasoning.

Length	Width	Area
1 foot	400 feet	400 square feet
2 feet	200 feet	400 square feet
4 feet	100 feet	400 square feet
5 feet	80 feet	400 square feet
8 feet	50 feet	400 square feet
10 feet	40 feet	400 square feet
16 feet	25 feet	400 square feet
20 feet	20 feet	400 square feet
25 feet	16 feet	400 square feet
40 feet	10 feet	400 square feet
50 feet	8 feet	400 square feet
80 feet	5 feet	400 square feet
100 feet	4 feet	400 square feet
200 feet	2 feet	400 square feet
400 feet	1 foot	400 square feet

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. a model or drawing based on a ratio of the dimensions for the model and the actual object it represents

______ 2. any number in the set {0, 1, 2, 3, 4…}

______ 3. any number written with a decimal point in the number

______ 4. a number that consists of both a whole number and a fraction

______ 5. any whole number that has more than two factors

______ 6. a closed plane figure whose sides are straight lines and do not cross

A. composite number

B. decimal number

C. mixed number

D. polygon

E. scale model

F. whole number

Practice

Use the list below to write the correct term for each definition on the line provided.

area
cubic units
customary units
diameter
estimation
formula
length
metric units
perimeter
rectangle
square units
triangle
volume
weight

__________________ 1. the use of rounding and/or other strategies to determine a reasonably accurate approximation without calculating an exact answer

__________________ 2. units for measuring volume

__________________ 3. the length of the boundary around a figure

__________________ 4. units for measuring area

__________________ 5. a polygon with four sides and four right angles

__________________ 6. a polygon with three sides

__________________ 7. a way of expressing a relationship using variables or symbols that represent numbers

__________________ 8. the units of measure developed in France and used in most of the world; uses the base 10, like the decimal system

__________________ 9. the units of measure developed in England and used in the United States

____________________ 10. measures that represent the force that attracts an object to the center of Earth

____________________ 11. a line segment from any point on the circle passing through the center to another point on the circle

____________________ 12. a one-dimensional measure that is the measurable property of line segments

____________________ 13. the amount of space occupied in three dimensions and expressed in cubic units; usually refers to solids

____________________ 14. the inside region of a two-dimensional figure measured in square units

Unit 3: Geometry

This unit emphasizes how models are used to sort, classify, make conjectures, and test geometric properties and relationships to solve problems.

Unit Focus

Number Sense, Concepts, and Operations

- Select the appropriate operation to solve problems involving ratios and proportions. (A.3.3.2)
- Add, subtract, multiply, and divide whole numbers and decimals, to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to predict results and to check the reasonableness of results. (A.4.3.1)

Measurement

- Use concrete and graphic models to derive formulas for finding perimeter, area, and volume. (B.1.3.1)
- Use concrete and graphic models to derive formulas for finding angle measures. (B.1.3.2)
- Understand and describe how the change of a figure in such dimensions as length, width, or radius affects its other measurements such as perimeter and area. (B.1.3.3)
- Use direct (measured) and indirect (not measured) to compare a given characteristic in customary units. (B.2.3.1)

Geometry and Spatial Relations

- Understand the basic properties of, and relationships pertaining to, geometric shapes in two dimensions. (C.1.3.1)
- Understand the geometric concepts of symmetry, reflections, congruency, similarity, perpendicularity, parallelism, and transformations, including flips, slides, turns, and enlargements. (C.2.3.1)
- Represent and apply geometric properties and relationships to solve real-world and mathematical problems. (C.3.3.1)
- Identify and plot ordered pairs of a rectangular coordinate system (graph). (C.3.3.2)

Algebraic Thinking

- Describe relationships through expressions and equations. (D.1.3.1)
- Create and interpret tables, equations, and verbal descriptions to explain cause-and-effect relationships. (D.1.3.2)
- Represent problems with algebraic expressions and equations. (D.2.3.1)

Farmer

- stores animal feed in a tower silo shaped like a cylinder
- determines the amount of feed that can be stored in a silo

Vocabulary

Study the vocabulary words and definitions below.

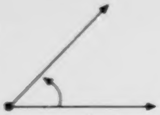
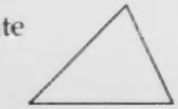
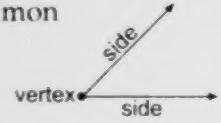
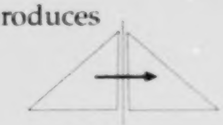
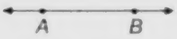
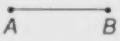
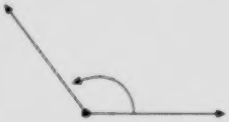
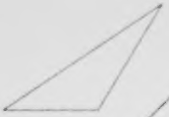
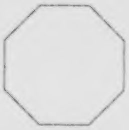
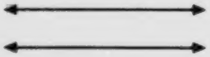
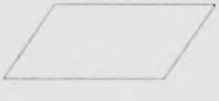
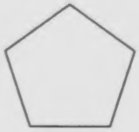
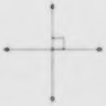
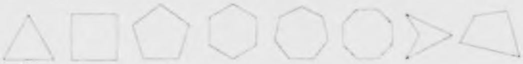
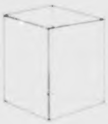
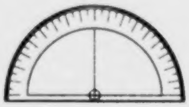
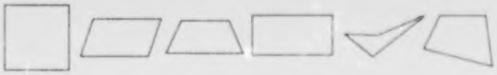
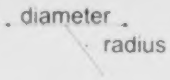
apex	the highest point of a triangle, cone, or pyramid; the vertex (corner) opposite a given base
acute angle	an angle with a measure of less than 90° 
acute triangle	a triangle with three acute angles 
angle	the shape made by two rays extending from a common endpoint, the vertex; measures of angles are described in degrees (°) 
area (*A*)	the inside region of a two-dimensional figure measured in square units *Example*: A rectangle with sides of four units by six units contains 24 square units or has an area of 24 square units.
axes (of a graph)	the horizontal and vertical number lines used in a rectangular graph or coordinate grid system as a fixed reference for determining the position of a point; (singular: *axis*)
base (*b*)	the line or plane upon which a figure is thought of as resting
center of a circle	the point from which all points on the circle are the same distance

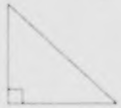
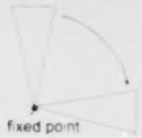
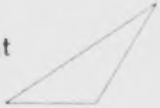
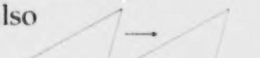
chart	see *table*
circle	the set of all points in a plane that are all the same distance from a given point called the center circumference
circumference (*C*)	the perimeter of a circle; the distance around a circle
congruent	figures or objects that are the same shape and the same size
coordinate grid or system	network of evenly spaced, parallel horizontal and vertical lines especially designed for locating points, displaying data, or drawing maps
coordinates	numbers that correspond to points on a graph in the form (x, y)
corresponding angles and sides	the matching angles and sides in similar figures
degree (°)	common unit used in measuring angles
diameter (*d*)	a line segment from any point on the circle passing through the center to another point on the circle
difference	the result of a subtraction *Example*: In 16 - 9 = 7, 7 is the difference.
equilateral triangle	a triangle with three congruent sides

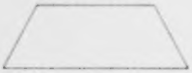
flip	a transformation that produces the mirror image of a geometric figure; also called a *reflection* 
grid	a network of evenly spaced, parallel horizontal and vertical lines
height (*h*)	a line segment extending from the vertex or *apex* (highest point) of a figure to its base and forming a right angle with the base or basal plane
hexagon	a polygon with six sides
intersection	the point at which two lines meet
isosceles triangle	a triangle with at least two congruent sides and two congruent angles
length (*l*)	a one-dimensional measure that is the measurable property of line segments
line	a straight line that is endless in length 
line of symmetry	a line that divides a figure into two congruent halves that are mirror images of each other
line segment	a portion of a line that has a defined beginning and end  *Example*: The line segment *AB* is between point *A* and point *B* and includes point *A* and point *B*.

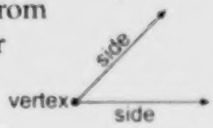
number line	a line on which numbers can be written or visualized
obtuse angle	the angle with a measure of more than 90° but less than 180° 
obtuse triangle	a triangle with one obtuse angle 
octagon	a polygon with eight sides 
ordered pairs	the location of a single point on a rectangular coordinate system where the digits represent the position relative to the *x*-axis and *y*-axis *Example*: (x, y) or (3, 4)
origin	the intersection of the *x*-axis and *y*-axis in a coordinate plane, described by the ordered pair (0, 0)
parallel	being an equal distance at every point so as to never intersect
parallel lines	two lines in the same plane that never meet; also, lines with equal slopes 
parallelogram	a polygon with four sides and two pairs of parallel sides 
pentagon	a polygon with five sides 

perimeter (*P*)	the length of the boundary around a figure; the distance around a polygon
perpendicular	forming a right angle
perpendicular lines	two lines that intersect to form right angles 
point	a location in space that has no length or width
polygon	a closed plane figure whose sides are straight lines and do not cross *Example*: triangle (3 sides), quadrilateral (4 sides), pentagon (5 sides), hexagon (6 sides), heptagon (7 sides), octagon (8 sides); concave, convex 
prism	a three-dimensional figure (polyhedron) with congruent, polygonal bases and lateral faces that are all parallelograms 
product	the result of a multiplication *Example*: In 6 x 8 = 48, 48 is the product.
proportion	a mathematical sentence stating that two ratios are equal *Example*: The ratio of 1 to 4 equals 25 to 100, that is $\frac{1}{4} = \frac{25}{100}$.
protractor	an instrument used for measuring and drawing angles 

quadrant	any of four regions formed by the axes in a rectangular coordinate system
quadrilateral	polygon with four sides *Example*: square, parallelogram, trapezoid, rectangle, concave quadrilateral, convex quadrilateral 
quotient	the result of a division *Example*: In $42 \div 7 = 6$, 6 is the quotient.
radius (r)	a line segment extending from the center of a circle or sphere to a point on the circle or sphere 
ratio	the quotient of two numbers used to compare two quantities *Example*: The ratio of 3 to 4 is $\frac{3}{4}$.
ray	a portion of a line that begins at a point and goes on forever in one direction
rectangle	a polygon with four sides and four right angles
reflection	see *flip*
reflectional symmetry	when a figure has at least one line which splits the image in half, such that each half is the mirror image or reflection of the other; also called *line symmetry* or *mirror symmetry*

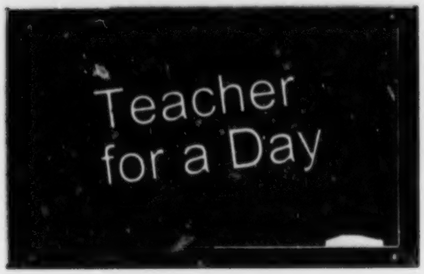
right angle	an angle whose measure is exactly 90°
right triangle	a triangle with one right angle 
rotation	a transformation of a figure by *turning* it about a center point or axis; also called a *turn*  *Example*: The amount of rotation is usually expressed in the number of degrees, such as a 90° rotation.
rotational symmetry	when a figure can be turned less than 360 degrees about its center point to a position that appears the same as the original position; also called *turn symmetry*
scale factor	the ratio between the lengths of corresponding sides of two similar figures
scalene triangle	a triangle with no congruent sides 
side	the edge of a geometric figure *Example*: A triangle has three sides.
similar figures	figures that have the same shape but not necessarily the same size
slide	to move along in constant contact with the surface in a vertical, horizontal, or diagonal direction; also called a *translation* 

square	a polygon with four sides the same length and four right angles
square units	units for measuring area; the measure of the amount of area that covers a surface
sum	the result of an addition *Example*: In 6 + 8 = 14, 14 is the sum.
symmetry	when a line can be drawn through the center of a figure such that the two halves are congruent
table (or chart)	an orderly display of numerical information in rows and columns
translation	see *slide*
translational symmetry	when a figure can slide on a plane (or flat surface) without turning or flipping and with opposite sides staying congruent
trapezoid	a polygon with four sides and exactly one pair of parallel sides 
triangle	a polygon with three sides; the sum of the measures of the angles is 180 degrees
turn	see *rotation*
units (of length)	measurement in inches, feet, yards, and miles

vertex	the common endpoint from which two rays begin or the point where two lines intersect; the point on a triangle or pyramid opposite to and farthest from the base; (plural: *vertices*); vertices are named clockwise or counter-clockwise 
volume (*V*)	the amount of space occupied in three dimensions and expressed in cubic units *Example*: Both capacity and volume are used to measure empty spaces; however, *capacity* usually refers to *fluids*, whereas *volume* usually refers to *solids*.
width (*w*)	a one-dimensional measure of something side to side
***x*-axis**	the horizontal (↔) axis on a coordinate plane
***x*-coordinate**	the first number of an ordered pair
***y*-axis**	the vertical (↕) axis on a coordinate plane
***y*-coordinate**	the second number of an ordered pair

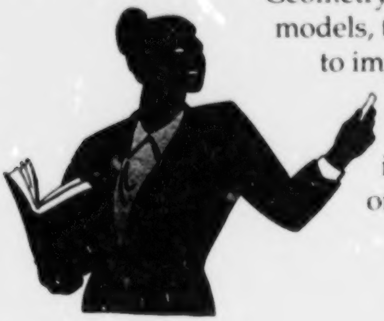
Unit 3: Geometry

Introduction



At Bos Middle School, one week was set aside for students to be the "Teacher for a Day." Geometry was chosen by the students to be the area of study during the week. Students organized and submitted lessons as part of their application to be the "Teacher for a Day."

Before you begin work on the lessons the students prepared, go to page 240, "Understanding Symmetry." This provides information on a lesson you are to prepare to share with your class. Look for examples in the world around you for the next few days to use in your lesson.

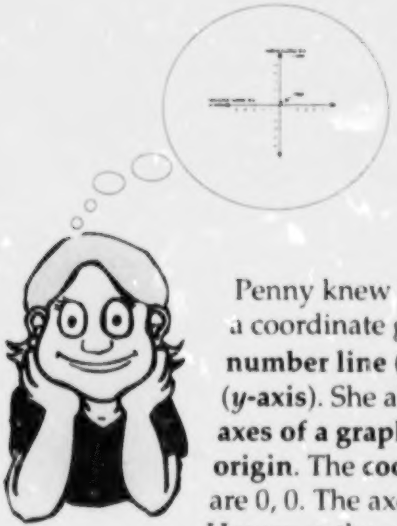


Geometry provides opportunities to draw and build models, to sort and classify, to conjecture and test, and to improve spatial visualization and reasoning skills. It can increase our appreciation of the physical world and our ability to function in it. Enjoy the lessons prepared for you and the one you will prepare for others.

Lesson One Purpose

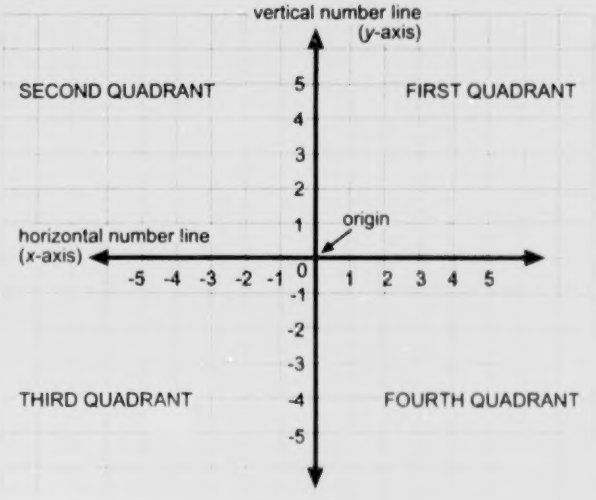
- Add and multiply whole numbers and decimals to solve problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Represent problems with algebraic expressions and equations. (D.2.3.1)
- Use concrete and graphic models to derive formulas for finding area, perimeter, and volume. (B.1.3.1)
- Understand the basic properties of, and relationships pertaining to, geometric shapes. (C.1.3.1)
- Understand the geometric concepts of reflections, congruency, perpendicularity, parallelism, and symmetry. (C.2.3.1)
- Identify and plot ordered pairs in a rectangular coordinate system (graph). (C.3.3.2)

Geometric Properties



Penny's challenge was to demonstrate as many geometric properties as possible using a **rectangle**. She made a series of *conjectures* (statements of opinions or educated guesses) and tested each one. But first she reviewed what she knew about a **coordinate grid or system** and geometric figures.

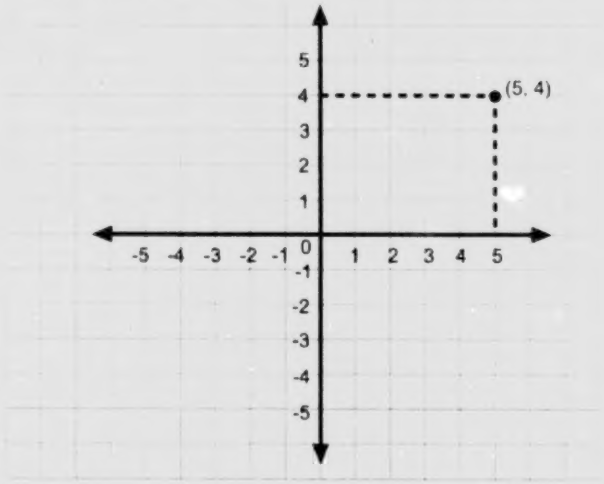
Penny knew that the **grid** on the following page is called a coordinate grid or system. It has a horizontal (⟷) **number line** (***x*-axis**) and a vertical (↕) number **line** (***y*-axis**). She also knew that these two number lines or **axes of a graph** *intersect* or meet at **point** zero (0) or the **origin**. The **coordinates** at the **intersection** of the origin are 0, 0. The axes form four regions or **quadrants**. However, the origin and the *x*- and *y*-axis are not in any quadrant.


vertical number line
(y-axis)
SECOND QUADRANT
FIRST QUADRANT
5
4
3
2
1
origin
horizontal number line
(x-axis)
-5 -4 -3 -2 -1 0 1 2 3 4 5
-1
-2
-3
-4
-5
THIRD QUADRANT
FOURTH QUADRANT

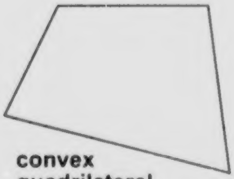
coordinate grid or system

Penny remembered that to place **ordered pairs** or *coordinates*, such as (5, 4) on a coordinate system, she had to do the following.

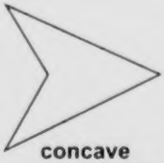
- start at the origin (0, 0) of the grid
- locate the first number of the ordered pair or the ***x*-coordinate** on the *x*-axis (⟷)
- then move **parallel** to the *y*-axis and locate the second number of the ordered pair or the ***y*-coordinate** on the y-axis (↕) and draw a dot


5
4
3
2
1
(5, 4)
-5 -4 -3 -2 -1 0 1 2 3 4 5
-1
-2
-3
-4
-5

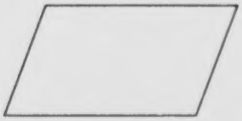
Here are the figures that Penny drew.



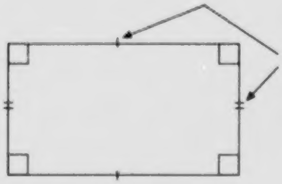
convex quadrilateral



concave quadrilateral

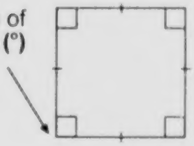


parallelogram



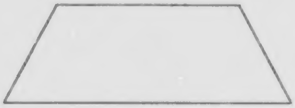
the tally marks (–) show **congruent** or same size **sides**

rectangle

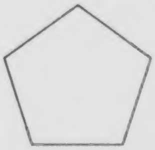


right angle of 90 **degrees (°)**

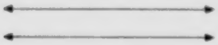
square



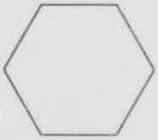
trapezoid



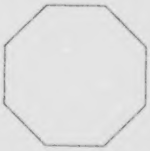
pentagon



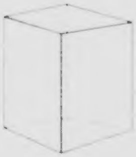
parallel lines



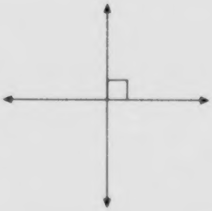
hexagon



octagon



prism



perpendicular lines

Practice

Use **Penny's figures** *on the previous page, the* **illustration below** *and the* **vocabulary words** *on pages 171-179 to test each of* **Penny's conjectures**. *Write* **True** *if the conjecture is correct. Write* **False** *if the conjecture is* not *correct.*

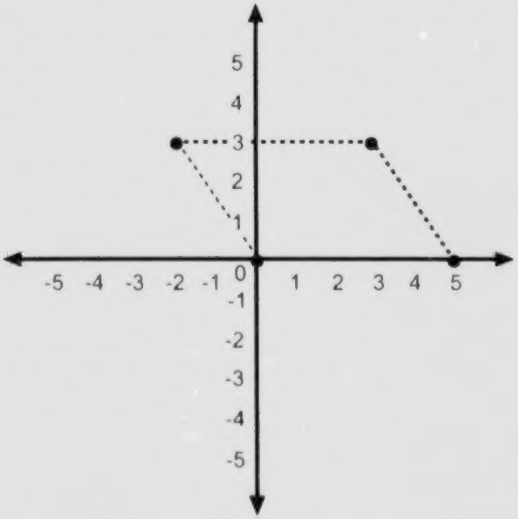
__________ 1. A rectangle is a four-sided **polygon** and is also a **quadrilateral**.

__________ 2. Opposite **sides** in a rectangle are **congruent** and **perpendicular**. A rectangle is also a **parallelogram**.

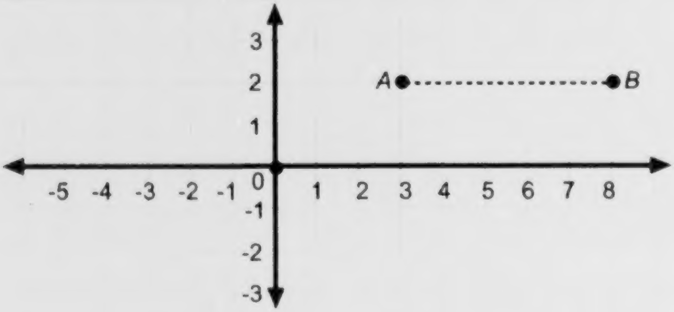
__________ 3. Adjacent sides are perpendicular in a rectangle and all **angles** are **right angles**. (*Adjacent* means next to each other.)

__________ 4. A rectangle can be a **square** but all rectangles are not squares.

__________ 5. On the illustration below, if two of the *vertices* of a parallelogram are at (0, 0) and (5, 0) or at (-2, 3) and (3, 3), the side determined by those vertices would have a **length** of 5 units..(*Vertices* is plural for **vertex**.)

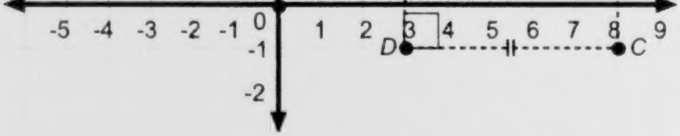


__________ 6. On the illustration below, point *A* is at (3, 2) and point *B* is at (8, 2). Points *A* and *B* are connected. The **line segment** is 8 units long.



__________ 7. If line segment *AB* above is the length of a rectangle, the sides representing the **width** must be perpendicular to line segment *AB*.

__________ 8. If the two vertices for the length of the rectangle below are located at (3, 2) and (8, 2) (as in number 6) and the width is three **units**, there are two choices for the other two vertices. They are (3, 5) and (8, 5) or (3, -1) and (8, -1).



__________ 9. If one of the other two vertices are named *C* and *D* as shown in the illustration above, the rectangle has many names including rectangle *ABCD*, *BCDA*, *CBAD*, and *BADC*. Names also include rectangle *ACBD*.

__________ 10. Drawing or tracing a small square at a vertex indicates that the angle measures 90 **degrees**.

__________ 11. Placement of one tally mark on one pair of opposite sides and two tally marks on the other pair of opposite sides indicates that opposite sides are the same length, or congruent.

__________ 12. If the length of the rectangle is 5 units and the width is 3 units, there are five rows with three unit squares in each row for a total **area** of 15 **square units**. Use the grid below to draw the figure.

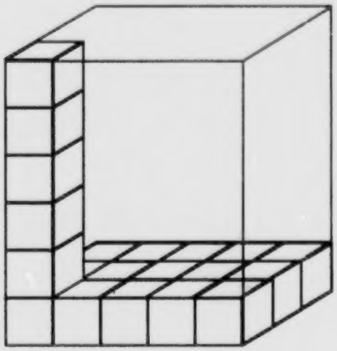
__________ 13. The **perimeter** of the figure in number 12 can be calculated by finding the **sum** of the four sides (5+3+5+3) for a total of 16 units.

- It could also be found by adding the length and width (5+3) and doubling the sum (2 x 8), for a total of 16 units.
- It could also be found by doubling the length (2 x 5), doubling the width (2 x 3), and adding the two **products** (10 + 6), for a total of 16 units.

__________ 14. The rules for finding perimeter can also be expressed as follows, where P represents the perimeter, l represents the length, and w represents the width.

- $P = l + w + l + w$ or
- $P = (l + w)2$ or
- $P = 2l + 2w$

__________ 15. If the rectangular **prism** below showed the **base** and had a **height** of six unit cubes, 15 unit cubes would be needed to fill the bottom layer.



__________ 16. The rectangular prism described in question 15 would have a **volume** of 90 square units because six layers with 15 cubes in each layer would be needed to fill the prism.

__________ 17. Other rectangles could be made having an area of 15 square units including a 1 by 15, a 2 by 7.5, a 4 by 3.5, a 6 by 2.5, a 9 by 1.5, or a 0.5 by 30.

__________ 18. The 0.5 by 30 rectangle would have a greater perimeter than a 5 by 3 rectangle.

________ 19. Four points are plotted at (4, 2), (4, 8), (9, 8), (9, 2). These points are connected and labeled *R*, *E*, *F*, and *L*. Four additional points are plotted at (4, -2), (4, -8), (9, -8) and (9, -2). These points are connected and labeled *R'*, *E'*, *F'*, and *L'*. One represents a **reflection** or mirror image of the other, and the x-axis is the line of **symmetry**. Use the grid below to plot the ordered pairs.

(**Remember:** The line of symmetry is the line that divides the figure into two congruent halves that are mirror images of each other. The labeled point *R'* is read *R* prime, *E'* is read *E* prime.)

_________ 20. If the coordinate grid in number 19 is folded along the *y*-axis, rectangle *REFL* fits exactly over rectangle *R'E'F'L'*.

_________ 21. Rectangles *REFL* and *R'E'F'L'* in number 19 are the same shape and the same size and are congruent.

Practice

Use the list below to complete the following statements. **One or more terms will be used more than once.**

area
congruent
parallel
parallelograms
perimeter
perpendicular
quadrilaterals
right
squares
square units
units

1. All rectangles are ______________________ and ______________________ .

2. Some rectangles are ______________________ .

3. A rectangle has four ______________________ angles.

4. Adjacent sides in a rectangle are ______________________ .

5. Opposite sides in a rectangle are ______________________ and ______________________ .

6. The product of the length and width is the ______________________ of a rectangle.

7. The sum of the measures of all sides is the ______________________ of a rectangle.

8. Area is reported in ______________________ .

9. Perimeter is reported in ______________________ .

Practice

Use the list below to write the correct term for each definition on the line provided.

- **axes**
- **coordinate grid or system**
- **coordinates**
- **intersection**
- **line**
- **line of symmetry**
- **number line**
- **ordered pairs**
- **origin**
- **parallel**
- **point**
- **quadrant**
- ***x*-axis**
- ***x*-coordinate**
- ***y*-axis**
- ***y*-coordinate**

________________ 1. the horizontal (↔) axis on a coordinate plane

________________ 2. the intersection of the *x*-axis and *y*-axis in a coordinate plane, described by the ordered pair (0, 0)

________________ 3. a straight line that is endless in length

________________ 4. numbers that correspond to points on a graph

________________ 5. a location in space that has no length or width

________________ 6. the second number of an ordered pair

________________ 7. the horizontal and vertical number lines used in a rectangular graph or coordinate grid system as a fixed reference for determining the position of a point

________________ 8. any of four regions formed by the axes in a rectangular coordinate system

________________ 9. being an equal distance at every point so as to never intersect

________________ 10. the first number of an ordered pair

______________________ 11. the location of a single point on a rectangular coordinate system where the digits represent the position relative to the x-axis and y-axis
Example: (x, y) or $(3, 4)$

______________________ 12. the vertical (↕) axis on a coordinate plane

______________________ 13. network of evenly spaced, parallel horizontal and vertical lines especially designed for locating points, displaying data, or drawing maps

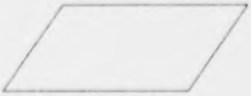
______________________ 14. a line on which numbers can be written or visualized

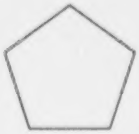
______________________ 15. the point at which two lines meet

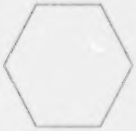
______________________ 16. a line that divides a figure into two congruent halves that are mirror images of each other

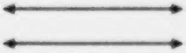
Practice

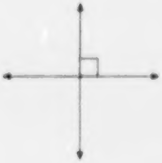
Match each **illustration** *with the correct term. Write the letter on the line provided.*

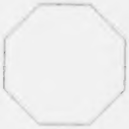
_____ 1. 

_____ 2. 

_____ 3. 

_____ 4. 

_____ 5. 

_____ 6. 

A. hexagon

B. octagon

C. parallel lines

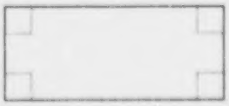
D. parallelogram

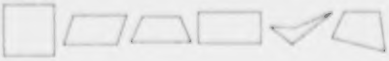
E. pentagon

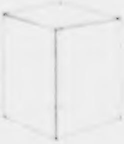
F. perpendicular lines

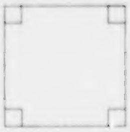
Practice

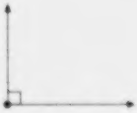
Match each **illustration** *with the* **most** *correct term. Write the letter on the line provided.*

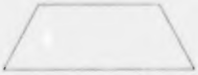
_____ 1. 

_____ 2. 

_____ 3. 

_____ 4. 

_____ 5. 

_____ 6. 

A. prism

B. quadrilaterals

C. rectangle

D. right angle

E. square

F. trapezoid

Practice

Use the list below to write the correct term for each definition on the line provided.

angle
congruent
degree
grid
length
line segment
perpendicular
polygon
side
units
vertex
width

____________________ 1. common unit used in measuring angles

____________________ 2. the shape made by two rays extending from a common endpoint, the vertex

____________________ 3. a one-dimensional measure that is the measurable property of line segments

____________________ 4. forming a right angle

____________________ 5. figures or objects that are the same shape and the same size

____________________ 6. measurement in inches, feet, yards, and miles

____________________ 7. a portion of a line that has a defined beginning and end

____________________ 8. the edge of a geometric figure

____________________ 9. a closed plane figure whose sides are straight lines and do not cross

____________________ 10. a one-dimensional measure of something side to side

____________________ 11. the common endpoint from which two rays begin or the point where two lines intersect

____________________ 12. a network of evenly spaced, parallel horizontal and vertical lines

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. the amount of space occupied in three dimensions and expressed in cubic units

______ 2. the result of a multiplication

______ 3. the inside region of a two-dimensional figure measured in square units

______ 4. a three-dimensional figure (polyhedron) with congruent, polygonal bases and lateral faces that are all parallelograms

______ 5. the line or plane upon which a figure is thought of as resting

______ 6. the length of the boundary around a figure; the distance around a polygon

______ 7. when a line can be drawn through the center of a figure such that the two halves are congruent

______ 8. a line segment extending from the vertex or *apex* (highest point) of a figure to its base and forming a right angle with the base or basal plane

______ 9. the result of an addition

______ 10. a transformation that produces the mirror image of a geometric figure; also called a *flip*

______ 11. units for measuring area

A. area

B. base

C. height

D. perimeter

E. product

F. prism

G. reflection

H. square units

I. sum

J. symmetry

K. volume

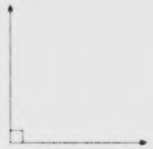
Lesson Two Purpose

- Add, subtract, and multiply whole numbers to solve problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to predict results and to check the reasonableness of results. (A.4.3.1)
- Use graphic models to derive formulas for finding angle measures. (B.1.3.2)
- Describe relationships through expressions and equations. (D.1.3.1)
- Understand the basic properties of, and relationships pertaining to, geometric shapes. (C.1.3.1)
- Identify and plot ordered pairs in a rectangular coordinate system (graph). (C.3.3.2)

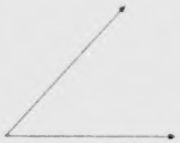
Measuring Angles

Sampson chose to develop a lesson on angles and **triangles**. You will need a **protractor** for measuring angles. You can use the straight edge on it to connect points.

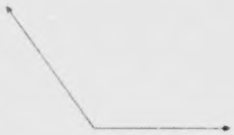
Sampson began his lesson with a quick review of angles and their measures. Angles are often classified as *right*, *acute*, and *obtuse*.



right angle - measures 90°



acute angle - measures between 0° and 90°



obtuse angle - measures between 90° and 180°

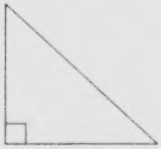
- The right angle is a good benchmark when estimating size of angles. The corner of a standard piece of paper fits perfectly in a *right angle*, and the measure is 90 degrees.

- If the angle is smaller than the right angle, it is an **acute angle** with a measure greater than 0 degrees and less than 90 degrees.
- If the angle is larger than the right angle, it is an **obtuse angle** with a measure greater than 90 degrees and less than 180 degrees.

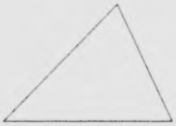
Classifying Triangles

Sampson also notes that triangles are classified in two different ways. Triangles are classified either by the measure of their angles or their sides.

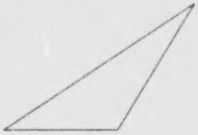
- Triangles classified by their angles are called **right triangles, acute triangles**, or **obtuse triangles**.



right triangle - has one right angle

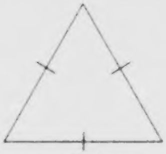


acute triangle - all angles are acute angles

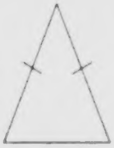


obtuse triangle - has one obtuse angle

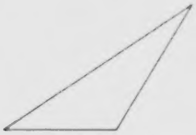
- Triangles measured by their sides are called **equilateral triangles, isosceles triangles**, or **scalene triangles**.



equilateral triangle - has three congruent sides



isosceles triangle - has at least two congruent sides

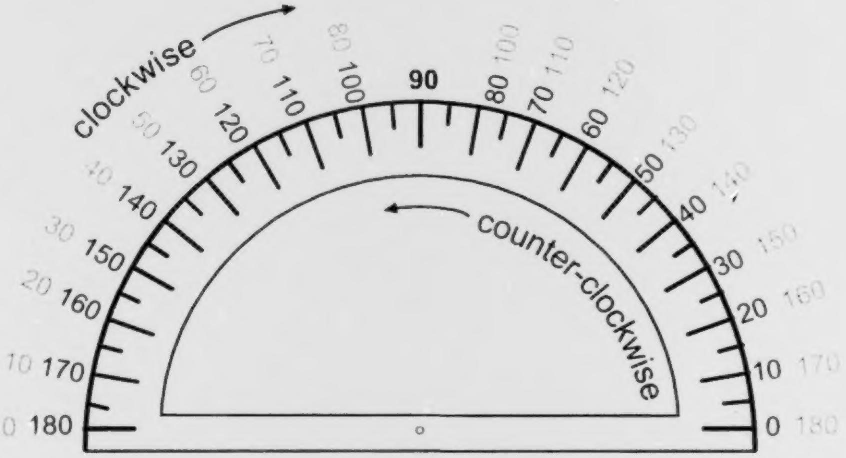


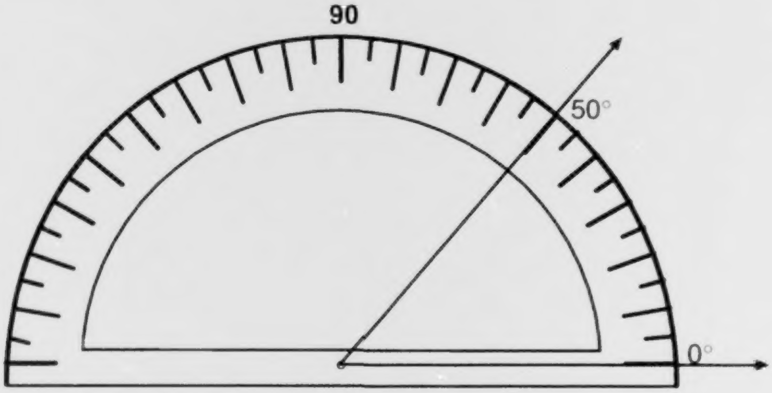
scalene triangle - has no congruent sides

(**Remember:** No matter how a triangle is classified, the sum of the measures of the angles in a triangle is 180 degrees.)

Using Protractors

Protractors are marked from 0 to 180 degrees in a clockwise manner as well as a counter-clockwise manner. We see 10 and 170 in the same position. We see 55 and 125 in the same position. If we estimate the size of the angle before using the protractor, there is no doubt which measure is correct.


clockwise
counter-clockwise
0 180
10 170
20 160
30 150
40 140
50 130
60 120
70 110
80 100
90



When using a protractor, make sure the vertex is lined up correctly and that one **ray** passes through the zero measure. A straightedge is often helpful to extend a ray for easier reading of the measure.

Practice

Draw a **right angle**, *an* **acute angle**, *and an* **obtuse angle**. *Label each of the angles and their measures. Practice measuring each with your* **protractor**. *Get a classmate to check your measures. If the two measures are within 3 to 5* **degrees** *of each other, you are ready to proceed. If not, more practice is needed.*

Practice

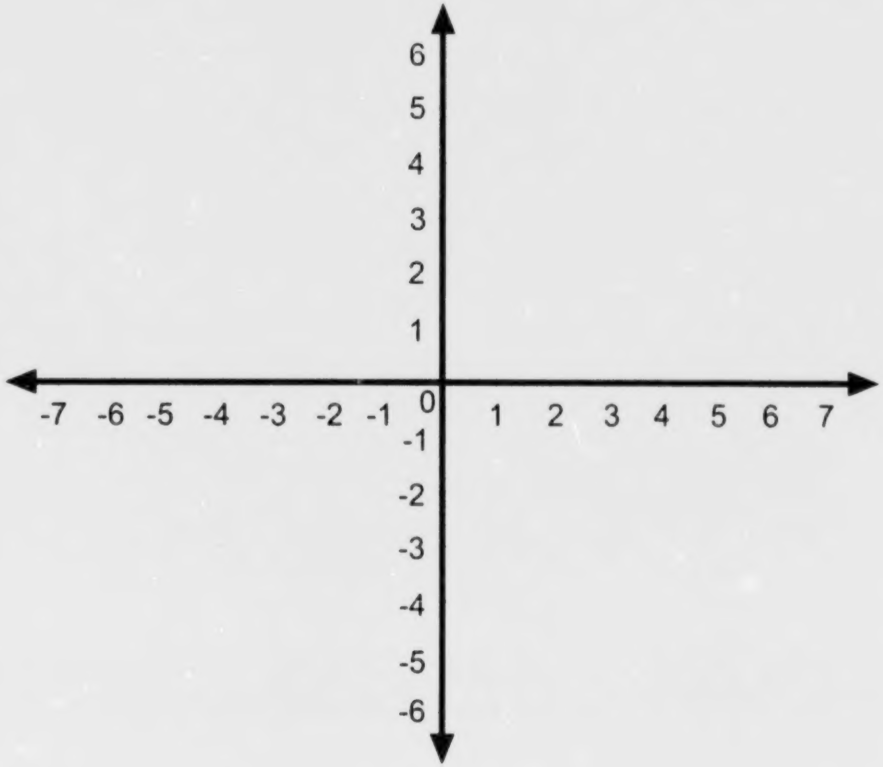
Use **Sampson's directions** *on pages 198-199 to create* **triangles** *and other* **polygons** *below. Then complete statements about your findings on pages 203-206.*

1. On a coordinate grid, plot, label, and connect each set of three points to create triangles.

   figure one *R*(1, 4) *I*(1, 1) *G*(4, 1)

   figure two *O*(1, -1) *B*(2, -3) *T*(5, -3)

   figure three *A*(-5, 4) *C*(-4, 1) *U*(-3,4)



Practice

Use the **triangles you drew** *on the previous pages, a* **protractor**, *and the list below to choose the correct term or angle measure(s) to complete the following statements.* **One or more terms will be used more than once.**

22
38
40
45
70
90
120
180
360
540
720
900
1080
acute
equilateral
five
greater
hexagon
isosceles
less
obtuse
pentagon
rectangle
right
scalene
triangles

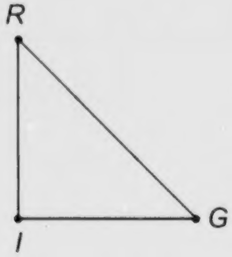
1. Angle *I* (also called angle *RIG* or angle *GIR**) is a

______________________ angle because it measures

______________________ degrees. Triangle *RIG* is

therefore a ______________________ triangle because it has

one ______________________ angle.



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* When angles are identified with three letters, the *vertex* label is in the center.

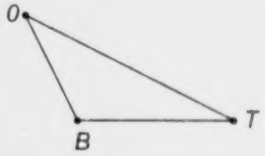
2. Angle *B* (also called angle *OBT* or angle *TBO*) is an

_________________________ angle because its measure is

_________________________ than 90 degrees. Triangle *OBT* is,

therefore, an _________________________ triangle because it has one

_________________________ angle.

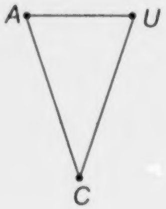


3. Angle *A*, angle *U*, and angle *C* are all _________________________

angles because each has a measure _________________________ than

90 degrees. Triangle *ACU* is, therefore, an _________________________

triangle because all angles are _________________________ angles.



4. Since no two sides in triangle *OBT* have the same measure, it is also called a ____________________ triangle.

5. Since two sides have the same measure in triangles *ACU* and *RIG*, they are also called ____________________ triangles.

6. None of the triangles are ____________________ because none have the same measure for all three sides.

7. The measures of angle *A*, angle *U*, and angle *C* are
____________________ degrees, ____________________
degrees, and ____________________ degrees, respectively, and
the sum of the angle measures is ____________________ degrees.

8. The measures of angle *R*, angle *I*, and angle *G* are
____________________ degrees, ____________________
degrees, and ____________________ degrees, respectively, and
the sum of the angle measures is ____________________ degrees.

9. The measures of angle *O*, angle *B*, and angle *T* are
____________________ degrees, ______________ degrees, and
____________________ degrees, respectively, and the sum of the
angle measures is ____________________ .

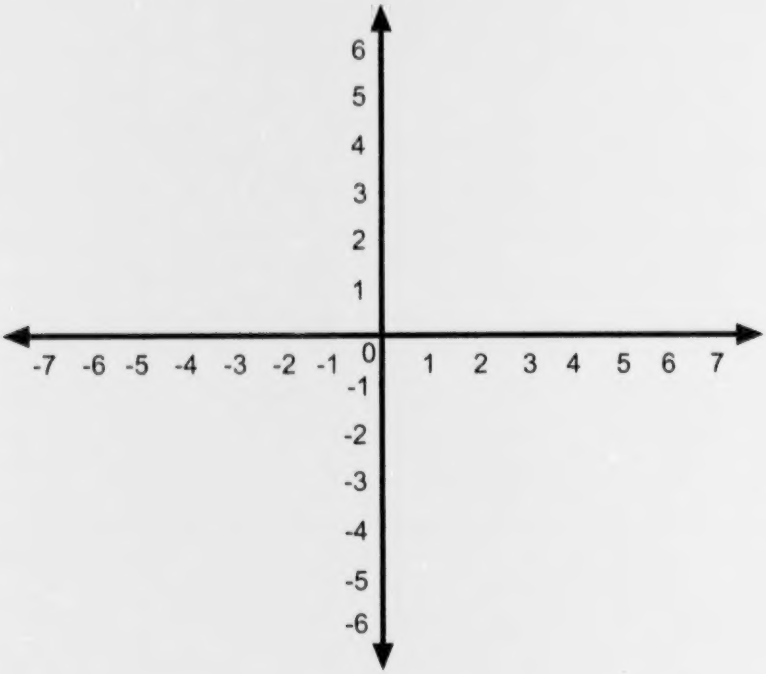
10. Draw several triangles, measure the angles in each and find the sum of the measures for each triangle. The sum of the measures for the angles in any triangle is ____________________ degrees.

11. On another coordinate grid, plot, label, and connect each set of points to form polygons.

figure four *R*(1, 2) *E*(5, 2) C(5, 5) *T*(1, 5)

figure five *P*(1, -1) *E*(3, -1) *N*(4, -2) *T*(3, -3) *A*(1, -3)

figure six *H*(-6, -2) *E*(-5, -3) *X*(-3, -3) *A*(-2, -2) G(-3, -1) O(-5, -1)



12. Figure *RECT* is a ____________________, and angles *R*, *E*, *C*, and *T* are all ____________________ angles.

13. Figure *PENTA* is a ____________________ , a five-sided polygon. Angles *E* and *T* are ____________________ angles while angle *N* is an ____________________ angle.

14. Figure *HEXAGO* is a ______________________, a six-sided polygon. Angles *O*, *E*, *G*, and *X* are ______________________ angles, and angles *A* and *H* are ______________________ angles.

15. In figure four, connect points *R* and *C* to form two right ______________________. Since the sum of the angle measures in any triangle is ______________________ degrees, the sum of the angle measures in figure four is ______________________ degrees.

16. In figure five, connect points *E* and *T* and points *E* and *A* to form three ______________________. Since the sum of the measures of the angles in any triangle is ______________________ degrees, the sum of the angle measures in figure five is ______________ degrees.

17. In figure six, connect points *G* and *X*, *G* and *E*, and *G* and *H* to form four ______________________. Since the sum of the measures of the angles in any triangle is ______________________ degrees, the sum of the measures of the angles in figure six is __________ degrees.

18. In the four-sided polygon, two ______________________ were formed. In the five-sided polygon, three ______________________ were formed. In the six-sided polygon, four ______________________ were formed. In a seven-sided polygon, ______________________ triangles would be expected, and the sum of the angle measures would be ______________________ degrees.

19. In general, if 2 is subtracted from the number of sides in a polygon and the **difference** is multiplied by ____________________, the sum of the angle measures in that polygon is the result. This can be expressed as a general rule: $S = (n - 2)180$ where S represents the sum of the angle measures and n represents the number of sides in the polygon.

20. Draw an 8-sided polygon (an octagon). Choose one vertex and make connections to all other vertices except the two already connected to this vertex as sides of the octagon. Six ____________________ are formed, and the sum of the angle measures in each is ____________________ degrees so the sum of the angle measures in the octagon is ____________________ degrees.

Practice

A classmate was absent and missed Sampson's lesson. For each statement below, make **drawings** *and* **comments** *to illustrate and explain each of the following points for the classmate. This will help the classmate and prove to Sampson that you understood his lesson.*

1. Angles may be classified as acute, obtuse, and right based on their measures.

2. Triangles may be classified as acute, obtuse, and right based on their angle measures.

3. Triangles may be classified as scalene, isosceles, or equilateral based on their side measures.

____

____

____

____

____

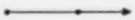
____

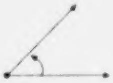
4. The sum of the angle measures for any triangle is 180 degrees.

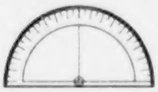
5. The sum of the angle measures for any polygon can be found by subtracting 2 from the number of sides and multiplying the difference by 180 degrees.

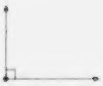
Practice

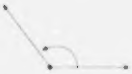
Match each illustration with the correct term. Write the letter in the line provided.

_____ 1. 

_____ 2. 

_____ 3. 

_____ 4. 

_____ 5. 

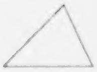
A. acute angle

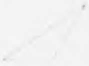
B. obtuse angle

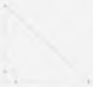
C. protractor

D. ray

E. right angle

_____ 6. 

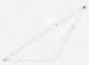
_____ 7. 

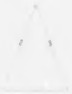
_____ 8. 

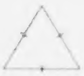
A. acute triangle

B. obtuse triangle

C. right triangle

_____ 9. 

_____ 10. 

_____ 11. 

A. equilateral triangle

B. isosceles triangle

C. scalene triangle

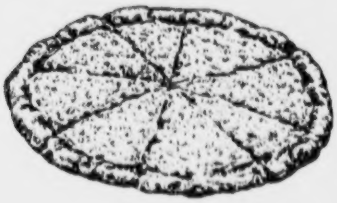
Lesson Three Purpose

- Add, subtract, multiply, and divide whole numbers and decimals to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to predict results. (A.4.3.1)
- Use direct (measured) and indirect measures (not measured) to compare a given characteristic in customary units. (B.2.3.1)
- Use graphic models to derive formulas for finding perimeter and area. (B.1.3.1)
- Understand and describe how the change of a figure in such dimensions as length, width, or radius affects its other measurements such as perimeter and area. (B.1.3.3)
- Create and interpret tables, equations, and verbal descriptions to explain cause-and-effect relationships. (D.1.3.2)
- Understand the basic properties of, and relationships pertaining to, geometric shapes in two dimensions. (C.1.3.1)
- Represent and apply geometric relationships to solve real-world problems. (C.3.3.1)
- Understand the geometric concept of enlargements. (C.2.3.1)

Analyzing Data to Make Comparisons

Lei works part-time for a pizza restaurant that makes square pizzas and round pizzas. Prices are shown in the **chart** for their Specialty Pizza.

Length of Side on Square Pizza	Price	Length or Diameter on Round Pizza	Price
6"	$6.00	6"	$5.00
9"	$9.00	9"	$8.00
12"	$12.00	12"	$11.00



Lei is often asked by customers which is the best buy. She has never taken the time to figure this out. The prices for square pizzas seem fair. As the side measure increases 3 inches, the cost increases by $3.00. When she serves pizzas, however, the 12-inch pizza looks bigger than two 6-inch pizzas.

The price for a 6-inch round pizza is less than the 6-inch square pizza, which seems fair since the "corners" of the square are missing in the round pizza. Round pizzas are measured by their **diameter**, not their **circumference**. Square pizzas are measured by their *sides*. The price increases by $3.00 as the *diameter* increases 3 inches. It all seems fair, but again, the 12-inch round looks like more pizza than two 6-inch rounds.

Lei asks Althamese to be her partner to develop a lesson that they will team teach. The lesson will feature the question concerning which pizza is the best buy.

Their lesson will help you look at some important relationships as you systematically analyze the data in the problem.

(**Remember:** Circumference is the distance or perimeter around a **circle**. Diameter is the line segment passing through the **center of a circle**.)

Practice

Draw **3 squares** *on your grid paper to represent the 3 square pizzas. The first should have a side measure of* **6 units**, *the second* **9 units**, *and the third* **12 units**. *Find the number of* **unit squares** *in each pizza and indicate this inside each drawing.*

(**Remember:** area = __________ square units.)

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Practice

Use your **3 squares** *drawn in the* **previous practice** *and the* **prices** *shown in the chart on page 217 to complete the following.*

1. Each side of the 12 by 12 square is __________ times as long as a side of the 6 by 6 square.
2. The perimeter of the 12 by 12 square is __________ times as much as the perimeter of the 6 by 6 square.
3. The area of the 12 by 12 square is __________ times as much as the 6 by 6 square.
4. Each side of the 9 by 9 square is __________ times as long as a side of the 6 by 6 square.
5. The perimeter of the 9 by 9 square is __________ times as much as the perimeter of the 6 by 6 square.
6. The area of the 9 by 9 square is __________ times as much as the area of the 6 by 6 square.
7. If the perimeter of the 12 by 12 square is divided by the perimeter of the 6 by 6 square, the **quotient** is __________ .
8. If the perimeter of the 9 by 9 square is divided by the perimeter of the 6 by 6 square, the quotient is __________ .
9. If the area of the 12 by 12 square is divided by the area of the 6 by 6 square, the quotient is __________ .

10. If the area of the 9 by 9 square is divided by the area of the 6 by 6 square the quotient is ________ .

11. There are 36 unit squares in the 6 by 6 square. On your 9 by 9 square, place check marks (√) on 36 unit squares. Place *x* marks on 36 additional unit squares. Number the unit squares left over from 1 to 9. Explain how this illustrates your answers from number 6 and 10. ____

____

____

12. There are 144 unit squares in the 12 by 12 square. Place check marks (√) on 36, of the unit squares, *x* marks on another 36, equal marks (=) on another 36, and percent signs (%) on another 36. You should have none left over. Explain how this illustrates your responses to number 3 and 9. ________________________

____

____

For numbers 13-18, refer to the chart on page 217.

13. The area of the 9 by 9 square is ________ times the area of the 6 by 6 square. The price of the 9" pizza is ________ times the price of the 6" pizza. Which would be the better buy? ________

____

14. The area of the 12 by 12 square is ____________ times the area of the 6 by 6 square. The price of the 12" pizza is ____________ times the price of the 6" pizza. Which would be the better buy?

____

15. The area of the 12 by 12 square is ____________ times the area of the 9 by 9 square. The price of the 12" pizza is ____________ times the price of the 9" pizza. Which would be the better buy?

____

16. If $6.00 is divided by the area of the 6 by 6 square, the quotient is the cost per square unit of this pizza which is ____________ .

17. If $9.00 is divided by the area of the 9 by 9 square, the quotient is the cost per square unit of this pizza which is ____________ .

18. If $12.00 is divided by the area of the 12 by 12 square, the quotient is the cost per square unit of this pizza which is ____________ .

19. Explain how your responses to numbers 16-18 support your responses to 13-15. ________________________________

____

____

____

____

____

Practice

Study the following **tables** *and* **statements**. *Copy each of the statements one time on the lines provided.*

Square Pizza Side Measure	**Growth Factor**	**Increase in Area**	**Increase in Perimeter**
6"	—	—	—
9"	1.5 or $\frac{3}{2}$	$2\frac{1}{4}$ or $\frac{9}{4}$ times	1.5 or $\frac{3}{2}$ times

Square Pizza Side Measure	**Growth Factor**	**Increase in Area**	**Increase in Perimeter**
6"	—	—	—
12"	2	4 times	2 times

- If the growth factor is 2, the area increases 4 times.

  ____

  ____

- If the growth factor is 3, the area increases 9 times.

  ____

  ____

- If the growth factor is 10, the area increases 100 times.

  ____

  ____

- If the growth factor is 2.5 or $\frac{5}{2}$, the area increases 6.25 or $\frac{25}{4}$.

  ____

  ____

- Area increases by the *square* of the growth factor, or the growth factor times itself.

- If the growth factor is 2, the perimeter doubles.

- If the growth factor is 3, the perimeter increases 3 times.

- The number of times the perimeter increases is the same as the growth factor.

Practice

Test the statements of the **previous practice** *by* **drawing** *each pair of figures and finding the* **area** *and* **perimeter** *of each.*

Draw a 2 by 3 rectangle and a 4 by 6 rectangle representing a growth factor of 2.

2 x 3 rectangle:

1. Perimeter = _____ units
2. Area = _____ square units

4 x 6 rectangle:

3. Perimeter = _____ units
4. Area = _____ square units

Draw a 1 by 3 rectangle and a 3 by 9 rectangle representing a growth factor of 3.

1 x 3 rectangle:

5. Perimeter = ________units

6. Area = _____ square units

3 x 3 rectangle:

7. Perimeter = _____ units

8. Area = ______ square units

Draw a 1 by 2 rectangle and a 5 by 10 rectangle representing a growth factor of 5.

1 x 1 rectangle:

9. Perimeter = ______ units

10. Area = _____ square units

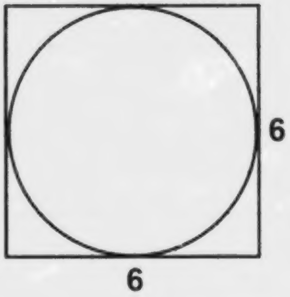
5 x 10 rectangle:

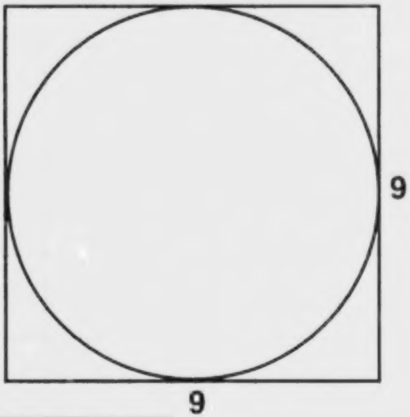
11. Perimeter = ______ units

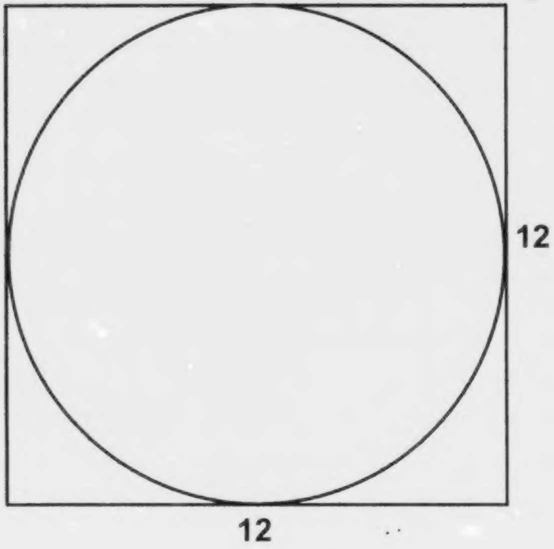
12. Area = _____ square units

Practice

A circle has been drawn in each of the squares below to represent three round pizzas having diameters of 6″, 9″, and 12″. **Estimate** *the number of* **square units** *in the* **four corners** *of each square that are* **outside** *the circle.*





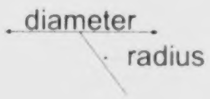


1. There are about __________ squares outside the 6″ pizza.
2. There are about __________ squares outside the 9″ pizza.
3. There are about __________ squares outside the 12″ pizza.

Practice

Use the **formula for area of a circle**, *$A = 3.14\ r^2$ and the* **areas found for the squares** *in practice on pages 222-223 to complete the following table.*

(**Remember:** To find the area (A) of a circle, you need to know its **radius** (r). The radius of a circle is $\frac{1}{2}$ of the diameter. So if you are given the diameter, you must first multiply it by $\frac{1}{2}$ to find the radius. Then use the formula for area of a circle, $A = 3.14\ r^2$. **Also remember:** r^2 is the radius times itself, in other words, $r^2 = r \times r$.)



Square Pizza Side Measure	Area of Square	Diameter of Circle	Radius of Circle	Area of Circle	Difference in Areas
6 units	____ square units	6 units	____ units	____ square units	____ square units
9 units	____ square units	9 units	____ units	____ square units	____ square units
12 units	____ square units	12 units	____ units	____ square units	____ square units

Practice

In questions 16-18 of the practice on pages 119-221, you found the **cost per unit square** *of pizza by* **dividing** *the* **cost** *of the pizza by the* **area** *of the pizza. Do this for the* **three sizes of round pizzas** *below to answer the following statements. Refer to* **chart** *on page 217 for* **pizza prices**.

1. __________ per square inch in the 6″ round at $5.00 a pizza.
2. __________ per square inch in the 9″ round at $8.00 a pizza.
3. __________ per square inch in the 12″ round at $11.00 a pizza.
4. The area of the 12″ round pizza is __________ times the area of the 6″ round pizza.
5. The area of the 9″ round pizza is __________ times the area of the 6″ round pizza.
6. Organize a table of cost per units for square and round pizzas. Compare the answers you got in questions 1-3 above with answers in questions 16-18 of the practice page 221. Which is the better buy, squares or rounds?

____

____

____

Practice

Answer the following using complete sentences.

Lei has learned a lot as she and Althamese have worked on this lesson together.

- She can now answer the questions asked by customers concerning pizza sizes, cost, and best buy.
- She wonders if the owner of the pizza business did this kind of analysis before pricing the pizzas.
- She believes that the owner would appreciate knowing what she has learned.
- She decides to summarize her findings in writing for her boss. She will also offer to show all of their work if more information is wanted.

1. Write a paragraph explaining the difference in the areas of the pizzas.

____

____

____

____

____

____

____

____

____

____

2. Write a paragraph explaining the cost per square unit for the pizzas.

3. Assume the price of the 6″ square will remain $6.00 and the 6″ round will remain $5.00. Write a paragraph proposing new prices for the 9″ and 12″ pizzas and the reason(s) for your proposal.

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Lesson Four Purpose

- Understand the geometric concepts of congruency and similarity. (C.2.3.1)
- Represent and apply geometric properties and relationships to solve real-world and mathematical problems. (C.3.3.1)
- Select the appropriate operation to solve problems involving ratios and proportions. (A.3.3.2)
- Multiply and divide whole number to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)

Applying Geometric Properties

The knowledge of **similar figures** is often helpful. Applications of this knowledge may help us to do the following tasks.

- Find the height of a building, pole, or basketball backboard without actually measuring it.
- Determine the height of a suspect recorded by a surveillance camera.
- Enlarge pictures or other printed materials.
- Reduce the size of pictures or other printed materials.
- Use scale drawings such as maps and blueprints.

We will begin with a definition of similar figures. Two figures are similar if

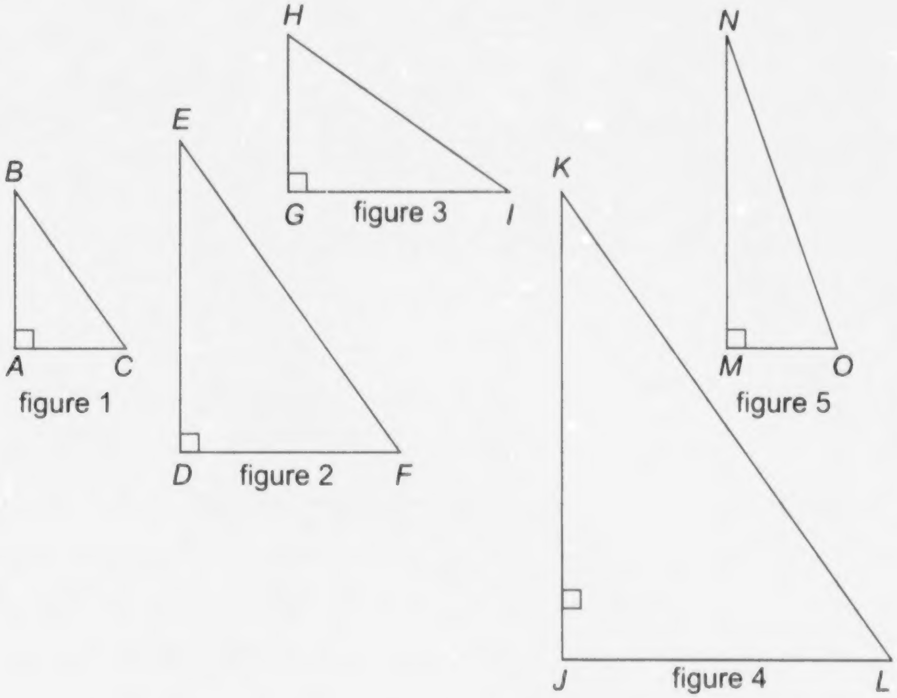
- the measures of their **corresponding angles** are equal and
- the lengths of **corresponding sides** increase by the same factor. (Meaning the sides are in **proportion** and have equal **ratios**.)

This factor is called the **scale factor**.

Practice

Use the directions above each section to answer the following.

Trace figure 1 below on a sheet of transparency and cut it out. Use the transparency cutout to check for congruency of corresponding angles in each of the other figures. To do this, place your transparent copy of figure 1 on figure 2 so that angle *B* in figure 1 is directly over angle *E* in figure 2.


B
A
C
figure 1
E
D
figure 2
F
H
G
figure 3
I
K
J
figure 4
L
N
M
O
figure 5

1. Is angle *B* in figure 1 congruent with angle *E* in figure 2?

   (yes, no) ____________

2. Is angle *B* in figure 1, congruent with angle *H* in figure 3?

   (yes, no) ____________

3. Is angle *B* in figure 1 congruent with angle *K* in figure 4?

   (yes, no) ____________

4. Is angle *B* in figure 1 congruent with angle *N* in figure 5?

   (yes, no) ____________

5. We know that angle *A*, angle *D*, angle *G*, angle *J*, and angle *M* are all ____________ angles because of the presence of the symbol for ____________ angles. We, therefore, know they are congruent and do not need to test them.

6. We know that the sum of the measures of the angles in any triangle is ____________ degrees from our work in Lesson Two and page 199.

If angle *E* was congruent to angle *B* when you checked it, then angle *C* has to be congruent to angle *F* because of statements on the previous page. Use your transparent copy of figure 1 to verify this.

7. We now know that corresponding angles in figures 1 and ____________ are congruent and that corresponding angles in figures 1 and ____________ are congruent.

8. We also know that figures ____________ and ____________ do not meet the corresponding angles being congruent criteria so they cannot be similar to figure 1.

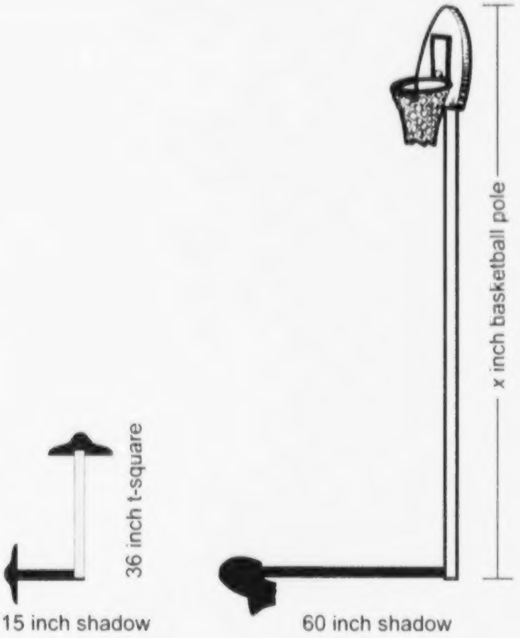
9. Line segment *DE* in figure 2 corresponds to line segment *AB* in figure 1. It is twice as long as *AB*. Line segment *DF* in figure 2 corresponds to line segment *AC* in figure 1. It is twice as long as *AC*. If I use a straightedge to measure line segments *BC* and *EF*, I will find that the same is true for *BC* and *EF*. The scale factor from figure 1 to figure 2 is __________. The triangles are __________.

10. The scale factor from figure 1 to figure__________ is __________, and these two triangles are also similar.

11. In figure 5, line segment *MO* is the same length as line segment *AC* in figure 1, but line segment *MN* in figure 5 is __________ times as long as line segment *AB* in figure 1.

12. In figure 3, line segment *GI* is __________ times as long as segment *AC* in figure 1, but line segment *GH* in figure 3 is the same length as line segment *AB* in figure 1.

The shadow method is commonly used to estimate the height of an object that may be inconvenient to measure directly.


x inch basketball pole
36 inch t-square
15 inch shadow
60 inch shadow

13. A basketball backboard and its supporting pole casts a shadow 60″ long. The pole is *perpendicular* to the ground so a ____________ angle is formed.

14. A t-square (or a yardstick) held perpendicular to the ground at the same time of day casts a shadow 15 inches long.
Draw the third side of each triangle on the illustration above. The t-square and shadow form two sides of a ____________ triangle just as the pole and its shadow form two sides of a larger ____________ triangle.

15. The sun's rays are at the same angle since the shadows are cast at the same time of day. We have two corresponding angles that are ____________ angles and two corresponding angles that are formed by the sun's rays and are congruent.

16. The length of the shadow of the t-square times what number equals 60? **Note:** 60 is the length of the shadow of the pole on the previous page.

(15 x ? = 60) To find this, divide 60 by 15 and get ____________ , which is the scale factor. Multiply the height of the t-square by this scale factor to get the height of the pole.

(36 x ____________ = ____________)

17. A person standing at a table of sweaters in a department store appears to be shoplifting a sweater. The action is recorded on a surveillance camera, and a photograph is made from the recording. The height of the table holding the sweaters is 30" tall but in the photograph, it is 3" tall. The scale factor is ____________ . To determine the approximate height of the suspected shoplifter, the height of the suspect in the photograph must be multiplied by the same scale factor.

Practice

Interview *someone with knowledge of copy machines or someone with knowledge of enlarging photographs. Find out all you can about* **enlargements** *and* **size reductions** *made with* **photocopiers** *or the* **enlarging** *or* **reducing** *of* **photographs**. *Write a* **summary** *of your findings. Make a* **scale drawing** *of a picture from a coloring book or greeting card by drawing a 2-centimeter (cm) by 2-cm grid on the picture. Create a 1-cm by 1-cm grid and a 3-cm by 3-cm grid. Duplicate the original picture one square at a time. Provide a* **key showing the scale** *used.*

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. the result of a subtraction

______ 2. the perimeter of a circle; the distance around a circle

______ 3. the ratio between the lengths of corresponding sides of two similar figures

______ 4. the quotient of two numbers used to compare two quantities

______ 5. a line segment from any point on the circle passing through the center to another point on the circle

______ 6. the result of a division

______ 7. a line segment extending from the center of a circle or sphere to a point on the circle or sphere

______ 8. the matching angles and sides in similar figures

______ 9. a mathematical sentence stating that two ratios are equal

______ 10. the set of all points in a plane that are all the same distance from a given point called the center

______ 11. the point from which all points on the circle are the same distance

______ 12. an orderly display of numerical information in rows and columns

A. center of a circle

B. circle

C. circumference

D. corresponding angles and sides

E. diameter

F. difference

G. proportion

H. quotient

I. radius

J. ratio

K. scale factor

L. table

Lesson Five Purpose

- Understand the geometric concepts of symmetry and related transformations. (C.2.3.1)
- Use concrete models to derive formulas for finding angle measures. (B.1.3.2)

Understanding Symmetry

You are to prepare a lesson on symmetry.

- It should have three parts.
- Part one will feature reflectional symmetry, part two will feature rotational symmetry, and part three will feature translational symmetry.
- Each part should include a definition.
- Each part should include examples found in the real world illustrating that type of symmetry. Illustrations may be drawn, photographed, clipped from newspapers or magazines, or generated by computer.
- A brief explanation should accompany your illustrations in each part.
- Among the many places to look for your examples will be nature, art, fabrics, wallpaper, architecture, and letters of the alphabet.

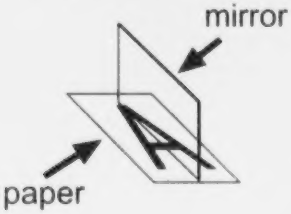
Part One: Reflectional Symmetry

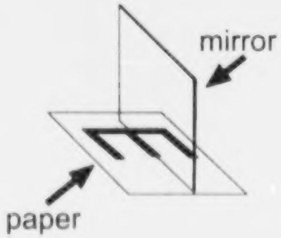
A figure has **reflectional symmetry** if there is at least one line which splits the image in half. Once split, one side is the mirror image or *reflection* of the other.

If a line is drawn through the center of a figure and the two halves are congruent, the figure has reflectional symmetry. This is often called *line symmetry* or *mirror symmetry*.

(**Remember:** It is possible to have more than one line of reflectional symmetry.)

Try this with the capital letters A and E.


mirror
paper


mirror
paper

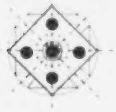
When testing figures for reflectional symmetry, look for a line of symmetry.

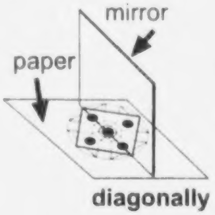
- You might fold the figure on a line to test for a match.
- You might use a mirror or reflecting device on a line to test for a match.
- You might use tracing paper to trace half of the figure and **flip** the tracing over a line to test the image for a match.

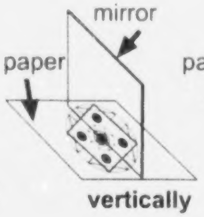
The following letters illustrate reflectional symmetry, either horizontally or vertically.

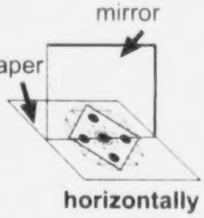
K M H

The following design also illustrates reflectional symmetry horizontally, vertically, and diagonally.



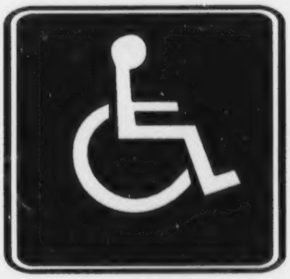

mirror
paper
diagonally

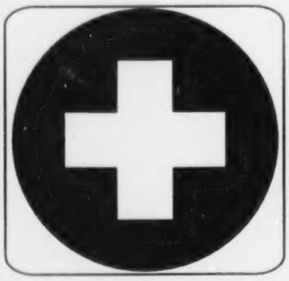

mirror
paper
vertically

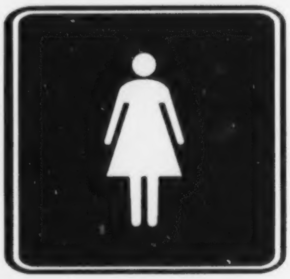

mirror
paper
horizontally

Practice

Determine if the **shapes** *of the following* **signs** *and their* **symbols** *illustrate* **reflectional symmetry**. *Below each sign write* **yes** *or* **no**.



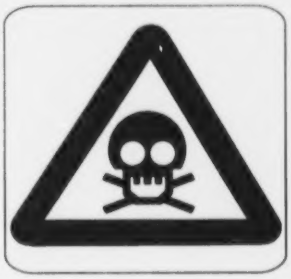


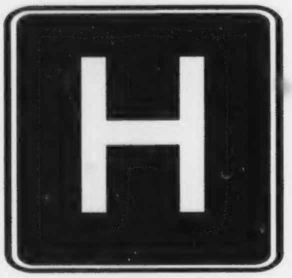


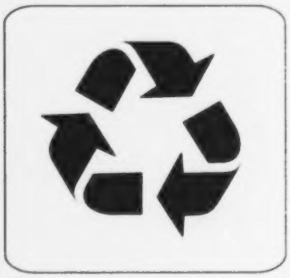
1. ____________

2. ____________

3. ____________







4. ____________

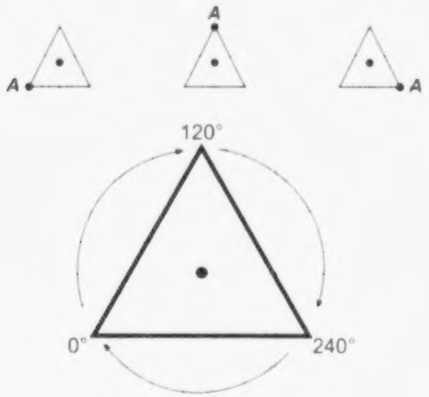
5. ____________

6. ____________

Part Two: Rotational Symmetry

If a figure can be rotated by turning it less than 360 degrees about its center point to a position that appears the same as the original position, then the figure has **rotational symmetry**. This is often called **turn** *symmetry*. Try this with an equilateral triangle.

- If you cut an identical triangle from a piece of paper,
  - — lay it on top of the original triangle, and
  - — turn the figure about its center point 120 degrees,
  - — the turned figure appears the same as the original.
- If you turn it another 120 degrees for a total of 240 degrees the result is the same.
- A third turn of 120 degrees returns the triangle to its original position. The angle of **rotation** for this figure is 120 degrees.



Finding the center point is sometimes challenging, but keep trying. Once you find the center and rotate successfully to find a match, the angle must be measured to determine the angle of rotation.

When testing figures for rotational symmetry, look for center point symmetry.

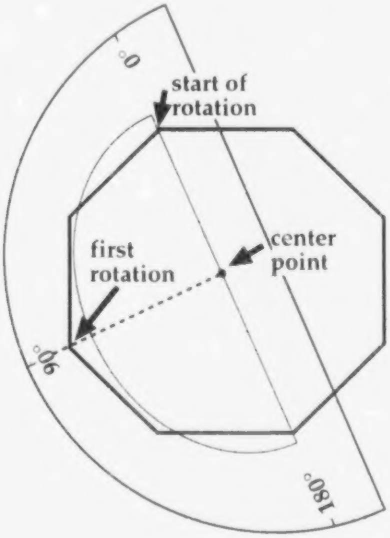
- You might trace the figure using tracing paper and
- rotate the tracing to test for a match.

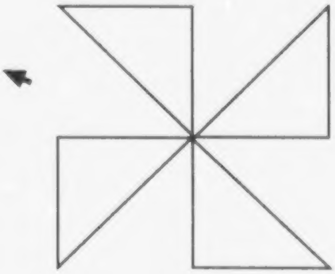
The following figures illustrate rotational symmetry. Each figure can be rotated around a center point. The rotation leaves the figure looking exactly the same. Use a protractor to measure the angle of rotation.

- Place the center of the protractor on the center point of the figure.
- Line up the 0 degree mark with the start of the rotation.
- Use a straightedge to extend the line for easier reading of the measure.
- Rotate the figure and mark the place the rotated figure matches the original figure.
- Extend the line with a straightedge and note the degree on the protractor.

The degree the rotation stopped at is called the angle of rotation.

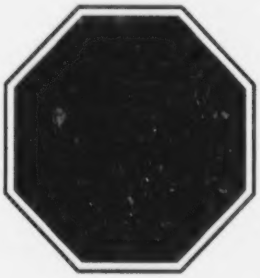
Note that the measure of the angle of rotation for each figure below is 90 degrees (see page 200, Using Protractors).

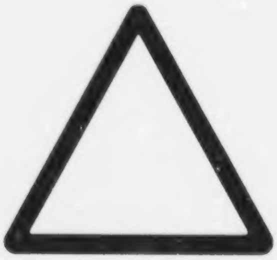


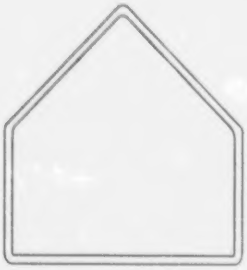


Practice

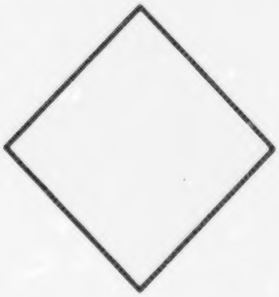
Determine if the **shapes** *of the following* **signs** *illustrate* **rotational symmetry**. *Below each sign write* **yes** *or* **no**. *If* **yes**, *write the measure of the* **angle of rotation**.

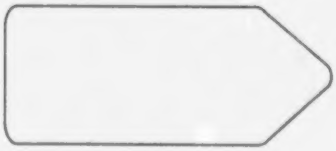


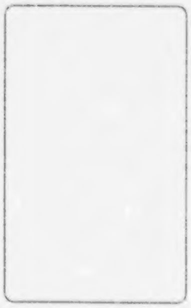




1. ____________________
2. ____________________
3. ____________________







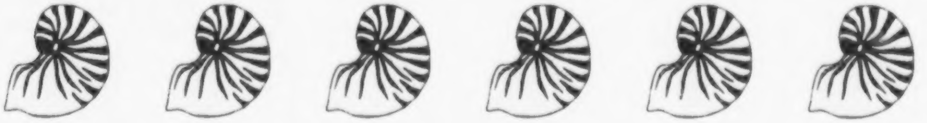
4. ____________________
5. ____________________
6. ____________________

Practice

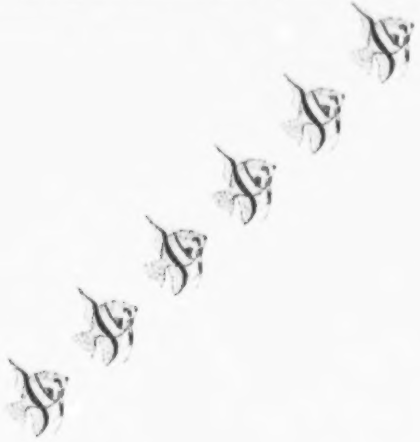
Draw **three figures** *that illustrate* **rotational symmetry**. *Write the* **angle of rotation** *below each of your illustrations.*

Part Three: Translational Symmetry

When one slides a figure a specific distance in a straight line from one place to another, **translational symmetry** occurs. The distance of the **slide**, or **translation**, and the direction of the slide are the important elements here. They are often illustrated by an arrow with its endpoint on one point of the first figure and an arrow on the same point in the next figure (see grid below).

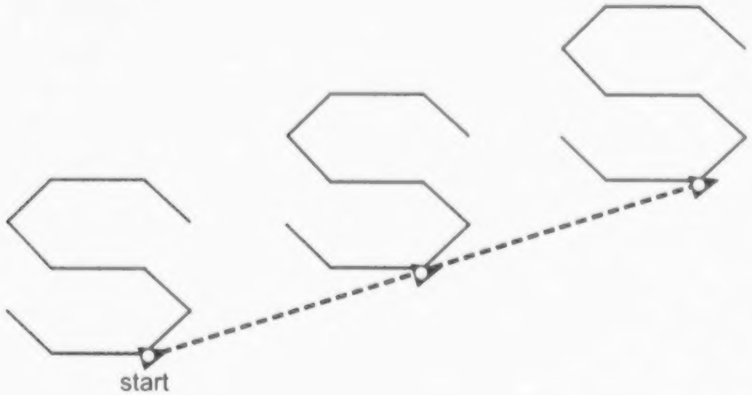


(The figure slides one space to the right.)



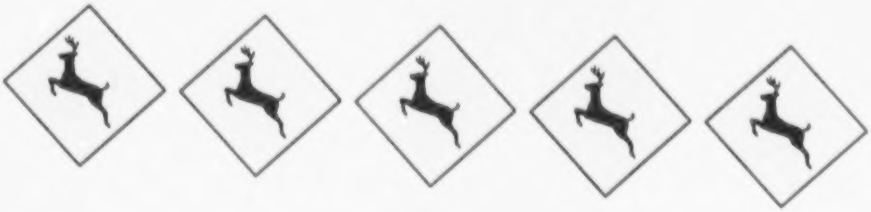
(The figure slides one space to the right and one up.)

The figure below illustrates translational symmetry.

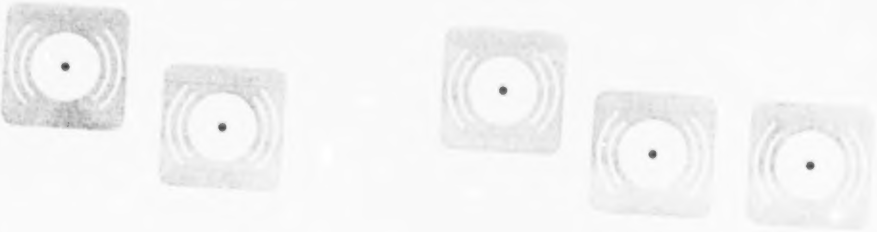


Practice

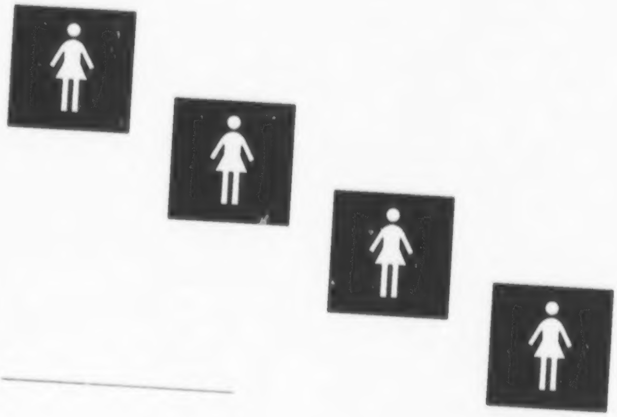
Determine if the following **signs** *illustrate* **translational symmetry**. *Below each grouping of signs write* **yes** *or* **no**.



1. ____________________



2. ____________________



3. ____________________

Practice

Draw **three figures** *that illustrate* **translational symmetry**. *Provide an arrow to specify the* **length** *and* **direction** *of your* **slide** *for each of your illustrations, like the bottom figure on page 247.*

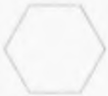
- You might use tracing paper to trace the figure and slide it in a straight line along its endpoint to test for a match.
- As you test figures for translational symmetry, an endpoint is sought to slide the figure along a straight line.

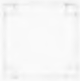
Practice

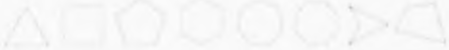
It's time to prepare and present your **lesson** *on* **symmetry**. *Include a definition of* **reflectional symmetry**, **rotational symmetry**, *and* **translational symmetry**. *Provide* **three real-world illustrations of each type of symmetry** *and give a brief* **explanation of each illustration**. *Illustrations may be drawn, photographed, clipped from newspapers or magazines, or generated by computer.*

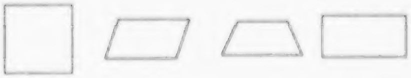
Practice

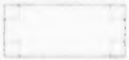
Match each **illustration** *with the* **most** *correct term. Write the letter on the line provided.*

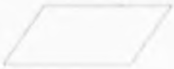
______ 1. 

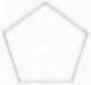
______ 2. 

______ 3. 

______ 4. 

______ 5. 

______ 6. 

______ 7. 

A. hexagon

B. parallelogram

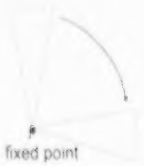
C. pentagon

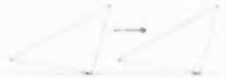
D. polygon

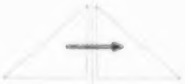
E. rectangle

F. square

G. quadrilateral

______ 8. 

______ 9. 

______ 10. 

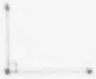
A. reflectional symmetry (flip)

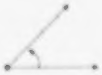
B. rotational symmetry (turn)

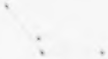
C. translational symmetry (slide)

Practice

Match each **illustration** *with the correct term. Write the letter on the line provided.*

______ 1. 

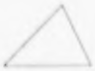
______ 2. 

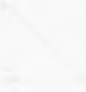
______ 3. 

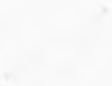
A. acute angle

B. obtuse angle

C. right angle

______ 4. 

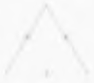
______ 5. 

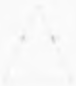
______ 6. 

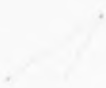
A. acute triangle

B. obtuse triangle

C. right triangle

______ 7. 

______ 8. 

______ 9. 

A. equilateral triangle

B. isosceles triangle

C. scalene triangle

Practice

Use the list below to write the correct term for each definition on the line provided.

angle
area
congruent
coordinates
degree
length
perimeter
square units
sum

1. ____________________ a one-dimensional measure that is the measurable property of line segments

2. ____________________ numbers that correspond to points on a graph in the form (x, y)

3. ____________________ units for measuring area; the measure of the amount of area that covers a surface

4. ____________________ the shape made by two rays extending from a common endpoint, the vertex

5. ____________________ the inside region of a two-dimensional figure measured in square units

6. ____________________ the length of the boundary around a figure; the distance around a polygon

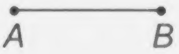
7. ____________________ common unit used in measuring angles

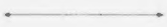
8. ____________________ the result of an addition

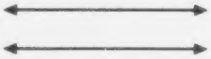
9. ____________________ figures or objects that are the same shape and the same size

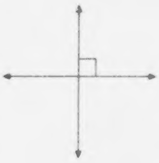
Practice

Match each **illustration** *with the correct term. Write the letter on the line provided.*

_____ 1. 

_____ 2. 

_____ 3. 

_____ 4. 

A. diameter

B. line segment

C. parallel lines

D. perpendicular lines

Unit 4: Creating and Interpreting Patterns and Relationships

This unit emphasizes how patterns of change and relationships are used to describe, estimate reasonableness, and summarize information with algebraic expressions or equations to solve problems.

Unit Focus

Number Sense, Concepts, and Operations

- Use exponential and scientific notation. (A.2.3.1)
- Add, subtract, multiply, and divide whole numbers and decimals to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculators. (A.3.3.3)
- Use estimation strategies to predict results and to check for reasonableness of results. (A.4.3.1)
- Use concepts about numbers to build number sequences. (A.5.3.1)

Measurement

- Use a model to derive the formula for circumference. (B.1.3.1)
- Derive and use formulas for finding rate, distance, and time. (B.1.3.2)

Geometry and Spatial Relations

- Understand the basic properties of, and relationships to, geometric shapes in two dimensions. (C.1.3.1)
- Identify and plot ordered pairs in a rectangular coordinate system. (C.3.3.2)

Algebraic Thinking

- Describe a wide variety of patterns and relationships through tables and graphs. (D.1.3.1)
- Create and interpret tables, graphs, and verbal descriptions to explain cause-and-effect relationships. (D.1.3.2)
- Represent and solve real-world problems graphically and with algebraic expressions and equations. (D.2.3.1)
- Use algebraic problem-solving strategies to solve real-world problems. (D.2.3.2)

Market Researcher

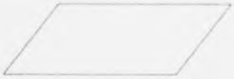
- collects data about customers' preferences, attitudes, and interests
- uses data to predict which types of products people might buy
- companies use data and predictions to decide how to invest their time and money

Vocabulary

Study the vocabulary words and definitions below.

algebraic expression	an expression containing numbers and variables and operations that involve numbers and variables but not containing equality or inequality symbols *Example*: $7x$ or $2x + y$
algebraic rule	a mathematical expression that contains variables and describes a pattern or relationship
area (A)	the inside region of a two-dimensional figure measured in square units *Example*: A rectangle with sides of four units by six units contains 24 square units or has an area of 24 square units.
axes (of a graph)	the horizontal and vertical number lines used in a rectangular graph or coordinate grid system as a fixed reference for determining the position of a point; (singular: *axis*)
base (of an exponent)	the number that is used as a factor a given number of times *Example*: (base) $2^{3\text{ (exponent)}}$
center of a circle	the point from which all points on the circle are the same distance
chart	see *table*
circle	the set of all points in a plane that are all the same distance from a given point called the center

circumference (*C*)	the perimeter around a circle; the distance around a circle circumference
composite number	any whole number that has more than two factors *Example*: 16 has five factors—1, 2, 4, 8, and 16.
coordinate grid or system	network of evenly spaced, parallel horizontal and vertical lines especially designed for locating points, displaying data, or drawing maps
data	information in the form of numbers gathered for statistical purposes
data display	different ways of displaying data in tables, charts, or graphs *Example*: pictographs; circle graphs; single, double, or triple bar and line graphs; histograms; stem-and-leaf plots; and scatterplots
diameter (*d*)	a line segment from any point on the circle passing through the center to another point on the circle
equation	a mathematical sentence that equates one expression to another expression *Example*: $2x = 10$
estimation	the use of rounding and/or other strategies to determine a reasonably accurate approximation without calculating an exact answer

exponent (exponential form)	the number of times the base occurs as a factor *Example*: 2^3 is the exponential form of 2 x 2 x 2. The numeral two (2) is called the *base*, and the numeral three (3) is called the *exponent*.
factor	a number or expression that divides exactly another number *Example*: 1, 2, 4, 5, 10, and 20 are factors of 20.
formula	a way of expressing a relationship using variables or symbols that represent numbers
graph	a drawing used to represent data *Example*: bar graphs, double bar graphs, circle graphs, and line graphs
labels (for a graph)	the titles given to a graph, the axes of a graph, or the scales on the axes of a graph
multiples	the numbers that result from multiplying a given number by the set of whole numbers *Example*: the multiples of 15 are 0, 15, 30, 45, 60, 75, etc.
ordered pair	the location of a single point on a rectangular coordinate system where the digits represent the position relative to the *x*-axis and *y*-axis *Example*: (x, y) or (3, 4)
parallelogram	a polygon with four sides and two pairs of parallel sides 

pattern (relationship)	a predictable or prescribed sequence of numbers, objects, etc.; also called a *relation* or *relationship*; may be described or presented using manipulatives, tables, graphics (pictures or drawings), or algebraic rules (functions) *Example*: 2, 5, 8, 11...is a pattern. The next number in this sequence is three more than the preceding number. Any number in this sequence can be described by the algebraic rule, $3n - 1$, by using the set of counting numbers for n.
perimeter (P)	the length of a boundary of a figure; the distance around a polygon
polygon	a closed figure whose sides are straight and do not cross *Example*: triangle (3 sides), quadrilateral (4 sides), pentagon (5 sides), hexagon (6 sides), heptagon (7 sides), octagon (8 sides); concave, convex
power (of a number)	an exponent; the number that tells how many times a number is used as a factor *Example*: $2^{3 \text{ (power)}}$
prime number	any whole number with only two factors, 1 and itself *Example*: 2, 3, 7, 11, etc.
prism	a three-dimensional figure (polyhedron) with congruent, polygonal, and lateral faces that are all parallelograms

product	the result of a multiplication *Example*: In $6 \times 8 = 48$, 48 is the product.
quadrant	any of the four regions formed by the axes in a rectangular coordinate system
quotient	the result of a division *Example*: In $42 \div 7 = 6$, 6 is the quotient.
relationship (relation)	see *pattern*
rule	a mathematical expression that describes a pattern or relationship, or a written description of the pattern or relationship
scales	the numeric values assigned to the axes of a graph
scientific notation	a shorthand method of writing very large or very small numbers using exponents in which a number is expressed as the product of a power of 10 and a number that is greater than or equal to one (1) and less than 10 *Example*: The number is written as a decimal number between 1 and 10 multiplied by a power of 10, such as $7.59 \times 10^5 = 759{,}000$. It is based on the idea that it is easier to read exponents than it is to count zeros. If a number is already a power of 10, it is simply written 10^{27} instead of 1×10^{27}.
sum	the result of an addition *Example*: In $6 + 8 = 14$, 14 is the sum.
table (or chart)	an orderly display of numerical information in rows and columns

triangle	a polygon with three sides
variable	any symbol that could represent a number
volume (*V*)	the amount of space occupied in three dimensions and expressed in cubic units *Example*: Both capacity and volume are used to measure empty spaces; however, *capacity* usually refers to *fluids*, whereas *volume* usually refers to *solids*.

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Unit 4: Creating and Interpreting Patterns and Relationships

Introduction

You have experienced many patterns of change in your life. In Florida, the pattern of change in temperatures from season to season is different from the pattern in Nebraska. The rate of speed when learning to ride a bicycle is likely to be quite different from the speed of an experienced rider. The time it takes to run 100 meters is expected to decrease as one prepares for an important race over a period of time.

If you are learning to ride a bicycle, your rate of speed is likely to be quite different from the speed of an experienced rider.

Patterns of change can be described in words, tables, and graphs. It is sometimes useful to summarize patterns and relationships with algebraic expressions or equations.

As you study this unit, make notes on one or more patterns of change important to you. You may know your length at birth and how tall you were at various ages up to now. You could continue this data collection over the next few years. While some things continue to increase, we know our height reaches a maximum at some point in our lives and remains about the same.

Unit 4 Assessment Requirement

You will be asked to attach the following to your unit assessment. Begin now to make notes of a pattern of change important to you.

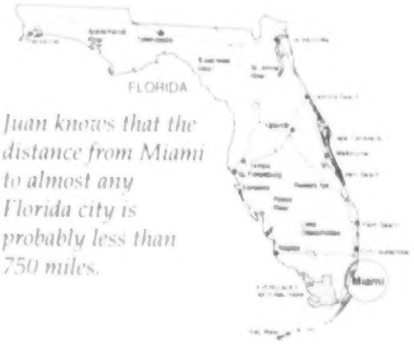
1. Describe a pattern or relationship of interest to you.
2. Display your data in a table and a graph.
3. Summarize conclusions or projections important to you.

Lesson One Purpose

- Create and interpret tables to explain cause-and-effect relationships. (D.1.3.2)
- Describe patterns and relationships through tables. (D.1.3.1)
- Derive formulas for finding rates, distance, and time. (B.1.3.2)
- Add, subtract, multiply, and divide whole numbers to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)

Creating and Interpreting Tables

Juan lives in Miami. He would like to know about travel time by automobile to some other Florida cities. He knows that to average 50 miles per hour (mph), the driver must travel more than 50 mph at times to make up for necessary reductions in speed at other times. He also knows that maximum speed on many two-lane highways in Florida is 55 mph.



Juan knows that the distance from Miami to almost any Florida city is probably less than 750 miles.

- He decides to display his **data** in a **data display** called a **table**.
- He will use 50 mph for the average rate of speed.
- He knows that the distance from Miami to almost any Florida city is probably less than 750 miles.

Time in Hours	Distance Traveled at 50 mph
0	0
1	50
2	100
3	150
4	200
5	250
6	300
7	350
8	400
9	450
10	500
11	550
12	600
13	650
14	700
15	750

Juan made the observations on the next page from looking at his table showing distance traveled at an average rate of 50 mph for 0 to 15 hours.

Practice

Use the **table** *on the previous page to determine if each* **observation** *is* **True** *or* **False**. *If* false, *explain why.*

__________ 1. I can get the next entry in the Distance Traveled column by adding 50 to the previous one.

____

__________ 2. I can also get each entry in that column by multiplying the number of hours by 50.

____

__________ 3. I can also get entries in that column by counting by 50s.

____

__________ 4. All entries in the Distance Traveled column are even numbers and are **multiples** of 50. (Note: *multiples* are the numbers that result from multiplying a number by the set of whole numbers.)

____

__________ 5. The units or ones digit of every entry in that column is 0.

____

__________ 6. The **pattern** or predictable *relationship* for the tens digit for entries is 0, 5, 0, 5….

____

__________ 7. The pattern for the hundreds digit for entries is 0, 0, 1, 1, 2, 2….

____

__________ 8. All of the entries in this column are divisible by 2, 5, 10, 25, 50, and 100.

____

__________ 9. If I had the **chart** and wanted to know the travel time for 1,000 miles, I could double the time for 500 miles.

____

__________ 10. If I wanted to know the distance traveled at 50 mph for any number of hours, I could multiply 50 by the number of hours. This can be written as 50 times *n* or simply as 50*n*. (Note: *n* would be the **variable** or symbol representing a number, and in this case, the number of hours and 50*n* would be the **algebraic expression**.)

____

__________ 11. If the distance to be traveled is 250 to 275 miles, a good **estimation** (or reasonably accurate approximation) would be 5 to 5.5 hours.

____

__________ 12. The distance traveled increases by 50 miles each hour.

____

__________ 13. The distance traveled in 18 hours would be 850 miles.

____

__________ 14. If I travel for 4.5 hours, the distance would be about 225 miles.

____

________ 15. Exactly two of the above 14 statements were marked false by me. If I can respond true to this statement, I should go to the next problem. If not, I should read and consider the statements again.

____

Practice

Use the **table** *below to complete a column for* **distance traveled** *at 65 mph. Then answer the items that follow.*

Juan knows that the maximum speed on many interstate highways is 70 mph. He thinks he might be able to travel at an average speed of 65 mph if the maximum speed is 70 mph.

Time in Hours	Distance Traveled at 50 mph	Distance Traveled at 65 mph
0	0	
1	50	
2	100	
3	150	
4	200	
5	250	
6	300	
7	350	
8	400	
9	450	
10	500	
11	550	
12	600	
13	650	
14	700	
15	750	

Practice

Use your 65 mph column to write a set of five **observations** *as Juan did for 50 mph that would be interesting for someone to read. Indicate by each observation if it is* **True** *or* **False**.

________ 1. ____

________ 2. ____

________ 3. ____

________ 4. ____

________ 5. ____

Practice

Use the **table** *on page 269 to complete the following statements.*

Pensacola
Apalachicola River
Tallahassee
10
Jacksonville
95
Suwannee River
St. Johns River
FLORIDA
Daytona Beach
75
Orlando
Cape Canaveral
4
Melbourne
Tampa
95
St. Petersburg
Vero Beach
Sarasota
Florida's Tpk.
Peace River
75
Palm Beach
Lake Okeechobee
Naples
75
Fort Lauderdale
Miami
EVERGLADES NATIONAL PARK
Key West

1. To travel 487 miles from Miami to Tallahassee requires about

_______________ hours at 50 mph and _______________ hours at 65 mph.

2. To travel 674 miles from Miami to Pensacola requires about

_______________ hours at 50 mph and _______________ hours at 65 mph.

3. To travel 283 miles from Miami to Tampa requires about

_______________ hours at 50 mph and _______________ hours at 65 mph.

Deriving Formulas

Juan knew that each entry in the 65 mph column could be obtained by multiplying:

65 x 0,
65 x 1,
65 x 2,
and so on.

He remembered that he had previously learned about the **relationship** or predictable *pattern* between multiplication and division.

- **If** 4 x 7 = 28,
- **then** 28 ÷ 7 = 4
- **and** 28 ÷ 4 = 7.

He thought

- **if** 65 mph times 2 hours = 130,
- **then** 130 miles ÷ 2 hours = 65 mph

  and 130 miles ÷ 65 mph = 2 hours.

B I N G O ! ! !

- If the rate of speed multiplied by the time gives the distance,
- then the distance divided by the time must give the rate of speed,
- and the distance divided by the rate of speed must give the time.

Study the **algebraic rules** or **formulas** written in words, symbols, and **equations** below.

Words	Symbols	Algebraic Equations
rate multiplied by time equals distance	rate x time = distance	$rt = d$
distance divided by rate equals time	distance ÷ rate = time	$d/r = t$
distance divided by time equals rate	distance ÷ time = rate	$d/t = r$

Practice

Try Juan's **Bingo discovery** *on the work you did in the table on pages 269.*

1. If you choose a distance and divide it by that column's rate of speed, do you get the travel time? ______________________

   Give an example. ______________________________

   ____

   ____

2. If you choose a distance and divide it by the travel time, do you get the rate of speed? ______________________________

   Give an example. ______________________________

   ____

   ____

Practice

Answer the following using complete sentences.

1. Give one example of how the table you made would be helpful in solving a problem. ________________________________

2. Give one example of how dividing the distance by the rate would be helpful. ________________________________

3. What effect does decreasing speed have on travel time? ________

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4. Give one example of how you used mental mathematics in this lesson. ______________________________

5. Give one example of how you used a calculator or paper and pencil in this lesson. ______________________________

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Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. different ways of displaying data in tables, charts, or graphs, including pictographs; circle graphs; single, double, or triple bar and line graphs; histograms; stem-and-leaf plots; and scatterplots

______ 2. the numbers that result from multiplying a given number by the set of whole numbers *Example*: the multiples of 15 are 0, 15, 30, 45, 60, 75, etc.

______ 3. a predictable or prescribed sequence of numbers, objects, etc.

______ 4. the use of rounding and/or other strategies to determine a reasonably accurate approximation without calculating an exact answer

______ 5. an orderly display of numerical information in rows and columns

______ 6. information in the form of numbers gathered for statistical purposes

A. data

B. data display

C. estimation

D. multiples

E. pattern (relationship)

F. table (or chart)

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. a mathematical expression that contains variables and describes a pattern or relationship

______ 2. an expression containing numbers and variables and operations that involve numbers and variables but not containing equality or inequality symbols
Example: $7x$ or $2x + y$

______ 3. a mathematical sentence that equates one expression to another expression
Example: $2x = 10$

______ 4. any symbol that could represent a number

______ 5. a way of expressing a relationship using variables or symbols that represent numbers

A. algebraic expression

B. algebraic rule

C. equation

D. formula

E. variable

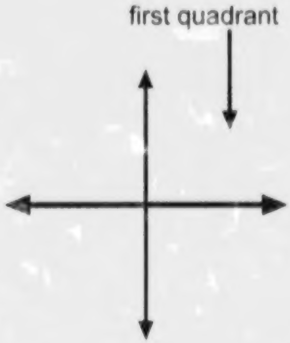
Lesson Two Purpose

- Create and interpret graphs to explain cause-and-effect relationships. (D.1.3.2)
- Describe patterns and relationships through graphs. (D.1.3.1)
- Identify and plot ordered pairs in a rectangular coordinate system (graph). (C.3.3.2)
- Use estimation strategies to predict results and to check for reasonableness of results. (A.4.3.1)

Identifying and Plotting Pairs

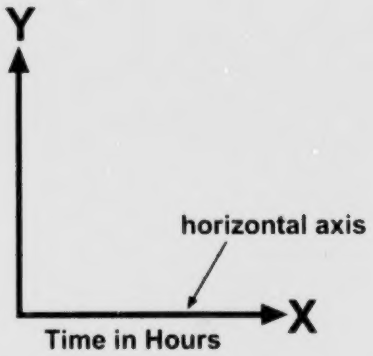
Juan decides he would like to display his data differently and decides to make a **graph**.

- He makes some plans before making his graph on a **coordinate grid or system** of evenly spaced, parallel horizontal and vertical lines. (Refer to pages 182-183 for more descriptions and illustrations of coordinate grids or systems.)
- Since his time and distance are always positive numbers, he only needs the first **quadrant** or region of the graph.

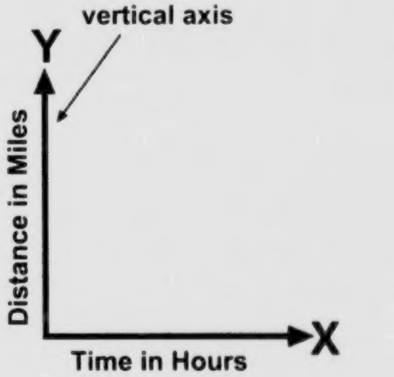


coordinate graph

- He knows that distance changes over time, so he decides to put time on the horizontal **axis** (often called the *x*-axis). He will **label** this axis "Time in Hours."

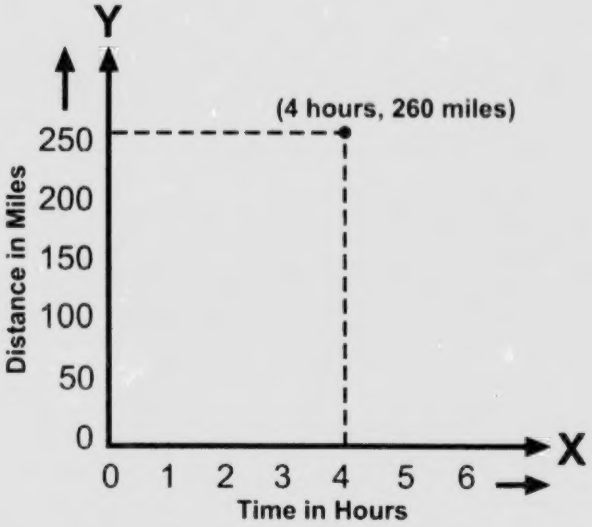

Y
horizontal axis
X
Time in Hours

- He will put distance traveled on the vertical axis (often called the *y*-axis). He will label this axis "Distance in Miles."


vertical axis
Y
Distance in Miles
X
Time in Hours

- He decides to title his graph "The Effect of the Rate of Speed on Distance Traveled" and places this title above the graph.
- He will use zero as his minimum value and 15 as his maximum value for time. He will use a **scale** or *assigned numeric value* of one on the *x*-axis.
- He will use zero as his minimum value and 1000 as his maximum value for distance. He does this since speeds greater than 50 mph yield distances greater than 750 miles. He will use a scale of 50 on this axis. This is not because one of the rates is 50 mph but because counting by 50s from 0 to 1000 is convenient. He would be likely to use the same scale if the rate were 40 mph.
- He wants to plot the data for 65 mph first and then compare it with the data for the 50 mph rate from practice on page 269.
- He plots the **ordered pairs** or points for the data in the table for 65 mph. He does this by first locating the number of hours on the horizontal axis and moving from that point straight up to a point aligned with the correct distance on the vertical axis. For the point representing 4 hours, 260 miles, he would plot the ordered pairs as shown below.

The Effect of the Rate of Speed on Distance Traveled



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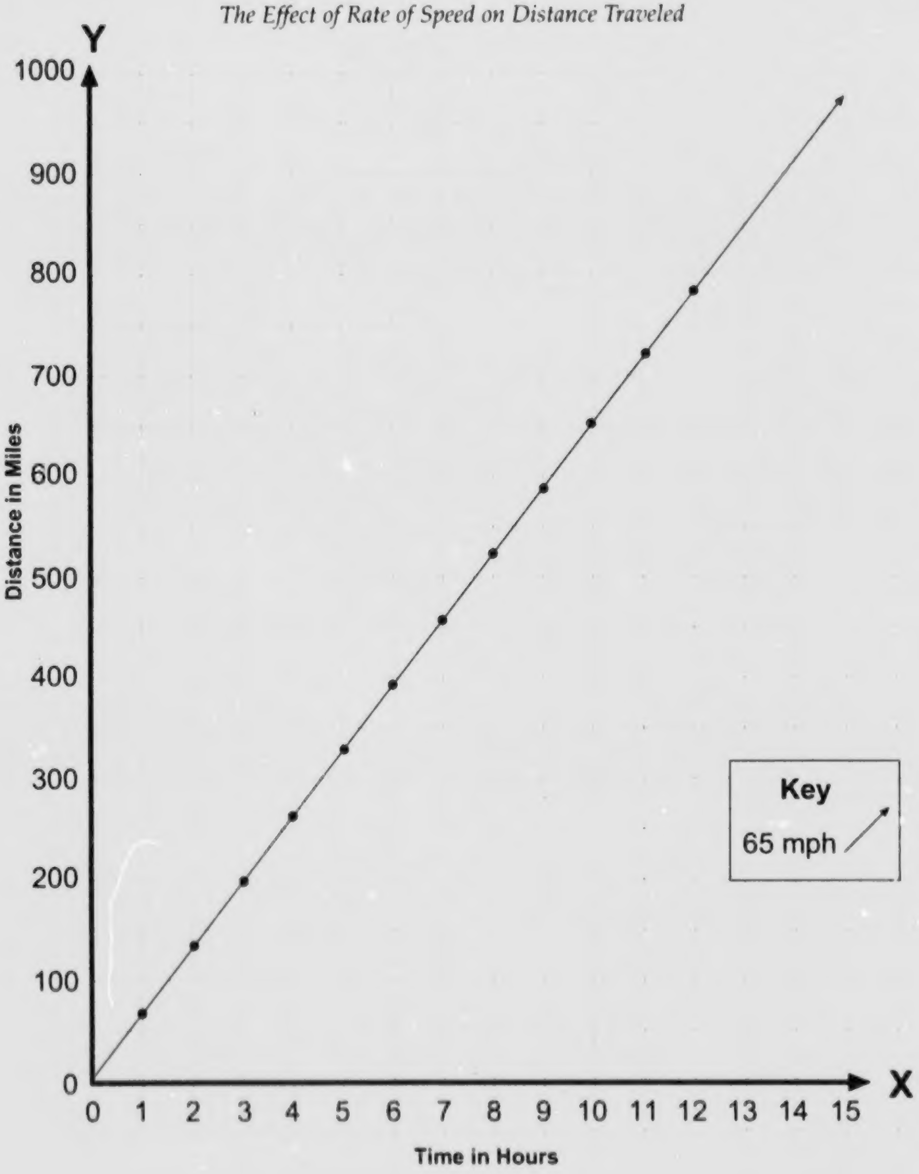
- This is the graph Juan made.



Practice

Juan made some **observations** *from looking at his* **graph** *on the previous page showing* **distance** *traveled at a rate of 65 mph for 0-15 hours. Determine if each observation is* **True** *or* **False**. *If the observation is* **false**, *explain why.*

__________ 1. I plotted points for 16 *ordered pairs* of data and one line passes through all of them.

____

__________ 2. I would get the same line if I picked two pairs of data such as (2 hours, 130 miles) and (8 hours, 520 miles), plotted them correctly, connected the points and extended the line.

____

__________ 3. I could plot only two points, connect them and extend the line. If I made a mistake when plotting one or both of the points, my graph would not be correct.

____

__________ 4. Plotting three or more points would be more reliable than plotting only two points. If they all were on the same line, I'd be more confident.

____

__________ 5. Since I know I travel 65 miles for each hour of travel, the rate is constant, and I should get a line when making my graph.

____

__________ 6. Since the line passes through many points between the points I plotted for (1 hour, 65 miles) and (2 hours, 130 miles), I can estimate distances using the graph for times between one and two hours.

____

__________ 7. If I use the graph to find how far I can drive at 65 mph in 7 hours, I find it to be approximately 550 miles.

____

__________ 8. If I use the graph to find how long it takes to travel 250 miles at 65 mph, I find it takes about 5.5 hours.

____

__________ 9. If I plot points on the *coordinate grid* for the data for 50 mph, the title, labels on *axes*, and *scale* could remain the same.

____

__________ 10. I could use a different color for each rate of speed in my table. If I do so, a key is not necessary. The user should be able to figure out which line goes with which speed.

____

__________ 11. If I had no color markers, I could create another system with my pencil and provide a key to help the user.

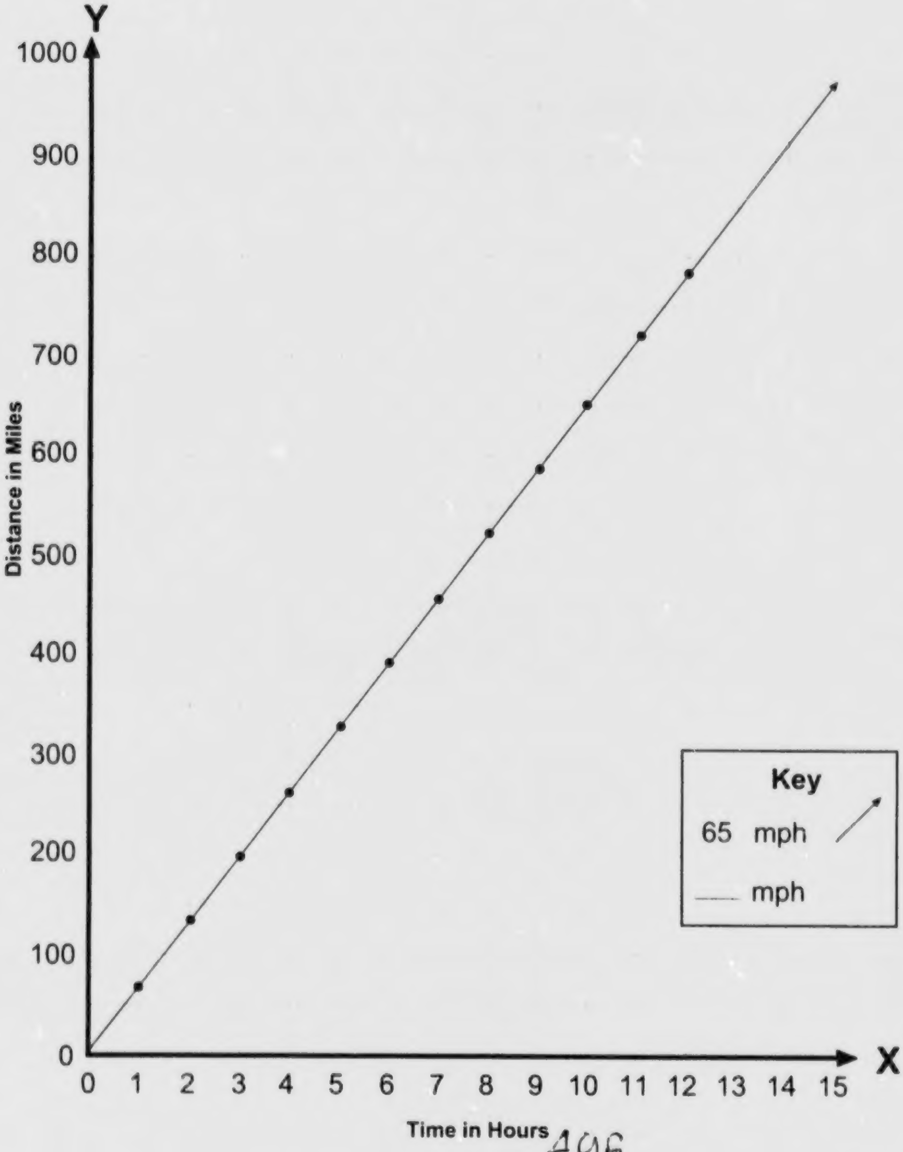
____

__________ 12. Exactly three of the above statements were marked false by me. If I can respond true to this statement, I should go to the next problem. If not, I should read and consider the statements again.

____

Practice

On the copy of **Juan's graph** *below,* **plot points** *for the* **data** *for the* **rate** *of* **50 mph** *from the practice on page 269. Include a* **key** *for the graph.*



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The Effect of Rate of Speed on Distance Traveled

Practice

Use your **graph** *on the previous page to complete the following statements.*

1. There is one point on both lines and the coordinates of that point are (________ , ________), representing ____________ hours and ____________ miles traveled.

2. Juan's line for distance traveled at 65 mph is ________ (more, less) steep than the line for the rate of 50 mph.

3. If I sketched in a line for 70 mph, it would be ________ (above, below) the line for 65 mph.

4. If I sketched in a line for 60 mph, it would lie between the lines for ____________ mph and ____________ mph.

5. The difference in distance traveled at the various rates of speed ____________ (decreases, increases) as the time traveled increases.

6. The distance between the lines ____________ (decreases, increases) as the time traveled increases.

7. According to the graph, the distance traveled at a rate of 65 mph for 11 hours is about ____________ miles.

8. According to the graph, the time required to travel a distance of 275 miles at 50 mph is about ____________ hours.

9. I could make a table and a graph for distances traveled by an airplane at rates of 200 mph and 250 mph. If I did, I would change the scale on the distance axis to accommodate distances from 0 to 3,750. I would expect the graphs to be ____________________ (somewhat like, very different from) the graph I made in this lesson.

10. I could make a table and a graph for distances traveled by a bicycle at rates of 7 mph and 10 mph. I would change the scale on the distance axis to accommodate distances from 0 to about ____________________ miles, and I would expect the graphs to be ____________________ (somewhat like, very different from) the graph I made in this lesson.

Practice

Answer the following using complete sentences.

1. Give one example of when you might prefer using the table from Lesson One to get information. ______________________________

____

____

____

____

2. Give one example of when you might prefer using the graph from this lesson to get information. ______________________________

____

____

____

____

3. Give one example of how you used estimation in this lesson.

____

____

____

____

____

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. the numeric values assigned to the axes of a graph

______ 2. the location of a single point on a rectangular coordination system where the digits represent the position relative to the *x*-axis and *y*-axis
Example: (x, y) or (3, 4)

______ 3. any of the four regions formed by the axes in a rectangular coordinate system

______ 4. the horizontal and vertical number lines used in a rectangular graph or coordinate grid system

______ 5. network of evenly spaced, parallel horizontal and vertical lines especially designed for locating points, displaying data, or drawing maps

______ 6. a drawing used to represent data

______ 7. the titles given to a graph, the axes of a graph, or the scales on the axes of a graph

A. axes (of a graph)

B. coordinate grid or system

C. graph

D. labels (of a graph)

E. ordered pair

F. quadrant

G. scale

Lesson Three Purpose

- Describe a wide variety of patterns. (D.1.3.1)
- Use exponential and scientific notation. (A.2.3.1)
- Use concepts about numbers to build number sequences. (A.5.3.1)
- Understand the basic properties of, and relationships pertaining to, geometric shapes in two dimensions. (C.1.3.1)
- Use a model to derive the formula for circumference. (B.1.3.1)
- Add, subtract, multiply, and divide whole numbers and decimals to solve problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)

Using Concepts about Numbers

Cassandra's math teacher told the class that they could have a pizza party if the class created a set of problems featuring patterns.

- Students would review all submissions,
- vote on the five they liked best and
- challenge another math class to solve them.

Practice

Solve the following problems dealing with **patterns** *in* **numbers**.

(**Remember:** When you have **completed** numbers 1 through 11, you will **vote** for the **five** patterns that you would recommend as **challenging** for students in another math class.)

1. Numbers associated with square arrangements are often called square numbers. For example:

	Square Number	Square Arrangement
1st	1	X
2nd	4	X X X X
3rd	9	X X X X X X X X X
4th	16	X X X X X X X X X X X X X X X X

Find the 7th square number. ______

2. **Prime numbers** have two and only two **factors** or numbers that divide exactly into that number. **Composite numbers** have three or more factors. Use these definitions to continue this pattern.

13, 17, 19, 23, ______ , ______ , ______

3. The expression 2^3 (or 2 ^ 3 on a graphing calculator) is the **exponential form** of 2 x 2 x 2 and means two used as a factor three times. It is read as two to the third **power**, and its value is 8. The two is called the **base**, and the three is called the **exponent**. The expression 2^5, read as two to the fifth power, means 2 X 2 X 2 X 2 X 2, or two used as a factor five times. Its value is 32. The expression 2^1 (or 2^1 on a graphing calculator) is read as two to the first power and means 2 used as a factor one time. Its value is 2. Use this definition to complete this pattern.

2^6	64
2^5	32
2^4	16
2^3	8
2^2	______
2^1	______
2^0	______

Note: Enter 1^0, 3^0, 4^0, 75^0, 1234^0 *(or 2^0, 3^0, 4^0, 75^0, 1234^0) on your calculator and see what results you get! Patterns like the one above may help you understand the results you got with your calculator.*

4. The work you did in the first problem may provide a clue to help you complete this pattern.

1, 2, 6, 15, 31, ______ , ______

The differences between the entries are always consecutive

______________________ numbers.

5. Numbers associated with triangular arrangements are often called triangular numbers. For example:

	Triangular Number	Triangular Arrangement
1st	1	X
2nd	3	X X X
3rd	6	X X X X X X
4th	______	
5th	______	

Find and illustrate the 4th and 5th triangular numbers and their triangular arrangement in the chart above.

6. Continue the following pattern.

1, 3, 9, 27, 81, ______ , ______

State the general **rule** or mathematical expression that describes the pattern or relationship in words. ______________________________

____

____

7. Continue the following pattern.

$\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4},$ ______ , ______

State the general rule in words. ____________________________________

____

____

8. Continue the following pattern.

0, 0.35, 0.70, 1.05, 1.40, ______ , ______

State the general rule in words. ____________________________________

____

____

9. Continue the following pattern.

$1\frac{1}{4}, 2\frac{1}{2}, 3\frac{3}{4}, 5, 6\frac{1}{4},$ ______ , ______

State the general rule in words. ____________________________________

____

____

10. The definition in problem number three may help you understand the following so you are able to continue the pattern.

$1^2 = 1$

$11^2 = 121$

$111^2 = 12321$

$1111^2 = 1234321$

$11111^2 = 123454321$

$111111^2 =$ ______________

Note: If you enter this on your calculator, the answer appears in **scientific notation** or shorthand method of writing numbers. It is fun to do with paper and pencil to see why the pattern occurs. Try 111111 X 111111 to verify your response.

11. Place an X by each of the five patterns you would recommend as challenging for other students.

Ballot

Now that you have completed the practice on pages 290-294, vote for the five patterns you would recommend as challenging by placing an X by the problem number.

______ 1.

______ 2.

______ 3.

______ 4.

______ 5.

______ 6.

______ 7.

______ 8.

______ 9.

______ 10.

______ 11.

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. the number of times the base occurs as a factor

______ 2. a number or expression that divides exactly another number

______ 3. a shorthand method of writing very large or very small numbers

______ 4. the number that is used as a factor a given number of times

______ 5. any whole number with only two factors, 1 and itself

______ 6. any whole number that has more than two factors

______ 7. an exponent; the number that tells how many times a number is used as a factor

______ 8. a mathematical expression that describes a pattern or relationship

A. base

B. composite number

C. exponent (exponential form)

D. factor

E. power

F. prime number

G. rule

H. scientific notation

Practice

Answer the following using short answers.

Interesting relationships exist in measurement and geometry.

- Study the data provided.
- Discover the relationship.
- Add appropriate entries.
- Explain in words a general rule for the relationship.

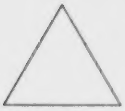
1. There is a relationship between the number of sides in a **polygon** and the **sum** of the measures of the interior angles. Study the table and determine the relationship.

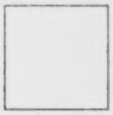
Polygon	**Number of Sides**	**Sum of Interior Angle Measures**
triangle	3	180°
quadrilateral	4	360°
pentagon	5	540°
hexagon	6	720°
heptagon	7	_____°
octagon	8	_____°

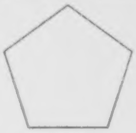
State the general rule in words. ______________________________

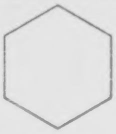
____

____

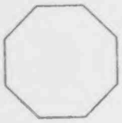

triangle


quadrilateral


pentagon


hexagon

heptagon


octagon

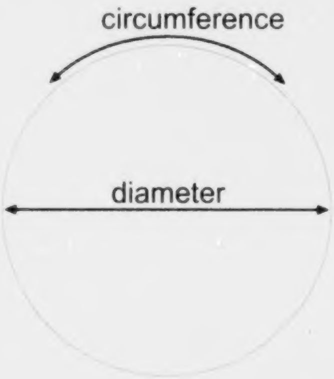
2. There is a relationship between the **diameter** or line passing through the **center of a circle** and the **circumference** or the **perimeter** around the **circle**. Study the table and determine the relationship.

Measure of Diameter of Circle	**Measure of Circumference of Circle**
1 unit	3.14 square units (units^2)
2 units	6.28 square units
3 units	9.42 square units
4 units	12.56 square units
5 units	______ square units
6 units	______ square units

State the general rule in words. ____________________________

____

____



circle

3. There is a relationship between numbers that are powers of two such as 2^1, 2^2, and 2^3 and the sum of the proper factors of the number. Proper factors are all of the factors of the number except the number itself. Study the table and determine the relationship.

Powers of Two	**Sum of Proper Factors**
$2^1 = 2$	1
$2^2 = 4$	$1 + 2 = 3$
$2^3 = 8$	$1 + 2 + 4 = 7$
$2^4 =$ _____	_____________
$2^5 =$ _____	_______________
$2^6 =$ _____	____________________

State the general rule in words. ______________________________

____

____

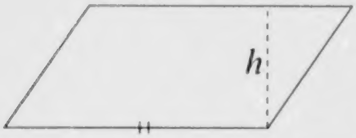
4. There is a relationship between the **areas** or the inside regions of a **triangle** and a **parallelogram** if their bases are the same measure and their heights are the same measure. We will say that the triangle is "related" to the parallelogram when this is true. Study the table and determine the relationship.

Area of Parallelogram	**Area of "Related" Triangle**
3 square units	1.5 square units
4 square units	2 square units
5 square units	2.5 square units
6 square units	3 square units
7 square units	
8 square units	

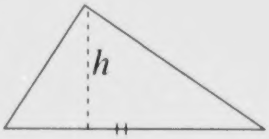
State the general rule in words. ______________________________

____

____



parallelogram

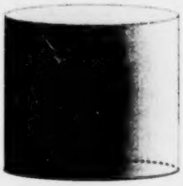


triangle

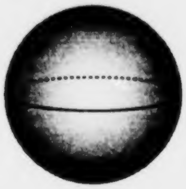
5. There is a relationship between the amount of space occupied, or the **volume** of a cylinder and the volume of a sphere, if the diameter and height of the cylinder are the same measure as the diameter of the sphere. We will say that the sphere is "related" to the cylinder when this is true. Study the table to determine the relationship and continue the pattern.

Volume of Cylinder	Volume of "Related" Sphere
3 cubic units	2 cubic units
6 cubic units	4 cubic units
9 cubic units	6 cubic units
12 cubic units	8 cubic units
15 cubic units	__________
18 cubic units	__________

What fractional part of the volume of the cylinder is the volume of the "related" sphere? __________________



cylinder



sphere

6. A rectangle is enlarged by doubling its length and doubling its width. Study the table to determine the relationship between the area of the original rectangle and the area of the enlargement.

Area of Original Rectangle	Area When Enlarged as Stated
3 square units	12 square units
4 square units	16 square units
5 square units	20 square units
6 square units	24 square units
7 square units	____________
8 square units	____________

When the length and width of a rectangle are doubled, the area of the enlargement is ______ times the area of the original. Use grid paper to draw a rectangle and its enlargement by doubling its dimensions to illustrate why this is true.

7. A rectangle is enlarged by tripling its length and tripling its width. Study the table to determine the relationship of the area of the original with the area of the enlargement and continue the pattern.

Area of Original Rectangle	**Area When Enlarged as Stated**
3 square units	27 square units
4 square units	36 square units
5 square units	45 square units
6 square units	54 square units
7 square units	______________
8 square units	______________

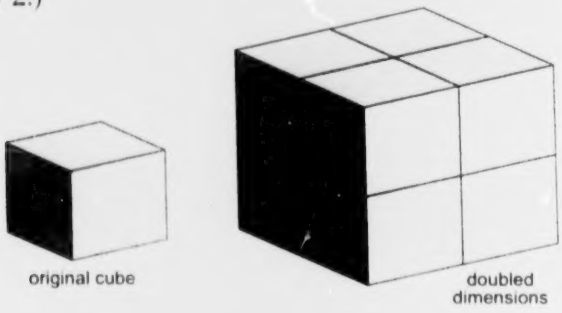
When a rectangle is enlarged by tripling its length and tripling its width, the area of the enlargement is ________ times the area of the original. Use grid paper to draw a rectangle and its enlargement by tripling dimensions to show why this is true.

8. A rectangular **prism** or three-dimensional figure is enlarged by doubling its length, doubling its width, and doubling its height. Study the table to determine the relationship of the volume of the original prism to the enlarged prism and continue the pattern.

Volume of Original Prism	**Volume When Enlarged as Stated**
3 cubic units	24 cubic units
4 cubic units	32 cubic units
5 cubic units	40 cubic units
6 cubic units	48 cubic units
7 cubic units	____________
8 cubic units	____________

If the length, width, and height of a rectangular prism are doubled, the volume of the enlarged prism is _____ times the volume of the original.

(If unit cubes are available in your classroom, start with a cube 1 by 1 by 1, and double its length, width, and height. How many unit cubes are needed to do this? Try starting with a cube that is 2 by 2 by 2.)



9. Give an example of how you used mental mathematics for computation in this lesson.

10. Create a question involving a pattern that might be interesting and challenging for another student.

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. the amount of space occupied in three dimensions and expressed in cubic units

______ 2. the inside region of a two-dimensional figure measured in square units

______ 3. a line segment from any point on the circle passing through the center to another point on the circle

______ 4. a three-dimensional figure (polyhedron) with congruent, polygonal, and lateral faces that are all parallelograms

______ 5. the perimeter around a circle; the distance around a circle

______ 6. the result of an addition

______ 7. a closed figure whose sides are straight and do not cross

______ 8. a polygon with three sides

______ 9. a polygon with four sides and two pairs of parallel sides

______ 10. the length of a boundary of a figure; the distance around a polygon

______ 11. the point from which all points on the circle are the same distance

______ 12. the set of all points in a plane that are all the same distance from a given point called the center

A. area

B. center of a circle

C. circle

D. circumference

E. diameter

F. parallelogram

G. perimeter

H. polygon

I. prism

J. sum

K. triangle

L. volume

Lesson Four Purpose

- Create and interpret tables and verbal descriptions. (D.1.3.2)
- Represent problems with algebraic expressions. (D.2.3.1)
- Use exponential and scientific notation. (A.2.3.1)
- Add, subtract, multiply, and divide whole numbers and decimals to solve problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculators. (A.3.3.3)

Using Algebraic Expressions

Levi and his classmates enjoyed the challenge with patterns sent to them by Cassandra's class. Levi's teacher has taught his class to play a game called "What's My Rule?"

- Before learning to play the game, the class was given a rule, and each student had to apply it to the numbers 1 through 5.
- They also let the letter n stand for any number and wrote the rule as an *algebraic expression*.

One rule that you use every day is the rule concerning sales tax. The amount may vary from one county to another in Florida, but all Florida counties have a sales tax. The rule for the sales tax in Leon County is seven cents for each dollar you spend. If the price of an item is $1.00, the customer pays $1.07 with tax. If the item is $5.00, the customer pays $5.35 with tax. If the item is $10,000, the customer pays $10,700 with tax. The rule could be written as $1.07p$ where p represents price.

Leon County sales tax is seven cents for each dollar.

Rule: 1.07 times the price of $1.07p$

1.07 x p or price of item = total cost
1.07 x $1.00 = $1.07
1.07 x $5.00 = $5.35
1.07 x $10,000 = $10,700

Practice

Use the **rule** *provided to complete a* **table** *for each problem. The rule is also expressed as an* **algebraic expression** *at the bottom of the right column of the table in* **odd** *numbered problems. You should provide the algebraic expression on* **even** *numbered problems at the bottom of the right column of the table.*

1. Rule: Double the number and then add 1.
   Example: Using 6 as your given number, either double the number or multiply the number by 2 and add 1.

   $6 + 6 + 1 \quad = \quad 13$ *or*

   $2(6) + 1 \quad = \quad 13$

1	______
2	______
3	______
4	______
5	______
n	$2n + 1$

2. Rule: Double the number and then subtract 1.

1	______
2	______
3	______
4	______
5	______
n	______

3. Rule: Find the value of the number to the 4th power.

1	______
2	______
3	______
4	______
5	______
n	n^4

4. Rule: Triple the number.

1	______
2	______
3	______
4	______
5	______
n	______

5. Rule: Multiply the number by 100 and subtract 5 from the **product**.

1	______
2	______
3	______
4	______
5	______
n	$100n - 5$

6. Rule: Divide the number by 4.

1	______
2	______
3	______
4	______
5	______
n	______

7. Rule: Divide the number by 10.

1	______
2	______
3	______
4	______
5	______
n	$\frac{n}{10}$

8. Rule: Multiply the number by 7 and add 3 to the product.

1	______
2	______
3	______
4	______
5	______
n	______

9. Rule: Add 3 to the number and multiply the sum by 7.

1	______
2	______
3	______
4	______
5	______
n	$(n + 3)7$

10. Rule: Divide by 4 and multiply the **quotient** by 4.

1	______
2	______
3	______
4	______
5	______
n	______

Practice

Use the **rule** *of* **1.07p** *(from page 306) where* **p** *represents* **price** *and use* **newspaper advertisements for electronic equipment**. *Choose five items and determine their total cost, with a sales tax of 7 percent. List the* **name** *of the item, the* **price** *of the item, and the* **total cost including tax**.

	Name of the Item	**Price**	**Total Cost**
1.			
2.			
3.			
4.			
5.			

Writing Algebraic Expressions

After Levi and his classmates had done a number of problems applying the rule provided by their teacher and writing it as an algebraic expression, they were ready for the challenge of the game, "What's My Rule?"

- The teacher provided the table.
- The numbers 1-5 were called the *input*.
- The result after the rule was applied was called the *output*.
- The students were challenged to discover the rule and to write it as an algebraic expression.

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Practice

Play the game "What's My Rule?" by discovering the **rule** *for each* **table** *and writing it as an* **algebraic expression**. *You may want to write it in words first, but that is not required.*

1.

Input	Output
1	6
2	7
3	8
4	9
5	10
n	________

2.

Input	Output
1	0
2	1
3	2
4	3
5	4
n	________

3.

Input	Output
1	4
2	8
3	12
4	16
5	20
n	________

4.

Input	Output
1	1
2	4
3	9
4	16
5	25
n	________

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5.

Input	Output
1	4
2	7
3	10
4	13
5	16
n	______

6.

Input	Output
1	0
2	2
3	4
4	6
5	8
n	______

7.

Input	Output
1	10
2	20
3	30
4	40
5	50
n	______

8.

Input	Output
1	0.5
2	1
3	1.5
4	2
5	2.5
n	______

9.

Input	Output
1	1
2	8
3	27
4	64
5	125
n	______

531

Practice

Create a **rule** *and use it to complete a* **table**.

Rule: ____

____

Table:

1	______
2	______
3	______
4	______
5	______
n	______

Practice

Determine the **percentages** *withheld from Mariah's paycheck.*

Mariah looked forward to her first paycheck from her summer job. She was paid $7.00 per hour and worked 20 hours her first week. She had big plans for the $140.00 she had made. When she got her check, $10.71 had been withheld for Social Security, and $21.00 had been withheld for federal income tax. She asked her boss about the rules concerning pay and withholding. Mariah's boss said, "You're a good student and a good employee. The withholding for Social Security and for federal income tax were percentages of your pay. I'd love to see you figure out the rules for yourself." Mariah wanted no more surprises on future paydays, so she used mathematics to determine the rules.

Can you estimate the percentage withheld for Social Security? For federal income tax? About what percentage of Mariah's paycheck is withheld for both Social Security and federal income tax?

Mariah's pay before deduction	Amount withheld for Social Security	Amount withheld for Federal Income Tax
$140.00	$10.71	$21.00

To find what part of her pay is withheld for Social Security, Mariah divided $140 into $10.71.

$$\begin{array}{r} 0.0765 \\ 140\overline{)10.7100} \\ \underline{9\,80} \\ 910 \\ \underline{840} \\ 700 \\ \underline{700} \end{array}$$

She then converted 0.0765 to a percent and now knows that 7.65% of her pay is withheld for Social Security. That means she pays $7.65 for each $100.00 earned.

1. If federal income tax withheld from Mariah's paycheck equals $21.00, and Mariah's pay equals $140.00, then

$\frac{21.00}{140}$ or $140\overline{)21.00}$ = ______ = ______ % is being withheld.

2. How much would be withheld from Mariah's paycheck for Social Security all year, assuming Mariah gets paid every week? (Note: One year has 52 weeks.) ________________________________

3. How much would be withheld from Mariah's paycheck for federal income tax all year, assuming Mariah gets paid every week?

   ____

4. How much does Mariah earn in one year before any deductions are made? ____

5. How much does Mariah earn in one year minus the amount withheld for Social Security and federal income tax? __________

6. What is the total percentage of Social Security and federal income tax withheld from Mariah's paycheck in a year? _____________

Lesson Five Purpose

- Represent and solve real-world problems graphically and with algebraic expressions and equations. (D.2.3.1)
- Create and interpret tables. (D.1.3.2)
- Use algebraic problem-solving situations to solve real-world problems. (D.2.3.2)
- Add, subtract, multiply, and divide whole numbers and decimals to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use the formulas for finding rate, time, and distance. (B.1.3.2)

Summary

Let's summarize what has been covered in Unit 4.

- Guessing the rule requires analysis and is sometimes difficult.
- Applying a rule requires computational skills and often is not difficult.
- In Lesson One, you applied rules to find the distance traveled at 65 miles per hour.
- In Lesson Two, you made a graph of data to illustrate a relationship.
- In Lesson Three, you studied patterns and relationships to make discoveries and extended patterns and relationships. You also stated rules in words.
- In Lesson Four, you applied rules, determined rules, and wrote rules as algebraic expressions.
- A graphing calculator will accept a rule in equation form and provide a table and a graph for the relationship represented by the rule.

Have you ever wondered when you will ever use this? Following are two examples of real-life experiences. Now use what you have learned in this unit to solve real-world problems.

Practice

Read to answer the items that follow.

- Mr. Walker is purchasing a cellular phone and must decide on one of two plans for monthly charges.
- Plan A charges $15 per month plus 50 cents per minute of usage.
- Plan B charges $30 per month plus 25 cents per minute for usage.

1. Write a rule or an algebraic expression using a variable for minutes used representing the cost of each plan.

   Plan A: ______________________________

   ____

   Plan B: ______________________________

   ____

2. If Mr. Walker plans to use the phone for emergencies only and estimates that his usage will be 10 minutes or less per month, which plan should he use? Explain your answer.

   ____

   ____

   ____

   ____

   ____

3. If Mr. Walker plans to use the phone frequently and estimates that his usage will be more than 60 minutes per month, which plan should he use? Explain your answer.

4. Make a table for the plans for 0 to 70 minutes.

Minutes of Usage	**Plan A**	**Plan B**
0		
10		
20		
30		
40		
50		
60		
70		

5. Make a graph of the data. Be sure to title your graph, label your axes, and provide a key.

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6. Explain how the rules you wrote in item 1 helped you write the explanations in items 2 and 3.

Explain how the table and graph might help you write those explanations.

Which do you prefer and why?

Practice

Read to answer the items that follow. Recall the work you did earlier in this unit related to **time**, **rate,** *and* **distance** *and answer the following using short answers.*

In March of 1999, a Swiss doctor and British pilot became the first aviators to circle the world in a balloon. Newspapers reported that the 26,000 mile-plus, nonstop navigation took 19 days.

1. What was the average rate per *day* based on 19 full days and 26,000 miles? ______________________________
2. What was the average rate per *hour* based on 19 days of 24 hours each and 26,000 miles? ______________________

Interesting Notes

- The names of the two men are Bertrand Piccard and Brian Jones.
- One of the men has twice visited a cousin in Tallahassee, Florida.
- In 1522 Juan Sebastian de Elcano of Spain led the remnants of Ferdinand Magellan's expedition to complete the first voyage around the world.
- In 1783 Pilatre de Rozier and Francois Laurent of France made the first balloon flight in Paris, going $5\frac{1}{2}$ miles.
- In 1898 Captain Joshua Slocum of the United States became the first to sail around the world alone.
- In 1931 Auguste Piccard (grandfather of round-the-world balloonist Bertrand Piccard) and Paul Kipfer were the first to reach the stratosphere in a balloon, rising to 51,775 feet.
- In 1961 Yuri Gagarin of the Soviet Union was the first human to orbit Earth in a spacecraft.

Unit 5: Probability and Statistics

This unit emphasizes how statistical methods and probability concepts are used to gather and analyze data to solve problems.

Unit Focus

Numbers Sense, Concepts, and Operations

- Understand the relative size of whole numbers and fractions. (A.1.3.2)
- Understand concrete and symbolic representations of rational numbers in real-world situations. (A.1.3.3)
- Understand and explain the effects of addition and multiplication on whole numbers. (A.3.3.1)
- Add, subtract, multiply, and divide whole numbers, decimals, and fractions to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to predict results and to check for reasonableness of results. (A.4.3.1)

Measurement

- Use concrete and graphic models to derive formulas for finding angle measures. (B.1.3.2)
- Solve problems involving units of measure and convert answers to a larger or smaller unit. (B.2.3.2)

Algebraic Thinking

- Create and interpret tables, graphs, and vertical descriptions to explain cause-and-effect relationships. (D.1.3.2)

Data Analysis and Probability

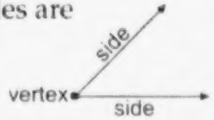
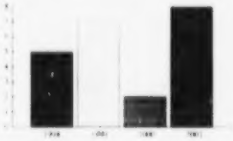
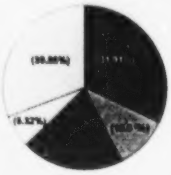
- Collect, organize, and display data in a variety of forms, including tables, charts, and bar graphs, to determine how different ways of presenting data can lead to different interpretations. (E.1.3.1)
- Understand and apply the concepts of range and central tendency (mean, median, and mode). (E.1.3.2)
- Analyze real-world data by applying appropriate formulas for measures of central tendency and organizing data in a quality display, using appropriate technology, including calculators and computers. (E.1.3.3)
- Compare experimental results with mathematical expectations of probabilities. (E.2.3.1)
- Determine the odds for and the odds against a given situation. (E.2.3.2)

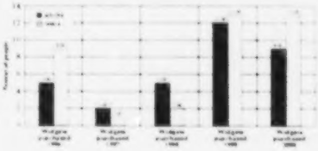
Store Owner

- determines markup for an item's selling price
- pays expenses of operating the store, such as rent, salaries, insurance, and cost of goods

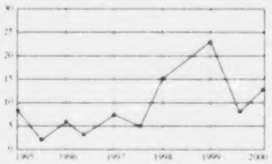
Vocabulary

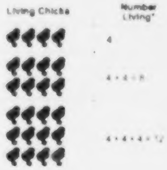
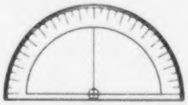
Study the vocabulary words and definitions below.

angle	the shape made by two rays extending from a common endpoint, the vertex; measures of angles are described in degrees (°) 
axes (of a graph)	the horizontal and vertical number lines used in a rectangular graph or coordinate grid system as a fixed reference for determining the position of a point; (singular: *axis*)
bar graph	a graph used to compare quantities in which lengths of bars are used to compare numbers 
center of circle	the point from which all points on the circle are the same distance
chart	see *table*
circle	the set of all points in a plane that are all the same distance from a given point called the center
circle graph	a graph used to compare parts of a whole; the whole amount is shown as a circle, and each part is shown as a percent of the whole 

congruent	figures or objects that are the same shape and the same size
cube	a rectangular prism that has six square faces
data	information in the form of numbers gathered for statistical purposes
data display	different ways of displaying data in tables, charts, or graphs *Example*: pictographs; circle graphs; single, double, or triple bar and line graphs; histograms; stem-and-leaf plots; and scatterplots
degree (°)	common unit used in measuring angles
denominator	the bottom number of a fraction, indicating the number of equal parts a whole was divided into *Example*: In the fraction $\frac{2}{3}$ the denominator is 3, meaning the whole was divided into 3 equal parts.
difference	the result of a subtraction *Example*: In 16 - 9 = 7, 7 is the difference.
double bar graph	a graph used to compare quantities of two sets of data in which length of bars are used to compare numbers 

equally likely	outcomes in a situation where each outcome is assumed to occur as often as every other outcome
estimation	the use of rounding and/or other strategies to determine a reasonably accurate approximation without calculating an exact answer
even number	any whole number divisible by 2 *Example*: 2, 4, 6, 8, 10, 12...
experimental or empirical probability	the likelihood of an event happening that is based on experience and observation rather than theory
face	one of the plane surfaces bounding a three-dimensional figure; a side
fraction	any numeral representing some part of a whole; of the form $\frac{a}{b}$ *Example*: One-half written in fractional form is $\frac{1}{2}$.
graph	a drawing used to represent data *Example*: bar graphs, double bar graphs, circle graphs, and line graphs
grid	a network of evenly spaced, parallel horizontal and vertical lines
labels (for a graph)	the titles given to a graph, the axes of a graph, or the scales on the axes of a graph

line graph (or line plot)	a graph used to show change over time in which line segments are used to indicate amount and direction 
mean (or average)	the arithmetic average of a set of numbers
median	the middle point of a set of ordered numbers where half of the numbers are above the median and half are below it
mode	the score or data point found most often in a set of numbers
numerator	the top number of a fraction, indicating the number of equal parts *Example*: In the fraction $\frac{2}{3}$, the numerator is 2.
odd number	any whole number not divisible by 2 *Example*: 1, 3, 5, 7, 9, 11...
odds	the ratio of one event occurring to the event *not* occurring; the ratio of favorable outcomes to unfavorable outcomes
outcome	a possible result of a probability experiment

percent	a special-case ratio in which the second term is always 100 *Example*: The ratio is written as a whole number followed by a percent sign, such as 25% which means the ratio of 25 to 100.
pictograph	a graph used to compare data in which picture symbols represent a specified number of items 
probability	the ratio of the number of favorable outcomes to the total number of outcomes
product	the result of a multiplication *Example*: In 6 x 8 = 48, 48 is the product.
protractor	an instrument used in measuring and drawing angles 
range (of a set of numbers)	the difference between the highest (H) and the lowest value (L) in a set of data; sometimes calculated as H - L + 1
ratio	the quotient of two numbers used to compare two quantities *Example*: The ratio of 3 to 4 is $\frac{3}{4}$.
scales	the numeric values assigned to the axes of a graph

stem-and-leaf plot	a way of organizing data to show their frequency

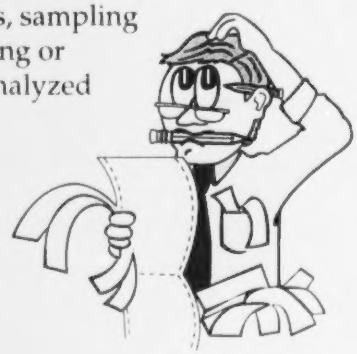
Stem	Leaf
1	5 9
2	1 3 7 7 7
3	0 1 3 4 4 5 6 7
4	2 3 6 7 8

sum	the result of an addition *Example*: In 6 + 8 = 14, 14 is the sum.
table (or chart)	an orderly display of numerical information in rows and columns
theoretical probability	the likelihood of an event happening that is based on theory rather than on experience and observation
whole number	any number in the set {0, 1, 2, 3, 4...}

Unit 5: Probability and Statistics

Introduction

As a consumer, the ability to gather and analyze data plays an important role in decision making. Learning to do this for yourself will enable you to question data-gathering processes, sampling *biases* or *systematic errors* introduced into sampling or testing, and conclusions of data gathered and analyzed by others.

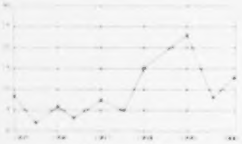


You may have participated in surveys, experiments, and simulations that all yield data for analysis to support decision making. Your continued experiences in these areas will strengthen your decision making in the real world.

Unit 5 Assessment Requirement

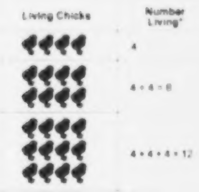
You will be asked to attach the following to your unit assessment. Begin now to collect necessary items.

1. Collect at least one example of each listed below from newspapers and magazines.
   - bar graph
   - circle graph
   - line graph, pictograph, or stem-and-leaf plot
   - table or chart
2. Write a summary showing how effective the data displays in your collection are in communicating information.

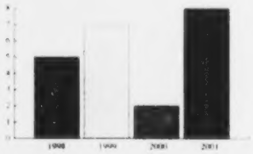


line graph

circle graph



pictograph



bar graph

Lesson One Purpose

- Add and subtract whole numbers to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use concrete and graphic models to derive formulas for finding angle measures. (B.1.3.2)
- Understand the relative size of fractions. (A.1.3.2)
- Understand concrete and symbolic representations of rational numbers in real-world situations. (A.1.3.3)
- Collect, organize, and display data in tables. (E.1.3.1)
- Compare experimental results with mathematical expectations of probabilities. (E.2.3.1)
- Determine the odds for and the odds against a given situation. (E.2.3.2)

Experimental and Theoretical Probability

Mr. and Mrs. Chance and their daughter, Hannah, like to vary their household chores, so they take turns doing the different chores in their home. Mr. Chance might sweep and mop this week, Mrs. Chance next week, and Hannah the following week. The one chore that doesn't appeal to any of them is taking out the garbage once a week.

- They decide to add some fun to this dreaded chore by using **probability**.
- Each Tuesday evening a probability activity is used to determine who takes out the garbage Wednesday morning.
- It is important that the activity be fair. To be fair, there must be an **equally likely** chance for any one of the three to be selected.

In the next set of practices:

- You will experiment with each activity to determine the **experimental or empirical probability**, and you will do an analysis of each activity to determine the **theoretical probability**.
- You will then summarize your findings and make a ruling as to the *fairness* of the activity. The Chance family will only use the ones that are found to be fair.

Practice

Use the directions below to answer the following **probability activity**.

Two coins will be flipped.

- Mr. Chance takes out the garbage if the result is two heads.
- Mrs. Chance takes out the garbage if the result is two tails.
- Hannah takes out the garbage if the result is one of each.

1. Toss two coins 30 times. Record your *experimental probability* results on the **table** below.

Outcomes	Tally Marks	Total of Tally Marks
two heads		
two tails		
one of each		

The *experimental probability* is written as a **fraction** with the number of times an **outcome** occurred in the **numerator** and the total number of trials in the **denominator**.

For example, if the coins are tossed 30 times and two heads occur 9 times, the experimental probability is $\frac{9}{30}$. ($\frac{9}{30} = \frac{\text{number of times outcome occurred}}{\text{total number of trials}}$)

Use your **data** to give the experimental probability for the following.

(**Remember:** P means probability; P(2 heads) means probability of two heads occurring.)

2. P(2 heads) = ________ P(2 tails) = ________

P(1 of each) = ________

Analysis:

3. If the first coin comes up heads, the second can come up

   ________ or ________ .

4. If the first coin comes up tails, the second can come up

   ________ or ________ .

5. This gives us four outcomes:

   heads, ________ or

   heads, ________ or

   tails, ________ or

   tails, ________

This *theoretical probability* is written as a fraction with the number of possible outcomes in the denominator and the number of times the desired outcome occurs in the numerator. Two heads occurs one time out of four in the analysis so the theoretical probability is $\frac{1}{4}$.

6. P(2 heads) = ________ P(2 tails) = ________

   P(1 of each) = ________

Make a ruling on whether or not the three family members are equally likely to be chosen to take out the garbage and include your reason.

7. This activity is ________ (fair, unfair) because ________________

   ____

   ____ .

Practice

Use the directions below to answer the following **probability activity**.

The Chance family will roll one number **cube** or die. The number cube will have one number, the numbers 1 through 6, on each **face** of the cube.

- If the result is 1 or 2, Mr. Chance takes out the garbage.
- If the result is 3 or 4, Mrs. Chances takes out the garbage.
- If the result is 5 or 6, Hannah takes out the garbage.

1. Roll one number cube 30 times and record your results on the table below.

Outcomes	Tally Marks	Total of Tally Marks
sum of 1 or 2		
sum of 3 or 4		
sum of 5 or 6		

2. Use your data to give the experimental probability for the following.

$P(1 \text{ or } 2) =$ ________ $P(3 \text{ or } 4) =$ ________ $P(5 \text{ or } 6) =$ ________

3. The number cube has 6 faces, and each is equally like to be the result of the roll. The number of possible outcomes is ________ . These outcomes can be ________ , ________ , ________ , ________ , ________ , or ________ .

4. The theoretical probability of a 1 or 2 is therefore ________ .

5. The theoretical probability of a 3 or 4 is therefore ________ .

6. The theoretical probability of a 5 or 6 is therefore ________ .

Make a ruling on whether or not the Chance family members have an equally likely chance of taking out the garbage with this activity and include your reason.

7. This activity is ________ (fair, unfair) because ________________

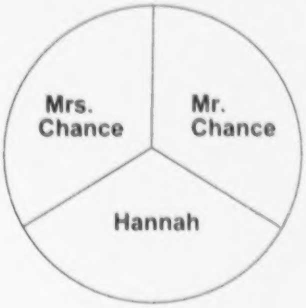
____

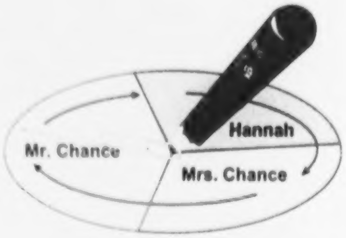
____ .

Practice

Use the directions below to answer the following **probability activity**.

The person named on the section on which the spinner stops will take out the garbage. Spin each spinner 30 times and record your results in the tables. (A bobbie pin makes a good spinner if anchored with your pencil point. Or if you straighten the outermost bend of a small paper clip, it also makes a convenient spinner.)





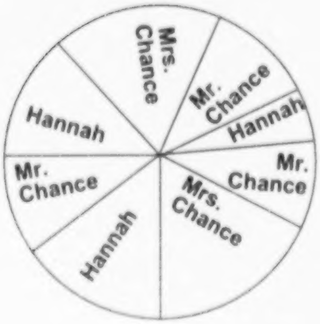
1. Spinner Card I: Using the above diagram, spin the spinner 30 times and record your results on the table below.

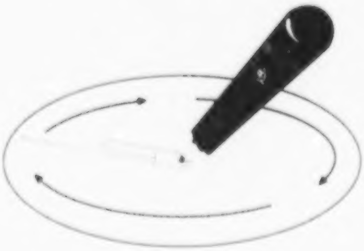
Outcomes	Tally Marks	Total of Tally Marks
Mr. Chance		
Mrs. Chance		
Hannah		

2. Use your data to give the experimental probability for the following.

P(Mr. Chance) = ________ P(Mrs. Chance) = ________

P(Hannah) = ________





3. Spinner Card II: Using the diagram, spin the spinner 30 times and record your results on the table below.

Outcomes	Tally Marks	Total of Tally Marks
Mr. Chance		
Mrs. Chance		
Hannah		

4. Use your data to give the experimental probability for the following.

   P(Mr. Chance) = ________ *P*(Mrs. Chance) ________

   P(Hannah) = ________

5. With Spinner Card I, the three sections appear to be **congruent**. If a **protractor** is used to measure the three **angles** formed at the **center of the circle**, the measures are ________ **degrees**, ________ degrees, and ________ degrees. (degrees = °)

The theoretical probability for each family member with Spinner Card I can be represented as a fraction where the number of degrees determining each member's section is the numerator, and 360 (the total degrees in the **circle**) is the denominator.

6. Use your data to give the experimental probability for the following.

P(Mr. Chance) = ________ *P*(Mrs. Chance) = ________

P(Hannah) = ________

7. If these fractions are simplified, each is equal to ________ .

With Spinner Card II, Mrs. Chance has two parts of the circle. Mr. Chance and Hannah have three parts, but the parts are not all the same size. Use a protractor to determine the **sum** of the angle measures for each family member's sections.

8. Mrs. Chance: ________ degrees + ________ degrees

= ________ degrees

9. Mr. Chance: ________ degrees + ________ degrees +

________ degrees = ________ degrees

10. Hannah: ________ degrees + ________ degrees +

________ degrees = ________ degrees

11. Use your data to give the experimental probability for the following.

P(Mr. Chance) = ________ *P*(Mrs. Chance) = ________

P(Hannah) = ________

Make a ruling on whether or not the members of the Chance family have an equally likely chance of being chosen to take out the garbage with one or more of these spinner cards and include your reasoning.

12. This activity is ________ (fair, unfair) when using Spinner Card I

because ____ .

13. This activity is ________ (fair, unfair) when using Spinner Card II

because ____ .

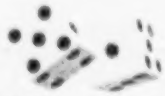
Practice

Use the directions below to answer the following **probability activity**.

A pair of dice will be rolled.

- If the sum is 2, 3, or 4, Mr. Chance takes out the garbage.
- If the sum is 5, 6, or 7, Mrs. Chance takes out the garbage.
- If the sum is 8, 9, or 10, Hannah takes out the garbage.
- If the sum is 11 or 12, keep rolling until a sum of 2 through 10 appears. Do **not** count extra rolls in the 30 times total.

1. Roll the dice 30 times and record the results on the table below.



Outcomes	**Tally Marks**	**Total of Tally Marks**
sum of 2, 3, or 4		
sum of 5, 6, or 7		
sum of 8, 9, or 10		

2. Use your data to give the experimental probability for the following.

P(Mr. Chance) = ________ *P*(Mrs. Chance) = ________

P(Hannah) = ________

To analyze this activity, we need to look at all possible outcomes.

If the first die comes up 1, the second die could come up 1, 2, 3, 4, 5, or 6. The sums would be: 1 + 1 = 2; 1 + 2 = 3; 1 + 3 = 4; 1 + 4 = 5; 1 + 5 = 6; 1 + 6 = 7. This first part of the analysis would appeal to Hannah since her parents have to take the garbage out every time!

3. Complete the table below to help with the full analysis.

- The left column represents the number rolled on the first die.
- The top row represents the number rolled on the second die.
- The other columns and rows represent the sums when the two numbers are rolled.

Sums Possible When Two Number Cubes or Dice Are Rolled

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3					
3						
4						
5						11, roll again
6					11, roll again	12, roll again

The table shows that 36 sums are possible, and 3 of these sums are 11 or 12. These are not used because we roll again.

Therefore, 33 sums remain.

4. ________ of the sums in the table are 2, 3, or 4, so the theoretical

P(Mr. Chance) is ________ .

5. ________ of the sums in the table are 5, 6, or 7, so the theoretical

P(Mrs. Chance) is ________ .

6. ________ of the sums in the table are 8, 9, or 10, so the theoretical

P(Hannah) is ________ .

Make a ruling on whether or not the members of the Chance family have an equally likely chance of taking out the garbage with this activity and include your reasoning.

7. This activity is ________ (fair, unfair) because ________________

____

____ .

Practice

Use the directions below to answer the following **probability activity**.

The two spinner cards on the following page will be used. The object will be to spin the spinners one time each and see what colors the two spins land on. Imagine that these two colors are to be combined. When two different colors are combined, a new color results.

- If the resulting color is red or blue, Mr. Chance takes out the garbage.
- If the resulting color is purple, Mrs. Chance takes out the garbage. (When red and blue are mixed, the result is purple.)
- If the resulting color is green or orange, Hannah takes out the garbage. (When blue and yellow are mixed, the result is green. When red and yellow are mixed, the result is orange.)

1. Spin each spinner once to determine the color that results. Repeat this for a total of 30 results. Record your results on the table below.

Outcomes	Tally Marks	Total of Tally Marks
red or blue		
purple		
green or orange		

2. The data from the experiment provide the following experimental probability.

P(Mr. Chance) = _______

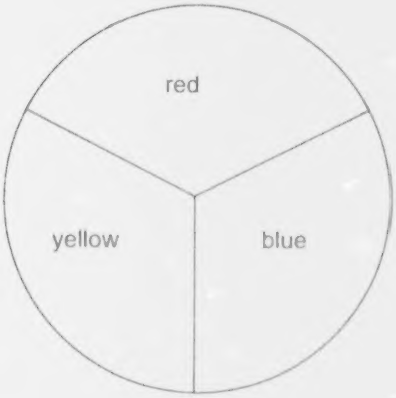
P(Mrs. Chance) = _______

P(Hannah) = _______

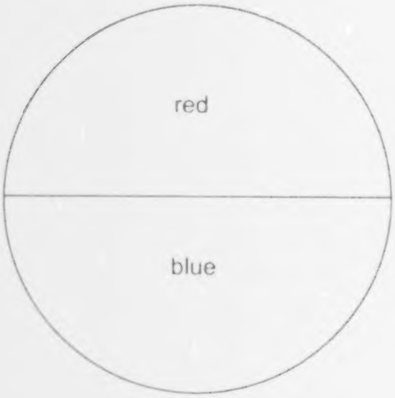
3. Use the table below for your analysis.

First spinner stops on the color:	Second spinner stops on the color:	Outcomes
red	red or blue	red and red = red red and blue = purple
blue	red or blue	blue and ______ = ______ blue and ______ = ______
yellow	red or blue	______ and ______ = ______ ______ and ______ = ______

First Spinner Card



Second Spinner Card



4. There are ________ possible outcomes.

5. Red or blue occur ________ times, so the theoretical P(Mr. Chance) is ________ .

6. Purple occurs ________ times, so the theoretical P(Mrs. Chance) is ________ .

7. Orange or green occur ________ times, so the theoretical P(Hannah) is ________ .

Make a ruling on whether or not the members of the Chance family are equally likely to be chosen to take out the garbage using this activity and include your reason.

8. This activity is ________ (fair, unfair) because ________________

____

____.

Practice

Choose the one **probability activity** *from pages 342-354 in which the* **difference** *in your* **experimental data** *and your* **theoretical analysis** *is the* **greatest**.

*(**Remember:** Doing an experiment 30 times, as you did, is considered limited experimentation. If 30 students in a class did an experiment 30 times, we would have 900 results rather than 30. The more times we do the experiment, the more likely our results are to be approximately the same as the theoretical analysis.)*

1. Draw an appropriate table below and do that experiment 70 more times. Add that data to the data from your original 30 trials. You now have 100 results.

2. Compare your experimental probability with the theoretical probability. Are your results closer than they were with 30 trials? Circle yes or no. If no, try another 100 trials. Add that data to your table on the previous page to see if your results are closer than before.

3. Suggest one additional activity the Chance family could use that would be fair. Explain why it would be fair.

4. Suggest one additional activity the Chance family could use that would not be fair. Explain why it would not be fair.

Practice

Use the directions below to answer the following statements.

The theoretical probability of getting 2 heads when two coins were tossed in the first probability activity on pages 342-343 was $\frac{1}{4}$ because there are four possible outcomes when we toss two coins. We will review them in the table below. Let H stand for heads and T stand for tails.

First Coin	Second Coin	Outcome
H (heads)	H	H, H
	T	H, T
T (tails)	H	T, H
	T	T, T

Of the four outcomes, one outcome is two heads: H, H. The probability of getting two heads is $\frac{1}{4}$.

The **odds** of getting 2 heads would be reported as 1 to 3 because the word odds is defined as the **ratio** of one event *occurring* to the event *not occurring*, or the *ratio* of *favorable outcomes* to *unfavorable outcomes*.

Two heads occur 1 time; they do not occur 3 times.

1. The odds of getting 2 tails would be ____________ .
2. The odds of getting 1 head and 1 tail would be ____________ .

The odds of not getting 2 heads would be reported as 3 to 1 since 2 heads do not occur 3 times and they do occur 1 time.

3. The odds of not getting 2 tails would be ____________ .
4. The odds of not getting 1 head and 1 tail would be ____________ .

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. outcomes in a situation where each outcome is assumed to occur as often as every other outcome

______ 2. the ratio of the number of favorable outcomes to the total number of outcomes

______ 3. the likelihood of an event happening that is based on experience and observation rather than theory

______ 4. the likelihood of an event happening that is based on theory rather than on experience and observation

______ 5. a possible result of a probability experiment

______ 6. the top number of a fraction, indicating the number of equal parts

______ 7. the bottom number of a fraction, indicating the number of equal parts a whole was divided into

______ 8. any numeral representing some part of a whole

______ 9. an orderly display of numerical information in rows and columns

A. denominator

B. equally likely

C. experimental or empirical probability

D. fraction

E. numerator

F. outcome

G. probability

H. table

I. theoretical probability

Practice

Use the list below to write the correct term for each definition on the line provided.

angle
center of circle
circle
congruent
cube
data
degree
face
odds
protractor
ratio
sum

__________________ 1. a rectangular prism that has six square faces

__________________ 2. one of the plane surfaces bounding a three-dimensional figure; a side

__________________ 3. figures or objects that are the same shape and the same size

__________________ 4. an instrument used in measuring and drawing angles

__________________ 5. the shape made by two rays extending from a common endpoint, the vertex; measures of angles are described in degrees (°)

__________________ 6. the point from which all points on the circle are the same distance

__________________ 7. common unit used in measuring angles

__________________ 8. the set of all points in a plane that are all the same distance from a given point called the center

__________________ 9. the result of an addition

__________________ 10. information in the form of numbers gathered for statistical purposes

____________________ 11. the ratio of one event occurring to the event *not* occurring; the ratio of favorable outcomes to unfavorable outcomes

____________________ 12. the quotient of two numbers used to compare two quantities

Lesson Two Purpose

- Add, subtract, multiply, and divide whole numbers to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Understand the relative size of whole numbers. (A.1.3.2)
- Organize and display data in bar graphs. (E.1.3.1)
- Understand and apply the concepts of range and central tendency (mean, median, and mode). (E.1.3.2)
- Analyze real-world data by applying appropriate formulas for measures of central tendency and organizing data in a quality display, using appropriate technology, including calculators and computers. (E.1.3.3)

Data Displays

For the school year 1999-2000, one middle school in Tallahassee operated on a year-round, optional school calendar. All other middle schools in Tallahassee operated on a traditional calendar.

The **data display** or chart on the following practice shows the number of school days for students and the number of holidays for students for each month, excluding weekends.

Practice

Use the **chart** *below to find the* **total** *for each* **column**. *Record the* **totals** *on the chart below.*

	Year-Round Calendar		Traditional Calendar	
Month	**Number of Days Students in School**	**Number of Days Students out of School***	**Number of Days Students in School**	**Number of Days Students out of School***
July	15	7	0	22
August	21	1	12	10
September	8	14	21	1
October	19	2	20	1
November	18	4	18	4
December	13	10	13	10
January	19	2	19	2
February	20	1	20	1
March	8	15	18	5
April	19	1	19	1
May	20	3	20	3
June	0	22	0	22
Total				

* *Does not include Saturdays and Sundays*

Mean, Mode, Median, and Range

- To find the **mean** or *average* for a set of data, we find the sum of the *numbers* or *data values* and divide the sum by the number of values in the set.
- To find the **mode** for a set of data, we review the values in a set of data and determine if one value appears more often than any other. If there is a tie, we say the data is *bimodal*. It has *two modes*.
- To find the **median** for a set of data, the values are written from smallest to largest. If the data set has an *odd number of values*, the value in the *center* will be the *median*. If the data set has an *even number of values*, the *mean* or *average of the two values in the center* will be the *median*.
- The **range** for a set of data may be reported two different ways. Sometimes it is reported as the **difference** in the highest and lowest values. Sometimes it is reported as ranging from the lowest to highest value.

Mean

Victor's test scores have been as follows.

90, 75, 85, 90, 100

To find the mean (or average), we add all of the numbers as follows.

90 + 75 + 85 + 90 + 100 = 440

Then we count how many numbers or values were added and divide the sum by the number of values that we added as follows.

sum divided by *number of values = average*

440 divided by 5 = 88

Mode

To find the mode, we review the data as follows.

90, 75, 85, 90, 100

Ordering the data from smallest to largest is helpful in finding the mode, especially if the data set has many members as follows.

75, 85, 90, 90, 100

We find that the value 90 appears twice, which is more often than any other value. The mode is therefore 90.

Median

To find the median, we write the values from smallest to largest as follows.

75, 85, 90, 90, 100

We have five values. The third value is in the center, and it is 90. Our median is 90.

Range

The range may be reported as the difference between the highest and the lowest score, which is 25.

75, 85, 90, 90, 100

It may also be reported that scores ranged from 75 to 100.

Practice

Use the **chart** *below and find the* **mean, mode, median** *and* **range** *for each of the four sets of data on the* **year-round calendar** *and* **traditional calendar** *on page 362.*

	Year-Round Calendar		Traditional Calendar	
Monthly	**Number of Days Students In School**	**Number of Days Students Out of School***	**Number of Days Students In School**	**Number of Days Students Out of School***
Mean				
Mode				
Median				
Range				

** Does not include Saturdays and Sundays*

Practice

Write **True** *if the statement is correct. Write* **False** *if the statement is* not *correct. If the statement is* **false**, **rewrite** *it to make it* **true** *on the lines provided.*

__________ 1. The year 2000 was a leap year, so the total number of days for the year 2000 was 366. That means that students also had 105 weekend days (Saturdays and Sundays) not to be in school.

__________ 2. During the months of November, December, January, February, April, May, and June, the number of days students were in school was the same for both calendars and that represents more than half of the year.

__________ 3. The traditional calendar has more months with only one day out of school (excluding weekends).

__________ 4. During July and August, the traditional calendar includes only 12 school days while the year-round calendar includes 26 school days.

____

____

____

____

__________ 5. The year-round calendar has 13 fewer school days than the traditional calendar for the months of September and June.

____

____

____

____

__________ 6. The *means* were the same for days in school for the year-round and traditional calendars because all of the students go to school for 180 days, and 180 divided by 12 represents the mean.

____

____

____

____

__________ 7. The *means* were different for the days not in school for the year-round and traditional calendars because students on the year-round calendar get more days out of school.

____

____

____

____

__________ 8. A year-round calendar student moving in late August and transferring to a school on the traditional calendar might question a requirement to attend school for 21 days in September and 18 days in March.

____

____

____

____

__________ 9. A traditional calendar student moving in late August and transferring to a school on the year-round calendar might ask what can be done to make up the work missed.

____

____

____

____

_________ 10. The *medians* were the same for the number of days in school for either calendar, but the values when listed from *smallest to largest* were not the same.

____

____

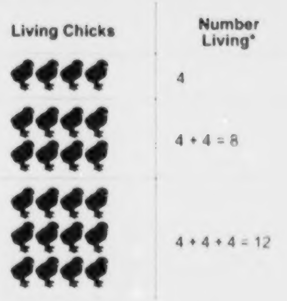
____

____

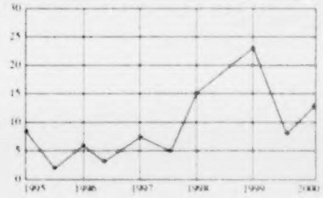
Graphing Data

A **graph** is a picture used to display *numerical facts* called *data*. Different graphs are used for different purposes.

Newspapers and magazines often use **pictographs** to *visually display* and *compare data*. Pictographs use *picture symbols* to represent a specified number of items.

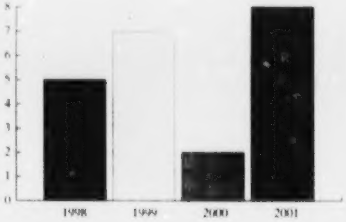


pictograph



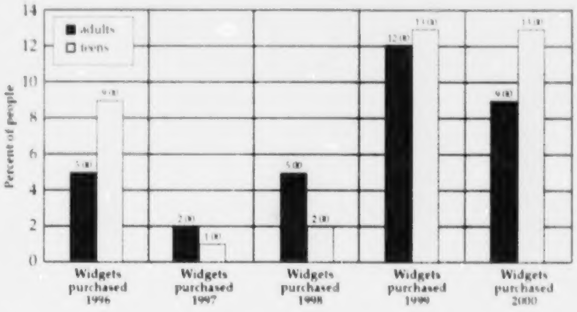
line graph

Line graphs use *line segments* to show *changes* in data over a period of time. The line graph shows both an *amount* and a *direction* of change. A line graph may be used to show a trend of increase or decrease in the data and may even be used to predict data for a future time.

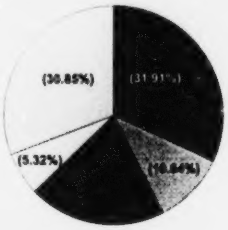


bar graph

Bar graphs use lengths of bars to *compare quantities* of data about different things at a given time. **Double bar graphs** are used to compare quantities of *two sets of data*. Bar graphs have two **axes**: a *horizontal axis* (—) and a *vertical axis* (|). One of these axes is **labeled** with a numerical **scale**.



double bar graph



circle graph

Circle graphs are used to *compare parts of a whole*. Circle graphs display data expressed as **percents** of a whole. The entire circle represents the whole, which is 100 percent. Each part is shown as a percent of the whole.

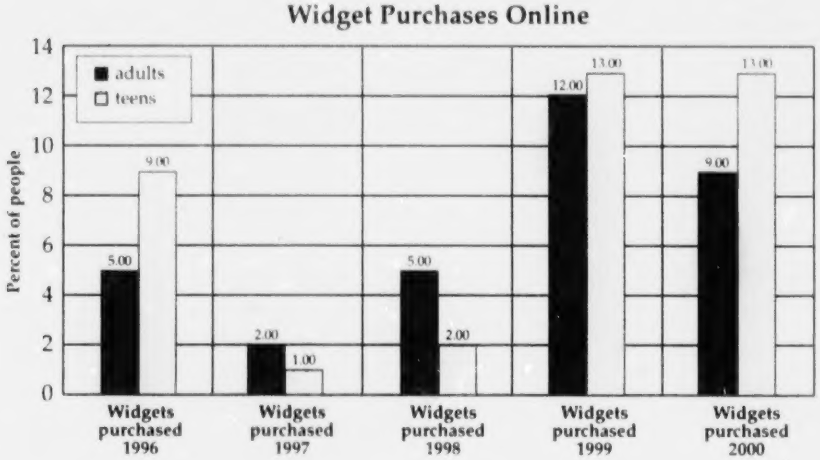
Double Bar Graphs

Parents and students are often interested in the number of *days* out of school each *month*. This helps in planning for trips, special appointments, or extracurricular activities. The chart on page 362 provides that information, but a double bar graph would be another way to display the data.

Bar graphs use vertical or horizontal bars to compare quantities of data of generally statistical information. Look at the example of a double bar graph below.

All bar graphs need to contain certain information. Note the list below and where each item is on the sample double bar graph.

1. appropriate title
2. labels for both axes
3. appropriate scales and labels of scales
4. key to indicate what the bars represent
5. space between the bars


Widget Purchases Online

Practice

Use the **grid paper** *below to make a* **double bar graph** *showing number of days students are out of school (excluding weekends) each month. Use the data on page 362 to complete the graph.*

Follow the directions below for your double bar graph. (Refer to the previous page, if necessary.)

- Title your graph.
- Label your axes.
- Create appropriate scales and labels of scales.
- Include a key to show which bar represents which calendar.
- Leave space between the bars for January and February, February and March, and so on.

title ____________________

key

____

Practice

Use your **graph** *from the* **previous practice** *to complete the following statements.*

1. A family of football fans who like to travel to out-of-town games in the fall might favor the ____________________ calendar because ____
____
____.

2. A family wishing to let the children spend a long summer with grandparents who have a beach home might prefer the ____________________ calendar because __________
____
____.

3. A family with a first grader on the traditional calendar might prefer the ____________________ calendar for their middle school student because ________________________________
____
____.

4. A farm family with the greatest need for help with planting in the spring might prefer the ____________________ calendar because ____
____
____.

5. A family concerned about some academic skills being forgotten if students are away from school for too long might prefer ______________________ calendar because ____________

____

____.

6. Take a position on which calendar you prefer and summarize the reasons you think it would be better for you.

____

____

____

____

____

____

____

____

____

____

____

____

____

____

7. If the school district were considering the year-round calendar for middle schools, a survey might be used to provide information in the decision-making process. Who should receive the survey? Add at least two groups of people on the lines provided.

Recommended to receive survey:

Parents of middle school students

Not recommended to receive survey:

Students, kindergarten to second grade

Practice

Use the list below to write the correct term for each definition on the line provided.

axes
data display
difference
graph
grid
labels (for a graph)
mean
median
mode
percent
range (of a set of numbers)
scales

________________ 1. different ways of displaying data in tables, charts, or graphs

________________ 2. the arithmetic average of a set of numbers

________________ 3. the score or data point found most often in a set of numbers

________________ 4. the middle point of a set of ordered numbers where half of the numbers are above the median and half are below it

________________ 5. the difference between the highest (H) and the lowest value (L) in a set of data

________________ 6. the result of a subtraction

________________ 7. a network of evenly spaced, parallel horizontal and vertical lines

________________ 8. the horizontal and vertical number lines used in a rectangular graph or coordinate grid system as a fixed reference for determining the position of a point

________________ 9. the titles given to a graph, the axes of a graph, or the scales on the axes of a graph

____________________ 10. the numeric values assigned to the axes of a graph

____________________ 11. a drawing used to represent data, such as bar graphs, double bar graphs, circle graphs, and line graphs

____________________ 12. a special-case ratio in which the second term is always 100

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. a graph used to compare quantities in which lengths of bars are used to compare numbers

______ 2. a graph used to compare data in which picture symbols represent a specified number of items

______ 3. a graph used to compare parts of a whole; the whole amount is shown as a circle, and each part is shown as a percent of the whole

______ 4. a graph used to show change over time in which line segments are used to indicate amount and direction

______ 5. a graph used to compare quantities of *two* sets of data in which length of bars are used to compare numbers

A. bar graph

B. circle graph

C. double bar graph

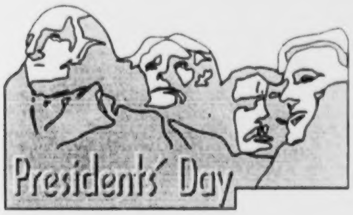
D. line graph

E. pictograph

Lesson Three Purpose

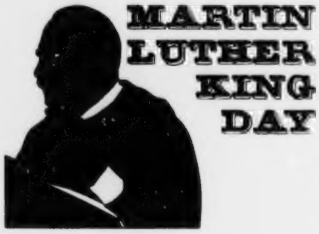
- Create and interpret tables, graphs, and verbal descriptions to explain cause-and effect relationships. (D.1.3.2)
- Collect, organize, and display data in a variety of forms, including tables, charts, and bar graphs, to determine how different ways of presenting data can lead to different interpretations. (E.1.3.1)
- Understand and apply the concepts of range and central tendency (mean, median, and mode). (E.1.3.2)
- Analyze real-world data by applying appropriate formulas for measures of central tendency and organizing data in a quality display, using appropriate technology, including calculators and computers. (E.1.3.3)
- Add, subtract, multiply, and divide whole numbers to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculators. (A.3.3.3)

Collecting, Organizing, and Displaying Data



In Lesson Two a comparison was made between the year-round calendar and the traditional calendar for the 1999-2000 school year in Tallahassee. In this lesson, a comparison will be made between your school calendar now and the work calendar for many adults.

Legal holidays for the whole United States do not exist. Our federal government determines which holidays it will recognize for federal employees. States, school systems, and individual businesses do likewise. Some companies give their employees their birthday off, and some have what is called a floating holiday. The employee chooses a day, and it becomes a holiday for that employee. It does not have to be the same day each year.



The chart below provides information about holidays for many government employees.

Holiday	When observed
Independence Day	July 4
Labor Day	1st Monday in September
Columbus Day	2nd Monday in October
Veterans Day	November 11
Thanksgiving	4th Thursday in November
Christmas	December 25
New Year's Day	January 1
Martin Luther King, Jr. Day	3rd Monday in January
Washington's Birthday or Presidents' Day	3rd Monday in February
Memorial Day	Last Monday in May

In addition to the holidays listed, many employees are given two (or more) weeks of vacation time a year or 10 work days. In some large factories, the entire factory closes for a two-week period, and all employees take their vacation at the same time. Most employees work in settings in which different people take vacation days at different times.

These factors make it difficult to show in a bar graph the number of days off work each month. A bar graph could be used to show the total number of work days, total number of weekend days, and total number of vacation/holidays. A *circle graph* could also show this.

Since the 1999-2000 school year included February 29, we will base our data on a 366-day year. Review the data in the table below.

	Number of days in school or at work	Number of holidays / vacation days	Number of Saturdays and Sundays
School	180	82	104
Work	242	10 + 10 = 20	104

Practice

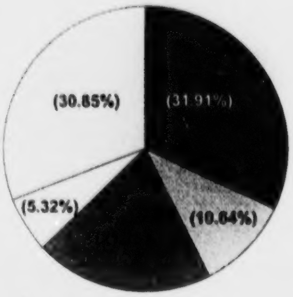
Use the **grid paper** *below to make a* **double bar graph** *showing the* **data** *from page 380. Be sure to do the following. (Refer to page 371 if needed.)*

- Title your graph.
- Label your axes.
- Create appropriate scales and labels of scales.
- Include a key to show which bar represents which group.
- Leave space between the three sets of bars.

key

Circle Graphs

You may prefer one data display over another depending on the purpose. A *circle graph* shows the relationship of the parts to a whole and to each other.



Review the following statements on constructing a circle graph using information from page 380.

- The 360 degrees in the circle should be divided among the sections to be graphed the same way the data is divided.
- Since there are 366 days in the year being considered, there is *almost* a one-to-one correspondence between the number of days and number of degrees.
- There are more days than degrees, so there will be slightly less than one degree per day.
- Students are in school 180 of the 366 days, which represents 49.2 percent of the time, or almost $\frac{1}{2}$ of the year. If we find 49.2 percent of the 360 degrees in the circle, we get 177 degrees, or almost $\frac{1}{2}$ of the circle. Do the same for other categories to be represented in each graph.

First, **estimate** changing the number of days in a year into the number of degrees in a circle.

180 school days equals less than 180°.

104 weekends days equals less than 104°.

82 holidays or vacation days equals less than 82°.

Next, make a more precise calculation. Determine how many degrees in a circle 180 school days would equal.

$$\frac{\text{school days}}{\text{days in a leap year}} = \frac{180}{366} = .492 = 49.2\%$$

$$49.2\% \text{ of } 360 \text{ degrees} = .492 \times 360 = 177 \text{ degrees}$$

or

$$\frac{180}{366} \times \frac{360}{1} = 177 \text{ degrees}$$

or

$$(180 \div 366) \times 360 = 177 \text{ degrees}$$

Practice

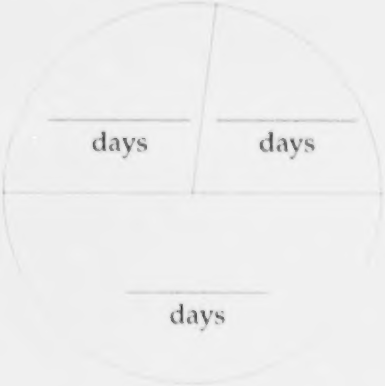
Choose one of the **methods** *from the previous page to determine the* **number of degrees** *each set of days equals in a* **circle graph**.

1. 104 days of weekends equals __________ degrees of a circle.
2. 82 days of holiday or vacation equals __________ degrees of a circle.

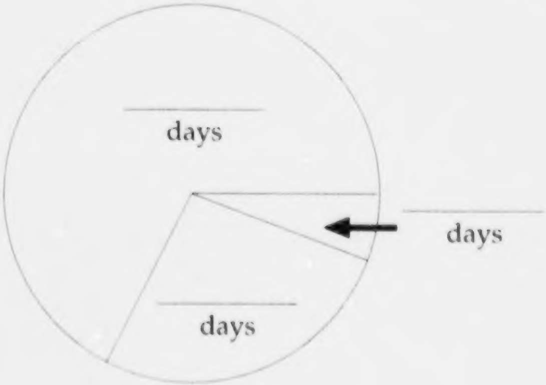
Practice

Use the **two circles** *below to correctly* **construct** *and* **label** *each* **circle graph**. *One circle has been divided to show* **school data** *and the other circle* **workforce data**. *Put the appropriate* **title** *on each graph and* **label** *the sections appropriately.*

title



title



Practice

Consider the following **information** *and then choose the* **data display** *you prefer. Explain why it is especially* **meaningful** *to you.*

Person A likes the *table format* because numbers tell the story well.

Person B likes the *double bar graph* because the heights of the bars clearly show the comparison of the three categories.

Person C likes *circle graphs* because they make it easy to see that students are in school slightly less than $\frac{1}{2}$ of the time, and people are at work almost $\frac{3}{4}$ of the time. Person C prefers seeing a part-to-whole comparison.

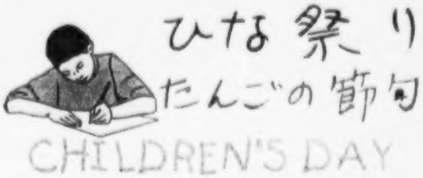
A note of interest:

A Special Day for Dad

Mother's Day is celebrated on the second Sunday in May each year, and Father's Day is celebrated on the third Sunday in June each year.

MOTHERS' DAY

Japan honors children with Children's Day on May 5 each year. Why do you think that the United States does not have a Children's Day?


ひな祭り
たんごの節句
CHILDREN'S DAY

Stem-and-Leaf Plot

Since 1910, membership of the United States House of Representatives has numbered 435. Each state gets at least one representative. The total number is based on state population. Adjustments are made every 10 years as a result of new census data.

The table below provides information on the number of representatives from each state as of 1999.

State	Number of representatives
Alabama	7
Alaska	1
Arizona	6
Arkansas	4
California	52
Colorado	6
Connecticut	6
Delaware	1
Florida	23
Georgia	11
Hawaii	2
Idaho	2
Illinois	20
Indiana	10
Iowa	5
Kansas	4
Kentucky	6
Louisiana	7
Maine	2
Maryland	8
Massachusetts	10
Michigan	16
Minnesota	8
Mississippi	5
Missouri	9

State	Number of representatives
Montana	1
Nebraska	3
Nevada	2
New Hampshire	2
New Jersey	13
New Mexico	3
New York	31
North Carolina	12
North Dakota	1
Ohio	19
Oklahoma	6
Oregon	5
Pennsylvania	21
Rhode Island	2
South Carolina	6
South Dakota	1
Tennessee	9
Texas	30
Utah	3
Vermont	1
Virginia	11
Washington	9
West Virginia	3
Wisconsin	9
Wyoming	1

A **stem-and-leaf plot** is another way to display numerical data. A stem-and-leaf-plot displays *all* the original data. For the data in our table, the *stem* would represent the tens digit and the *leaves* would represent the units or ones digit. Examine the stem-and-leaf plot below.

Number of State Representatives from the 50 States

Stem	**Leaf**
0	1 1 1 1 1 1 1 2 2 2 2 2 2 3 3 3 3 4 4 5 5 5 6 6 6 6 6 6 7 7 8 8 9 9 9 9
1	0 0 1 1 2 3 6 9
2	0 1 3
3	0 1
4	
5	2

- The stems are in bold print and are separated from the leaves by two spaces.
- The greatest place value of data is used for the stem, which is the tens digit. The next greatest place value of data is used for the leaves, which is the units or ones digit.
- The data range from 1 to 52. So the stems range from 0 to 5.
- Then each leaf is arranged in order from least to greatest.
- The stem of 5 and the leaf of 2 indicates one state has 52 representatives. 5|2 represents the number 52.
- The stem of 0 and the two leaves of 7 indicate that two states have 7 representatives.
- The stem of 2 and the leaf of 3 indicates one state has 23 representatives.

Practice

Use pages 388-389 to complete the following statements with the correct answer.

1. The number of states having 3 representatives is ________.

   The states are ________________, ________________,

   ________________, and ________________.

2. The number of states having 11 representatives is ________.

   The states are ________________ and

   ________________.

3. The number of representatives *ranges* from ____________

   to ________. The difference in the greatest number and smallest

   number is ________.

4. The *median* for the number of representatives is ________.

   I determined this by ________________________________

   ________________________________.

5. The *mode* for the number of representatives is ________.

   I determined this by ________________________________.

6. The *mean* for the number of representatives is ________.

   I determined this by ________________________________.

7. Florida ranks ________ in number of representatives, with the

   states of ________________, ________________,

   and ________________ having more representatives.

601

8. The stem-and-leaf plot organization of the data ____________ (was, was not) helpful in finding the mean, median, mode, and range.

9. The stem-and-leaf plot organization of the data ____________ (does, does not) let me know the number of representatives for each state.

10. Since 52 exceeds the other numbers significantly, a user might consult a dependable reference source to verify the data. ____________ (True or False)

Lesson Four Purpose

- Understand and explain the effects of addition and multiplication on whole numbers. (A.3.3.1)
- Add, subtract, multiply, and divide whole numbers and decimals to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to predict results and to check for reasonableness of results. (A.4.3.1)
- Solve problems involving units of measure and convert answers to a larger or smaller unit. (B.2.3.2)
- Collect and organize data in a variety of forms including tables and charts. (E.1.3.1)
- Understand and apply the concepts of range and central tendency (mean, median, and mode). (E.1.3.2)
- Analyze real-world data by applying appropriate formulas for measures of central tendency. (E.1.3.3)
- Determine the odds for and the odds against a given situation or determine the probability of an event occurring. (E.2.3.2)

Solving Problems Involving Statistics and Probability

In this lesson, you will solve a variety of problems involving *statistics* or *numerical data* and probability.

Practice

Use the directions below to answer the following statements.

1. Little League Baseball began in 1939 with 30 boys playing on three teams in Williamsport, Pennsylvania. Approximately 3,000,000 boys and girls were playing on approximately 200,000 teams in 1997. The mean for the number of players per team grew by how many?

   The mean in 1939 was ____________ .

   The mean in 1997 was ____________ .

   The increase in the mean was ____________ .

2. The state of California ranked first in population and third in area in 1997. The population was 32,268,301, and the area was 163,707 square miles. Population density is reported in terms of the mean number of people per square mile. Find the population density for the state of California to the nearest whole number.

   The total number of people was ____________ .

   The total area was ____________ .

   The number of people per square mile was ____________ .

   The population density for California was ____________ people per square mile.

3. The state of Alaska ranked 48th in population and 1st in area in 1997. The population was 609,311, and the area was 656,424 square miles. Find the population density for the state of Alaska to the nearest whole number.

The total number of people was __________ .

The total area was __________ .

The number of people per square mile was __________ .

The population density for Alaska was __________ people per square mile.

4. Among the world's tallest buildings are the Petronas Towers in Kuala Lumpur, Malaysia, at 1,483 feet tall with 88 stories, and the Sears Tower in Chicago, Illinois, at 1,450 feet tall with 110 stories. Find the difference in the mean height per story of the two buildings to the nearest tenth of a foot.

Petronas Towers is __________ feet tall and has __________

stories.

The mean height per story is __________ .

Sears Tower is __________ feet tall and has __________ stories.

The mean height per story is __________ .

The difference in the mean heights of the two buildings is

__________ .

5. On a flight from Los Angeles to Sydney, Australia, the cabin screen often showed facts about the flight. During the first hour of the flight, head winds ranged from 10 miles per hour to 58 miles per hour. When trying to describe a typical headwind rate, it might be helpful to find the mean, median, and mode and decide if one gives a better description than another. Find the mean, median, and mode for the set of data shown:

10, 12, 17, 26, 29, 29, 35, 42, 50, 58

To find the mean, I ____

____.

The mean is ____________.

To find the median, I ______________________________________

____.

The median is ____________.

To find the mode, I ____

____.

The mode is ____________.

If I were going to summarize the data and provide a "typical"

headwind rate, I would choose the ____________

because ____

____.

6. A contest is featuring scratch off cards. Each card has five spots. Two spots match on each card. If you scratch two and only two on a card and get a match, you win a prize.

To find the probability of winning a prize with one card, complete the outcome column in the following chart.

First Choice	Second Choice	Outcome
1st spot	2nd,	1st, 2nd
	or 3rd,	1st, ________
	or 4th,	1st, ________
	or 5th	1st, ________
2nd spot	3rd,	2nd, ________
	or 4th,	2nd, ________
	or 5th	2nd, ________
3rd spot	4th,	3rd, ________
	or 5th,	3rd, ________
4th spot	5th	4th, ________

The total number of possible outcomes is ____________ .

The chance to win is present on every card since there are

____________ matching spots on every card.

The probability of winning a prize with one card is ____________ .

7. Thomas Alva Edison claimed more than 1,000 inventions during his lifetime (1847-1931). Find the mean number of his inventions per month, rounded to the nearest **whole number**, for his lifetime.

To find the number of years he lived, I ________________________

____ .

To find the number of months he lived, I ____________________

____.

If he claimed 1,000 inventions and lived ____________ months, the

mean number of inventions per month, rounded to the nearest

whole number, is ____________.

8. As a plane departed Dallas for Honolulu, the cabin screen reported a distance of 3,782 miles to Honolulu and an estimated travel time of 8.25 hours. Find the average rate of speed this estimate was based upon, rounded to the nearest whole number.

The word *average* is commonly substituted for the word *mean*. To

find the mean, I ____.

The average rate of speed would be ____________ miles per hour (mph).

9. A student rolled a pair of dice to find its sum in an experiment. The student recorded the sums and *conjectured* or made an *educated guess* that the probability of getting an even sum was the same as getting an odd sum. Determine whether or not this was correct.

Use the table from a similar problem in Lesson One page 349 of this unit.

____________ sums are **even number** outcomes.

____________ sums are **odd numbers** outcomes.

The total number of sum outcomes is ____________.

The probability of an even sum outcome is ____________.

The probability of an odd sum outcome is ____________.

The student's conjecture was ____________ (true, false).

10. After exploring the possible sums when rolling dice, the student conjectured that the same would be true for **products** of the two numbers rolled.

Complete the table for products:

x	1	2	3	4	5	6
1	1	2	3			
2	2	4	6			
3	3					
4	4					
5						
6						

The total number of product outcomes possible is __________ .

__________ products are even number outcomes.

__________ products are odd number outcomes.

The probability of rolling an even product outcome is __________ .

The probability of rolling an odd product outcome is __________ .

The student's conjecture was __________ (true, false).

11. Use the work you did in questions 10 and 11 to complete the following statements.

The sum of two odd numbers is ____________ . (odd, even)

The sum of two even numbers is ____________ . (odd, even)

The sum of an even number and an odd number is ____________ . (odd, even)

The product of two odd numbers is ____________ . (odd, even)

The product of two even numbers is ____________ . (odd, even)

The product of an even number and an odd number is ____________ . (odd, even)

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. the use of rounding and/or other strategies to determine a reasonably accurate approximation without calculating an exact answer

______ 2. any number in the set {0, 1, 2, 3, 4...}

______ 3. the result of a multiplication

______ 4. any whole number not divisible by 2

______ 5. any whole number divisible by 2

______ 6. an orderly display of numerical information in rows and columns

______ 7. a graph used to compare quantities in which lengths of bars are used to compare numbers

______ 8. a graph used to compare parts of a whole; the whole amount is shown as a circle and each part is shown as a percent of the whole

______ 9. a graph used to compare quantities of *two* sets of data in which length of bars are used to compare numbers

______ 10. a graph used to show change over time in which line segments are used to indicate amount and direction

______ 11. a graph used to compare data in which picture symbols represent a specified number of items

______ 12. a way of organizing data to show their frequency

A. bar graph

B. chart

C. circle graph

D. double bar graph

E. estimation

F. even number

G. line graph

H. odd number

I. pictograph

J. product

K. stem-and-leaf plot

L. whole number

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. information in the form of numbers gathered for statistical purposes

______ 2. the numeric values assigned to the axes of a graph

______ 3. different ways of displaying data in tables, charts, or graphs

______ 4. a drawing used to represent data

______ 5. a network of evenly spaced, parallel horizontal and vertical lines

______ 6. the horizontal and vertical number lines used in a rectangular graph or coordinate grid system as a fixed reference for determining the position of a point

______ 7. the titles given to a graph, the axes of a graph, or the scales on the axes of a graph

A. axes

B. data

C. data display

D. graph

E. grid

F. labels

G. scales

Practice

Use the list below to write the correct term for each definition on the line provided.

equally likely
mean
median
mode
odds
probability
range
sum
theoretical probability

________________ 1. the result of an addition

________________ 2. the likelihood of an event happening that is based on theory rather than on experience and observation

________________ 3. the ratio of the number of favorable outcomes to the total number of outcomes

________________ 4. the ratio of one event occurring to the event *not* occurring

________________ 5. the arithmetic average of a set of numbers

________________ 6. the middle point of a set of ordered numbers where half of the numbers are above the median and half are below it

________________ 7. the difference between the highest (H) and the lowest value (L) in a set of data

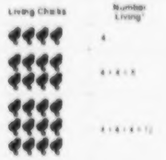
________________ 8. the score or data point found most often in a set of numbers

________________ 9. outcomes in a situation where each outcome is assumed to occur as often as every other outcome

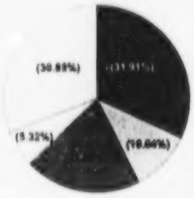
Practice

Match each **illustration** *with the correct term. Write the letter on the line provided.*

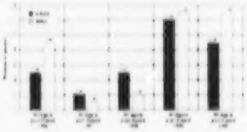
______ 1.



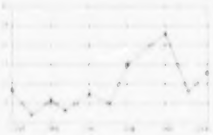
______ 2.



______ 3.



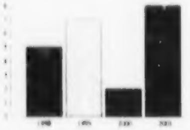
______ 4.



______ 5.

Stem	Leaf
1	5 9
2	1 3 7 7 7
3	0 1 3 4 4 5 6 7
4	2 3 6 7 8

______ 6.



A. bar graph

B. circle graph

C. double bar graph

D. line graph

E. pictograph

F. stem-and-leaf plot

Appendices

FCAT Mathematics Reference Sheet

Key

b = base

h = height

l = length

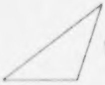
w = width

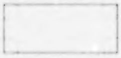
d = diameter

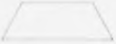
r = radius

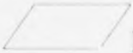
Use 3.14 or $\frac{22}{7}$ for π.

Formulas


triangle area = $\frac{1}{2}bh$


rectangle area = lw


trapezoid area = $\frac{1}{2}h(b_1 + b_2)$

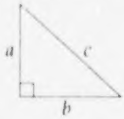

parallelogram area = bh

circle area = πr^2
circumference = $\pi d = 2\pi r$

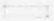
In a polygon, the sum of the measures of the interior angles is equal to $180(n - 2)$, where n represents the number of sides.

Pythagorean Theorem

$c^2 = a^2 + b^2$



right circular cylinder volume = $\pi r^2 h$ total surface area = $2\pi rh + 2\pi r^2$


rectangular solid volume = lwh total surface area = $2(lw) + 2(hw) + 2(lh)$

Conversions

1 yard = 3 feet = 36 inches
1 mile = 1,760 yards = 5,280 feet
1 acre = 43,560 square feet
1 hour = 60 minutes
1 minute = 60 seconds

1 liter = 1000 milliliters = 1000 cubic centimeters
1 meter = 100 centimeters = 1000 millimeters
1 kilometer = 1000 meters
1 gram = 1000 milligrams
1 kilogram = 1000 gram

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts

1 pound = 16 ounces
1 ton = 2,000 pounds

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